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Final state interaction effects in $B^+ \to D^+K^0$ decay

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In this research the exclusive decay of $B^+ \to D^+K^0$ is calculated by QCD factorization method (QCDF) and final state interaction (FSI). First, the $B^+ \to D^+K^0$ decay is calculated via QCDF method. The result that is found by using the QCDF method is less than the experimental result. So FSI is considered to solve the $B^+ \to D^+K^0$ decay. For this decay, the $D_s^+\pi^0$, $D_s^{+*}\rho^0$, $D_s^{+*}\phi$ via the exchange of $\bar{K}^0$, $\bar{K}^{0*}$, D^-, and D^{-*} mesons are chosen for the intermediate states. The above intermediate states are calculated by using the QCDF method. In the FSI effects, the results of our calculations depend on η as the phenomenological parameter. The range of this parameter is selected from 2 to 2.4. It is found that if $\eta = 2.4$ is selected, the numbers of the branching ratio are placed in the experimental range. The experimental branching ratio of this decay is less than $2.9 \cdot 10^{-6}$ and our results calculated by QCDF and FSI are $(0.16 \pm 0.04) \cdot 10^{-6}$ and $(2.8 \pm 0.09) \cdot 10^{-6}$ respectively.

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1. Introduction. B meson non-leptonic decays are significant for testing theoretical frameworks and searching new physics beyond the standard model. The next-to-leading order low-energy effective Hamiltonian is used for the weak interaction matrix elements and final state interaction(FSI). The importance of the FSI in hadronic processes has been identified for a long time. Recently, its applications in D and B decays have attracted extensive interests and attentions of theorists. Since the hadronic matrix elements are fully controlled by non-perturbative QCD, the most important problem is how to evaluate them properly. The factorization method enables one to separate the non-perturbative QCD effects from the perturbative parts and to calculate the latter in terms of the field theory order by order. Several factorization approaches have been proposed to analyze B meson decays, such as the naive factorization approach, QCDF approach, the perturbative QCD approach and soft-collinear-effective theory, but none provided an estimate of the FSI at the hadronic level. These approaches successfully explain many phenomenons; however, there are still some problems which are not easy to describe within this framework. These may be some hints for the need of FSI in B decays. FSI effects are non-perturbative in nature [1]. In many decay modes, FSI may play a crucial role [2]. In this way, the CKM matrix elements and the color factor are suppressed and the CKM's most favored two-body intermediate states are the only ones that have been taken into consideration [3]. The FSI can be considered as a soft re-scattering style for certain intermediate two-body hadronic channel $B^+ \to D_s^+\pi^0$ decay [4]. Therefore, FSI are estimated via the one particle exchange processes at the hadron loop level (HLL) as explained in Sec. 4. We calculated the $B^+ \to D^+K^0$ decay according to QCDF method. This process only occurs via annihilation between b and $\bar{u}$. We selected the leading order Wilson coefficients at the scale m_b [5, 6] and obtained the BR $(B^+ \to D^+K^0) = (0.16 \pm 0.04) \cdot 10^{-6}$. The decay is a pure annihilation decay channel. It is therefore, expected to be very small in factorization approach. The FSI can give sizable corrections. Re-scattering amplitude can be derived by calculating the absorptive part of triangle diagrams. In this decay, intermediate state are the $D_s^+\pi^0$, $D_s^{+*}\rho^0$, $D_s^{+*}\phi$. Then we calculated the $B^+ \to D^+K^0$ decay according to HLL method. By FSI method we obtain the branching ratio of $B^+ \to D^+K^0$ decay, $(2.8 \pm 0.09) \cdot 10^{-6}$ and the experimental result of this decay is less than $2.9 \cdot 10^{-6}$. We present the calculation of QCDF for $B^+ \to D^+K^0$ decay in Sec. 2. In Sec. 3, we calculate the amplitudes of the intermediate states. Then we present the calculation of HLL for $B^+ \to D^+K^0$ decay in Sec. 4. In Sec. 5, we give the numerical results, and in Sec. 6, we have conclusion.

2. QCD factorization of $B^+ \to D^+K^0$ decay. To compare QCDF with FSI, we explore QCDF analysis. In this case, we only have current–current, penguin and electroweak penguin annihilation effects. These contributions are small, but it is interesting and necessary to discuss them. For annihilation amplitude when all the

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basic building blocks equations are solved it is found that weak annihilation kernels exhibit endpoint divergence. Divergence terms are determined by $\int_0^1 dx/\bar{x}$ and $\int_0^1 dy/\bar{y}$. For the liberation of the divergence, a small ϵ of $\Lambda_{\mathrm{QCD}}/M_B$ order was added to the denominator. So the answer to the integral becomes $\ln(1+\epsilon)/\epsilon$ form, which is shown with X_A. Specifically, we treat X_A as a universal parameter obtained by using $\rho_A = 0.5$ and a strong phase for PP($M_1 M_2$) case, $\phi_A = -55°$ [7]. According to Fig. 1, we obtained the annihilation amplitude as

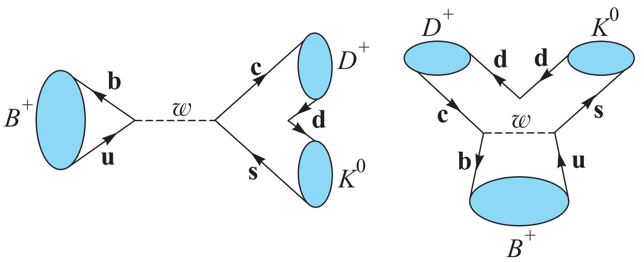

Fig. 1. Feynman annihilation diagrams for $B^+ \to D^+ K^0$ decay

$$A(B^+ \to D^+ K^0) = \frac{iG_F}{\sqrt{2}} f_B f_D f_K \{b_1 V_{cb} V_{us}^* + b_2 V_{ub} V_{cs}^*\}, \quad (1)$$

f_B, f_D, and f_K are the decay constants. $V_{pb}V_{ps}^*$ ($p = u, c$) are the CKM matrix elements and the quantities of the b_1 and b_2 correspond to the current–current annihilation. These quantities are given by

$$B_{1,2} = \frac{C_F}{N_C^2} C_{1,2} A_{1,2}^i, \quad (2)$$

$C_{1,2}$ are the Wilson coefficients, N_c is the color number and

$$A_1^i \approx -A_2^i =$$
$$= 6\pi\alpha_s \left[3\left(X_A - 4 + \frac{\pi^2}{3}\right) + r_X^D r_X^K (X_A^2 - 2X_A)\right],$$
$$C_F = \frac{N_C^2 - 1}{2N_C}. \quad (3)$$

There are large theoretical uncertainties related to the modeling of power corrections corresponding to weak annihilation effects, we parametrize these effects in terms of the divergent integrals X_A (weak annihilation)

$$X_A = (1 + \rho e^{i\phi}) \ln \frac{m_B}{\Lambda_h}, \quad \rho \le 1, \quad \Lambda_h = 0.5\,\mathrm{GeV}. \quad (4)$$

And for the pseudoscalar mesons of D^+ and K^0, the ration $r_X^{D^+}$ and $r_X^{K^0}$ are defined as

$$r_X^{D^+} = \frac{2m_D^2}{(m_b - m_c)(m_d + m_c)},$$

$$r_X^{K^0} = \frac{2m_K^2}{(m_b - m_d)(m_s + m_d)}. \quad (5)$$

3. Amplitudes of intermediate states. In this section, before analyzing FSI in the $B^+ \to D^+ K^0$ decay, we introduce the factorization approach in detail. The effective weak Hamiltonian for B decays consists of a sum of local operators Q_i multiplied by QCDF coefficients C_i and products of elements of the quark mixing matrix [7]. The factorization approach of the heavy-meson decays can be expressed in terms of different topologies of various decay mechanisms such as tree, penguin and annihilation.

3.1. $B^+ \to D_s^+ \pi^0$ decay. According to quark level diagrams in the next section, $D_s^+ \pi^0$ and $D_s^{+*} \rho^0$ are produced for intermediate states via exchange meson of $\bar{K}^0$, $\bar{K}^{0*}$, D^-, and D^{-*}. For $B^+ \to D_s^+ \pi^0$ decay, Feynman diagrams are shown in Fig. 2, and the amplitude reads

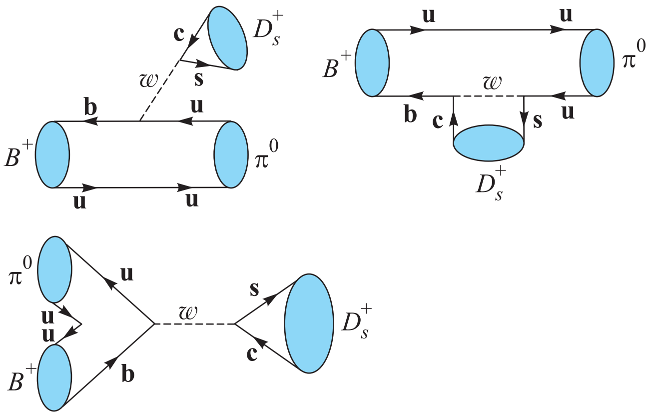

Fig. 2. $B^+ \to D_s^+ \pi^0$ decay diagrams

$$A(B^+ \to D_s^+ \pi^0) =$$
$$= \frac{iG_F}{2} f_{D_s^+} F_0^{B\to\pi} (m_B^2 - m_\pi^2)(a_1 V_{ub} V_{cs}^* + a_2 V_{cb} V_{us}^*). \quad (6)$$

Here

$$a_i = c_i^{\mathrm{eff}} + \frac{1}{N_c} c_{i+1}^{\mathrm{eff}} \quad (i = \mathrm{odd}),$$
$$a_i = c_i^{\mathrm{eff}} + \frac{1}{N_c} c_{i-1}^{\mathrm{eff}} \quad (i = \mathrm{even}). \quad (7)$$

Where i runs from $i = 1, \ldots, 10$ and a_1 and a_2 are both related to the coefficients c_1 and c_2, which are very large compared to other Wilson coefficients.

There are also similar diagrams such as $B^+ \to D_s^{+*} \rho^0$ decay. So the amplitude reads

$$A(B^+ \to D_s^{+*} \rho^0) = \frac{iG_F}{2} f_{D_s^*} m_{D_s^*} (a_1 V_{ub} V_{cd}^* + a_2 V_{cb} V_{us}^*) \times$$
$$\times \left[(\varepsilon_1^* \varepsilon_2^*)(m_B + m_\rho) A_1^{B\rho}(m_{D_s^*}^2) - \right.$$

$$- (\varepsilon_1^* p_B)(\varepsilon_2^* p_B)\frac{2A_2^{B\rho}(m_{D_s^*}^2)}{m_B + m_\rho}\Bigg]. \tag{8}$$

3.2. $B^+ \to D_s^{+*}\phi$ decay. When two intermediate mesons exchange s-quark and two final state mesons exchange d-quark, the D_s^{+*} and ϕ mesons are produced for intermediate state via exchange meson of ($\bar{K}^{0*}$, $\bar{K}^0$). The intermediate mesons cannot exchange d-quark because $B^+ \to D^+K^0 \to D^+K^0$ decay is impossible. For $B^+ \to D_s^{+*}\phi$ decay, Feynman diagrams are shown in Fig. 3, and the amplitude reads

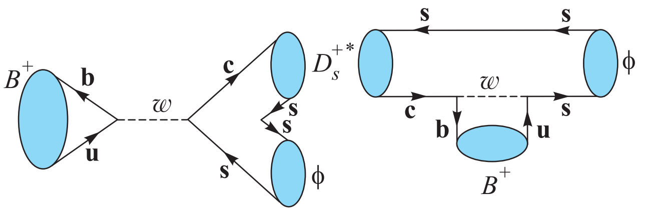

Fig. 3. $B^+ \to D_s^{+*}\phi$ decay diagrams

$$A(B^+ \to D_s^{+*}\phi) = \frac{-iG_F}{\sqrt{2}} f_B f_{D_s^*} f_\phi (b_1 V_{cb}V_{us}^* + b_2 V_{ub}V_{cs}^*). \tag{9}$$

Where

$$R_\chi^{D_s^*} = \frac{2m_{D_s^*}}{m_b}\frac{f_{D_s^*}^\perp}{f_{D_s^*}},$$

$$R_\chi^\phi = \frac{2m_\phi}{m_b}\frac{f_\phi^\perp}{f_\phi}. \tag{10}$$

4. Final state interaction of the $B^+ \to D^+K^0$ decay. For $B^+ \to D^+k^0$ decay, two body intermediate states such as $D_s^+\pi^0$, $D_s^{+*}\rho^0$, $D_s^{+*}\phi$ are produced. We can write out the decay amplitude involving HLL contributions with the following formula

$$\mathrm{Abs}\, M(B(p_B) \to M(p_1)M(p_2) \to M(p_3)M(p_4)) =$$

$$= \frac{1}{2}\int \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B - p_1 - p_2) \times$$

$$\times M(B \to M_1M_2)G(M_1M_2 \to M_3M_4), \tag{11}$$

for which both intermediate mesons (M_1, M_2) are pseudoscalar, and the absorptive part of the HLL diagrams for PP case can be calculated by

$$\mathrm{Abs}\, M(B(p_B) \to M(p_1)M(p_2) \to M(p_3)M(p_4)) =$$

$$= \frac{-iG_F}{2\sqrt{2}} V_{CKM} f_{V_2} m_2 \times$$

$$\times \int \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B - p_1 - p_2) \times$$

$$\times \Bigg[(\varepsilon_1^*\varepsilon_2^*)(m_B + m_1)A_1^{BM_1}(m_2^2) -$$

$$- (\varepsilon_1^* p_B)(\varepsilon_2^* p_B)\frac{2A_2^{BM_1}(m_2^2)}{m_B + m_1}\Bigg] G(M_1M_2 \to M_3M_4). \tag{12}$$

Where $M(B \to M_1M_2)$ is the amplitude of $B \to M_1M_2$ decay that calculated via QCDF method, and $G(M_1M_2 \to M_3M_4)$ involves hadronic vertices factor defined as

$$\langle D(p_2)\bar{K}^{0*}(p_3, \varepsilon_3)|i\mathcal{L}|D_s(p_1)\rangle = -ig_{D_sK^*D}\varepsilon(p_1 + p_2),$$

$$\langle D(\varepsilon_2, p_2)K^0(q)|i\mathcal{L}|D_s^*(\varepsilon_1, p_2)\rangle =$$

$$= -ig_{D_s^*D^*K}\varepsilon_{\mu\nu\alpha\beta}\varepsilon_1^\mu\varepsilon_2^{*\nu}q^\alpha p_1^\beta. \tag{13}$$

The dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation [1]:

$$\mathrm{Dis}\, M(m_B^2) = \frac{1}{\pi}\int_s^\infty \frac{\mathrm{Abs}\, M(s')}{s' - m_B^2}ds'. \tag{14}$$

Where s' is the square of the momentum carried by the exchanged particle and $s \sim m_B^2$ is the threshold of intermediate states, in this case s. Unlike the absorptive part, the dispersive contribution suffers from the large uncertainties arising from the complicated integration.

4.1. Final state interaction in the $B^+ \to D_s^+\pi^0 \to D^+K^0$ decay. The quark model diagram for $B^+ \to D_s^+\pi^0 \to D^+K^0$ via the exchange of D^+ is shown in Fig. 4. And the hadronic level diagrams are shown in Fig. 5.

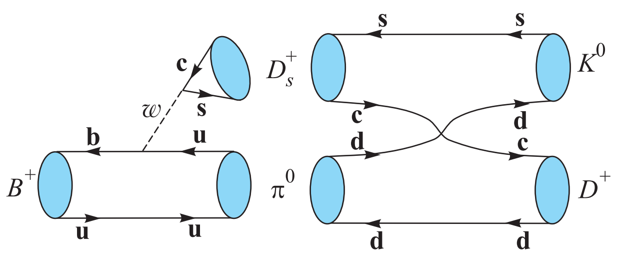

Fig. 4. Quark level diagram for $B^+ \to D^+K^0$

The amplitude of the mode $B^0 \to D_s^+(p_1)\pi^0(p_2) \to D^+(p_3)K^0(p_4)$ via the exchange of D^+ are given by

$$\mathrm{Abs(5a)} = \frac{1}{2}\int_{-1}^{1} \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B - p_1 - p_2) \times$$

$$\times M(B^+ \to D_s^+\pi^0)(-ig_{D_s^+D^*K})[\varepsilon_{D^*}(p_1 + p_3)] \times$$

$$\times (-ig_{D^*D\pi})[-\varepsilon_{D^*}(p_2 + p_4)]\frac{F^2(q^2, m_{D^*}^2)}{T_1} =$$

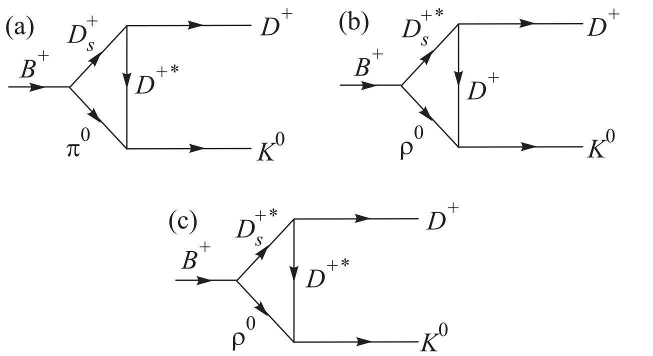

Fig. 5. HLL diagrams for long distance t-channel contribution to $B^+ \to D^+ K^0$

$$= -\int_{-1}^{1} |p_1| \frac{d(\cos\theta)}{16\pi m_B} (g_{D_s D^* K})(g_{D^* D\pi}) \times$$
$$\times H_1 M(B^+ \to D_s^+ \pi^0) \frac{F^2(q^2, m_{D^*}^2)}{T_1}. \tag{15}$$

Here

$$H_1 = \varepsilon_\mu (p_1 + p_3)^\mu \varepsilon_\nu (p_2 + p_4)^\nu =$$
$$= (p_1 + p_3)^\mu (p_2 + p_4)^\nu \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_{D^*}^2} \right) =$$
$$= -(p_1 p_2 + p_1 p_4 + p_2 p_3 + p_3 p_4) + \frac{(m_1^2 - m_3^2)(m_4^2 - m_2^2)}{m_{D^*}^2},$$
$$T_1 = (p_1 - p_3)^2 - m_{D^*}^2 =$$
$$= p_1^2 + p_3^2 - 2p_1^0 p_3^0 - 2p_1^0 p_3^0 + 2\mathbf{p}_1 \cdot \mathbf{p}_3 - m_{D^*}^2,$$
$$q^2 = m_1^2 + m_3^2 - 2E_1 E_3 + 2|\mathbf{p}_1||\mathbf{p}_3| \cos\theta =$$
$$= m_{D_s}^2 + m_{D_s^*}^2 - 2p_1^0 p_3^0 + 2|\mathbf{p}_1||\mathbf{p}_3| \cos\theta. \tag{16}$$

θ is the angle between $\mathbf{p}_1$ and $\mathbf{p}_3$, q is the momentum of the exchange D^* meson, and $F(q^2, m_{D^*}^2)$ is the form factor defined to take care of the off – shell of the exchange particles, which introduced as [8]

$$F(q^2, m_{D^*}^2) = \left(\frac{\Lambda^2 - m_{D^*}^2}{\Lambda^2 - q^2} \right)^n. \tag{17}$$

The form factor (i.e. $n = 1$) normalized to unity at $q^2 = m_{D^*}^2$. m_{D^*} and q are the physical parameters of the exchange particle and Λ is phenomenological parameter.

It is obvious that for $q^2 \to 0$, $F(q^2, m_{D^*}^2)$ becomes a number. If $\Lambda \gg m_{D^*}$ then $F(q^2, m_{D^*}^2)$ turns to be unity, whereas, as $q^2 \to \infty$ the form factor appraochs to zero and the distance becomes small and the hadron interaction is no longer valid. Since Λ shoud not be far from m_{D^*} and q, we choose

$$\Lambda = m_{D^*} + \eta \Lambda_{\text{QCD}}, \tag{18}$$

where the η is the phenomenological parameter that its value in the form factor is expected to be of the order of unity and can be determined from the measured rates, and

$$\text{Abs}(5b) =$$
$$= \frac{1}{2} \int_{-1}^{1} \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \times$$
$$\times (-ig_{D_s^* DK})(\varepsilon_1 q)(-ig_{\rho DD})(-\varepsilon_2 q) \left(\frac{iG_F}{2} \right) f_{D_s^*} m_{D_s^*} \times$$
$$\times \Bigg[\varepsilon_1^* \varepsilon_2^*)(m_B + m_\rho) A_1^{B\rho}(m_{D_s^*}^2) -$$
$$- (\varepsilon_1^* p_B)(\varepsilon_2^* p_B) \frac{2A_2^{B\rho}(m_{D_s^*}^2)}{m_B + m_\rho} \Bigg] \times$$
$$\times (a_1 V_{ub} V_{cs}^* + a_2 V_{cb} V_{us}^* \frac{F^2(q^2, m_D^2)}{T_2} =$$
$$= \frac{iG_F}{16\pi m_B} f_{D_s^*} m_{D_s^*} (a_1 V_{ub} V_{cs}^* + a_2 V_{cb} V_{us}^*) \times$$
$$\times g_{D_s^* DK} g_{\rho DD} \int_{-1}^{1} |p_1| d(\cos\theta) \times$$
$$\times \left[(m_B + m_\rho) A_1^{B\rho}(m_{D_s^*}^2) H_1 - \frac{2A_2^{B\rho}(m_{D_s^*}^2)}{m_B + m_\rho} H_2 \right] \times$$
$$\times \frac{F^2(q^2, m_D^2)}{T_2}. \tag{19}$$

Here

$$H_1 = (\varepsilon_1 q)(\varepsilon_2 q)(\varepsilon_1^* \varepsilon_2^*) =$$
$$= p_1^2 + p_3^2 - 2p_1 p_3 - \frac{(p_2 p_4 - m_2^2)^2}{m_2^2} - \frac{(m_1^2 - p_1 p_3)^2}{m_1^2} +$$
$$+ \frac{(m_1^2 - p_1 p_3)(p_2 p_4 - m_2^2)(p_1 p_2)}{m_1^2 m_2^2},$$
$$H_2 = (\varepsilon_1 q)(\varepsilon_2 q)(\varepsilon_1 p_B)(\varepsilon_2 p_B) =$$
$$= \frac{m_B p_1^0 (m_1^2 - p_1 p_3)}{m_1^2} \frac{m_B p_2^0 (p_2 p_4 - m_2^2)}{m_2^2},$$
$$T_2 = (p_1 - p_3)^2 - m_D^2 = p_1^2 + p_3^2 - 2p_1^0 p_3^0 + 2\mathbf{p}_1 \mathbf{p}_3 - m_D^2,$$
$$q^2 = m_1^2 + m_3^2 - 2E_1 E_3 + 2|\mathbf{p}_1||\mathbf{p}_3| \cos\theta =$$
$$= m_{D_s^*}^2 + m_D^2 - 2p_1^0 p_3^0 + 2|\mathbf{p}_1||\mathbf{p}_3| \cos\theta, \tag{20}$$

and

$$\text{Abs}(5c) =$$
$$= \frac{1}{2} \int_{-1}^{1} \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3} \frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2) \times$$
$$\times (-ig_{D_s^* D^* K})(\varepsilon_{\mu\nu\alpha\beta} \varepsilon_1^\mu \varepsilon_{D^*}^\nu p_3^\alpha p_1^\beta) \left(\frac{iG_F}{2} \right) \times$$
$$\times (-ig_{D^* D\rho})(\varepsilon_{\rho\sigma\lambda\eta} \varepsilon_2^\rho \varepsilon_{D^*}^\sigma P_4^\lambda p_2^\eta) f_{D_s^*} m_{D_s^*} \times$$

$$\times \left[(\varepsilon_1^*\varepsilon_2^*)(m_B+m_\rho)A_1^{B\rho}(m_{D_s^*}^2) - \right.$$

$$\left. -(\varepsilon_1^* p_B)(\varepsilon_2^* p_B)\frac{2A_2^{B\rho}(m_{D_s^*}^2)}{m_B+m_\rho}\right] \times$$

$$\times (a_1V_{ub}V_{cs}^* + a_2V_{cb}V_{us}^*)\frac{F^2(q^2,m_{D^*}^2)}{T_1} =$$

$$= \frac{iG_F\sqrt{2}}{32\pi m_B}(g_{D_s^*D^*K})(g_{D^*D\rho})f_{D_s^*}m_{D_s^*} \times$$

$$\times (a_1V_{ub}V_{cs}^* + a_2V_{cb}V_{us}^*)\int_{-1}^{1}|p_1|\frac{d(\cos\theta)}{16\pi m_B} \times$$

$$\times \left[(m_B-m_\rho)A_1^{B\rho}(m_{D_s^*}^2)H_1 - \frac{2A_2^{B\rho}(m_{D_s^*}^2)}{m_B+m_\rho}H_2\right] \times$$

$$\times \frac{F^2(q^2,m_{D^*}^2)}{T_1}. \qquad (21)$$

Here

$$H_1 = \varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\rho\sigma\lambda\eta}\varepsilon_1^\mu\varepsilon_{D^*}^\nu p_3^\alpha q^\beta\varepsilon_2^\rho\varepsilon_{D^*}^\sigma p_4^\lambda q^\eta(\varepsilon_1^*\varepsilon_2^*) =$$

$$= \{(p_1q)[(p_3q)(p_1p_4) - (p_3p_4)(p_1q)] +$$

$$+ (p_1p_3)[(p_4q)(p_1q) - q^2(p_1p_4)]\}/m_{D^*}^2 +$$

$$+ \{(p_2q)[(p_4q)(p_2p_3) - (p_3p_4)(p_2q)] +$$

$$+ (p_2p_4)[(p_3q)(p_2q) - q^2(p_2p_3)]\}/m_{D^*}^2 +$$

$$+ \{(p_1p_2)[(p_3q)(p_4q) - q^2(p_3p_4)] +$$

$$+ (p_1p_2)(p_2q)[(p_3p_4)(p_1q) - (p_1p_4)(p_3q)] +$$

$$+ (p_1p_2)(p_2p_3)[q^2(p_1p_4) - (p_4q)(p_1q)]\}/m_{D^*}^4,$$

$$H_2 = \varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\rho\sigma\lambda\eta}\varepsilon_1^\mu\varepsilon_{D^*}^\nu p_3^\alpha q^\beta\varepsilon_2^\rho\varepsilon_{D^*}^\sigma p_4^\lambda q^\eta(\varepsilon_1^* p_B)(\varepsilon_2^2 p_B) =$$

$$= (p_3q)[m_B^2(p_4q) - (p_4p_B)(qp_B)] + (p_3p_4) \times$$

$$\times [(qp_B)^2 - q^2m_B^2] + (p_3p_B)[q^2(p_4p_B) - (p_4q)(qp_B)] +$$

$$+ \{(qp_B)[(p_3q)(p_1p_4) - (p_1q)(p_3p_4)] +$$

$$+ (p_3p_B)[(p_1q)(p_4q) - q^2(p_1p_4)] +$$

$$+ (p_1p_B)[q^2(p_3p_4) - (p_3q)(p_4q)]\}(p_1p_B)/m_{D^*}^2 +$$

$$+ \{(p_2q)[(p_3q)(p_4p_B) - (p_3p_4)(qp_B)] +$$

$$+ (p_2p_3)[(p_4q)(qp_B) - q^2(p_4p_B)] +$$

$$+ (p_2p_B)[q^2(p_3p_4) - (p_4q)(p_3q)]\}(p_2p_B)/m_{D^*}^2 +$$

$$+ \{(p_1p_2)[(p_3q)(p_4q) - q^2(p_3p_4)] +$$

$$+ (p_2q)[(p_2p_3)(p_1q) - (p_1p_4)(p_3q)] +$$

$$+ (p_2p_3)[q^2(p_1p_4) - (p_1q)(p_4q)]\}[(p_1p_B)(p_2p_B)]/m_{D^*}^4,$$

$$T_1 = (p_1-p_3)^2 - m_{D^*}^2 = p_1^2 + p_3^2 - 2p_1^0p_3^0 + 2\mathbf{p}_1\mathbf{p}_3 - m_{D^*}^2,$$

$$q^2 = m_1^2 + m_3^2 - 2E_1E_3 + 2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta =$$

$$= m_{D_s^*}^2 + m_D^2 - 2p_1^0p_3^0 + 2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta. \qquad (22)$$

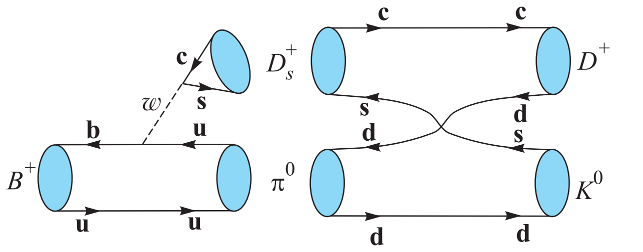

Fig. 6. Quark level diagram for $B^+ \to D^+K^0$

The dispersion relation is

$$\mathrm{Dis}\,5(m_B^2) = \frac{1}{\pi}\int_s^\infty \frac{\mathrm{Abs}\,5\mathrm{a}(s') + \mathrm{Abs}\,5\mathrm{b}(s')}{s'-m_B^2}ds'. \qquad (23)$$

The quark model diagram for $B^+ \to D_s^+\pi^0 \to D^+K^0$ decay via the exchange of $\bar{K}^0$ is shown in Fig. 6. And the hadronic level diagrams are shown in Fig. 7.

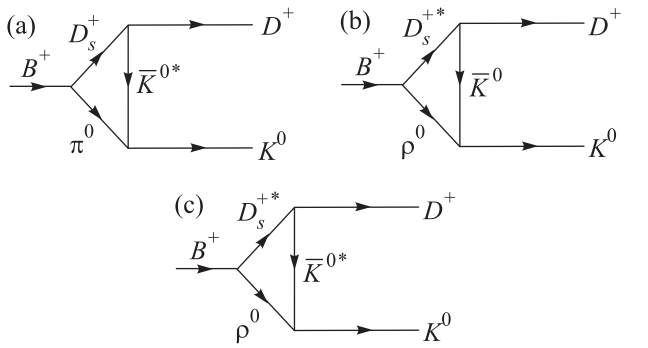

Fig. 7. HLL diagrams for long distance t-channel contribution to $B^+ \to D^+K^0$

And the amplitude of the mode $B^0 \to \to D_s^+(p_1)\pi^0(p_2) \to D^+(p_3)K^0(p_4)$ via the exchange of $\bar{K}^0$ are given by

$$\mathrm{Abs(7a)} =$$

$$= \frac{1}{2}\int_{-1}^{1}\frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B-p_1-p_2) \times$$

$$\times M(B^+ \to D_s^+\pi^0)(-ig_{D_s^+K^*D})[\varepsilon_{K^*}(p_1+p_3)] \times$$

$$\times (-ig_{K^*K\pi})[\varepsilon_{K^*}(p_2+p_4)]\frac{F^2(q^2,m_{K^*}^2)}{T_1} =$$

$$= \int_{-1}^{1}|p_1|\frac{d(\cos\theta)}{16\pi m_B}(g_{D_s^+K^*D})(g_{K^*K\pi}) \times$$

$$\times H_1M(B^+ \to D_s^+\pi^0)\frac{F^2(q^2,m_{K^*}^2)}{T_1}. \qquad (24)$$

Here

$$H_1 = \varepsilon_\mu(p_1+p_3)^\mu\varepsilon_\nu(p_2+p_4) =$$

$$= (p_1+p_3)^\mu(p_2+p_4)^\nu\left(-g_{\mu\nu} + \frac{q_\mu q_n u}{m_{K^*}^2}\right) =$$

$$= -(p_1p_2+p_1p_4+p_2p_3+p_3p_4)+\frac{(m_1^2-m_3^2)(m_4^2-m_2^2)}{m_{K^*}^2},$$
$$T_1 = (p_1-p_3)^2-m_{K^*}^2 = p_1^2+p_3^2-2p_1^0p_3^0+2\mathbf{p}_1\mathbf{p}_3-m_{K^*}^2,$$
$$q^2 = m_1^2+m_3^2-2E_1E_3+2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta =$$
$$= m_{D_s^*}^2+m_D^2-2p_1^0p_3^0+2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta, \qquad (25)$$

and

$$\mathrm{Abs}(7b) =$$
$$= \frac{-iG_F}{2\sqrt{2}}\int_{-1}^{1}\frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B-p_1-p_2)\times$$
$$\times(-ig_{D_s^*DK})(2\varepsilon_1p_3)(-ig_{\rho KK})(2\varepsilon_2p_4)\left(\frac{-iG_F}{2}\right)f_{D_s^*}m_{D_s^*}\times$$
$$\times\left[(\varepsilon_1^*\varepsilon_2^*)(m_B+m_\rho)A_1^{B\rho}(m_{D_s^*}^2)-\right.$$
$$\left.-(\varepsilon_1^*p_B)(\varepsilon_2^*p_B)\frac{2A_2^{B\rho}(m_{D_s^*}^2)}{m_B+m_\rho}\right]\times$$
$$\times(a_1V_{ub}V_{cs}^*+a_2V_{cb}V_{us}^*)\frac{F^2(q^2,m_K^2)}{T_2} =$$
$$= \frac{-iG_F}{16\pi m+B}f_{D_s^*}m_{D_s^*}(a_1V_{ub}V_{cs}^*+a_2V_{cb}V_{us}^*)\times$$
$$\times g_{D_s^*DK}g_{\rho KK}\int_{-1}^{1}|p_1|d(\cos\theta)\times$$
$$\times\left[(m_B+m_\rho)A_1^{B\rho}(m_{D_s^*})H_1-\frac{2A_2^{B\rho}(m_{D_s^*}^2)}{m_B+m_\rho}H_2\right]\times$$
$$\times\frac{F^2(q^2,m_K^2)}{T_2}. \qquad (26)$$

Here

$$H_1 = (\varepsilon_1q)(\varepsilon_2q)(\varepsilon_1^*\varepsilon_2^*) =$$
$$= p_1^2+p_3^2-2p_1p_3-\frac{(p_2p_4-m_2^2)^2}{m_2^2}-\frac{(m_1^2-p_1p_3)^2}{m_1^2}+$$
$$+\frac{(m_1^2-p_1p_3)(p_2p_4-m_2^2)(p_1p_2)}{m_1^2m_2^2},$$
$$H_2 = (\varepsilon_1q)(\varepsilon_2q)(\varepsilon_1p_B)(\varepsilon_2p_B) =$$
$$= \frac{m_Bp_1^0(m_1^2-p_1p_3)}{m_1^2}\frac{m_Bp_2^0(p_2p_4-m_2^2)}{m_2^2},$$
$$T_2 = (p_1-p_3)^2-m_K^2 =$$
$$= p_1^2+p_3^2-2p_1^0p_3^0+2\mathbf{p}_1\mathbf{p}_3-m_K^2,$$
$$q^2 = m_1^2+m_3^2-2E_1E_3+2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta =$$
$$= m_{D_s^*}^2+m_D^2-2p_1^0p_3^0+2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta, \qquad (27)$$

and

$$\mathrm{Abs}(7c) =$$
$$= \frac{1}{2}\int_{-1}^{1}\frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B-p_1-p_2)\times$$
$$\times(-i\sqrt{2}g_{D_s^*K^*D})\left(\frac{-iG_F}{2}\right)(\varepsilon_{\mu\nu\alpha\beta}\varepsilon_1^\mu\varepsilon_{K^*}^\nu p_3^\alpha p_1^\beta)\times$$
$$\times(-ig_{\rho K^*K})(\varepsilon_{\rho\sigma\lambda\eta}\varepsilon_2^\rho\varepsilon_{K^*}^\sigma p_4^\lambda p_2^\eta)f_{D_s^*}m_{D_s^*}\times$$
$$\times\left[(\varepsilon_1^*\varepsilon_2^*)(m_B+m_\rho)A_1^{B\rho}(m_{D_s^*}^2)-\right.$$
$$\left.-(\varepsilon_1^*p_B)(\varepsilon_2^*p_B)\frac{2A_2^{B\rho}(m_{D_s^*})}{m_B+m_\rho}\right]\times$$
$$\times(a_1V_{ub}V_{cs}^*+a_2V_{cb}V_{us}^*)\frac{F^2(q^2,m_{K^*}^2)}{T_1} =$$
$$= \frac{iG_F\sqrt{2}}{32\pi m_B}(g_{D_s^*K^*D})(g_{\rho K^*K})f_{D_s^*}m_{D_s^*}\times$$
$$\times\int_{-1}^{1}|p_1|\frac{d(\cos\theta)}{16\pi m_B}\times$$
$$\times\left[(m_B+m_\rho)A_1^{B\rho}(m_{D_s^*}^2)H_1-\frac{2A_2^{B\rho}(m_{D_s^*})}{m_B+m_\rho}H_2\right]\times$$
$$\times(a_1V_{ub}V_{cs}^*+a_2V_{cb}V_{us}^*)\frac{F^2(q^2,m_{K^*}^2)}{T_1}. \qquad (28)$$

Here

$$H_1 = \varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\rho\sigma\lambda\eta}\varepsilon_1^\mu\varepsilon_{D^*}^\nu p_3^\alpha q^\beta\varepsilon_2^\rho\varepsilon_{D^*}^\sigma p_4^\lambda q^\eta(\varepsilon_1^*\varepsilon_2^*) =$$
$$= \{(p_1q)[(p_3q)(p_1p_4)-(p_3p_4)(p_1q)]+$$
$$+(p_1p_3)[(p_4q)(p_1q)-q^2(p_1p_4)]\}/m_{K^*}^2+$$
$$+\{(p_2q)[(p_4q)(p_2p_3)-(p_3p_4)(p_2q)]+$$
$$+(p_2p_4)[(p_3q)(p_2q)-q^2(p_2p_3)]\}/m_{K^*}^2+$$
$$+\{(p_1p_2)[(p_3q)(p_4q)-q^2(p_3p_4)]+$$
$$+(p_1p_2)(p_2q)[(p_3p_4)(p_1q)-(p_1p_4)(p_3q)]+$$
$$+(p_1p_2)(p_2p_3)[q^2(p_1p_4)-(p_4q)(p_1q)]\}/m_{K^*}^4,$$

$$H_2 = \varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\rho\sigma\lambda\eta}\varepsilon_1^\mu\varepsilon_{D^*}^\nu p_3^\alpha q^\beta\varepsilon_2^\rho\varepsilon_{D^*}^\sigma p_4^\lambda q^\eta(\varepsilon_1^*p_B)(\varepsilon_2^*p_B) =$$
$$= (p_3q)[m_B^2(p_4q)-(p4p_B)(qp_B)]+$$
$$+(p_3p_4)[(qp_B)^2-q^2m_B^2]+(p_3p_B)[q^2(p_4p_B)-(p_4q)(qp_B)]+$$
$$+\{(qp_B)[(p_3q)(p_1p_4)-(p_1q)(p_3p_4)]+$$
$$+(p_3p_B)[(p_1q)(p_4q)-q^2(p_1p_4)]+$$
$$+(p_1p_B)[q^2(p_3p_4)-(p_3q)(p_4q)]\}(p_1p_B)m_{K^*}^2+$$
$$+\{(p_2q)[(p_3q)(p_4p_B)-(p_3p_4)(qp_B)]+$$
$$+(p_2p_3)[(p_4q)(qp_B)-q^2(p_4p_B)]+$$

$$+ (p_2p_B)[q^2(p_3p_4) - (p_4q)(p_3q)]\}(p_2p_B)/m^2_{K^*} +$$
$$+ \{(p_1p_2)[(p_3q)(p_4q) - q^2(p_3p_4)] +$$
$$+ (p_2q)[(p_2p_3)(p_1q) - (p_1p_4)(p_3q)] +$$
$$+ (p_2p_3)[q^2(p_1p_4) - (p_1q)(p_4q)]\}[(p_1p_B)(p_2p_B)]/m^4_{K^*},$$
$$T_1 = (p_1-p_3)^2 - m^2_{K^*} = p_1^2+p_3^2-2p_1^0p_3^0+2\mathbf{p}_1\mathbf{p}_3-m^2_{K^*},$$
$$q^2 = m_1^2 + m_3^2 - 2E_1E_3 + 2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta =$$
$$= m^2_{D_s^*} + m_D^2 - 2p_1^0p_3^0 + 2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta. \quad (29)$$

The dispersion relation is

$$\mathrm{Dis7}(m_B^2) = \frac{1}{\pi}\int_s^\infty \frac{\mathrm{Abs\,7a}(s') + \mathrm{Abs\,7b}(s')}{s' - m_B^2}\,ds'. \quad (30)$$

4.2. Final state interaction in the $B^+ \to D_s^{+*}\phi \to \to D^+K^0$ decay. The quark model diagram for $B^+ \to \to D_s^{+*}\phi \to D^+K^0$ decay is shown in Fig. 8. And

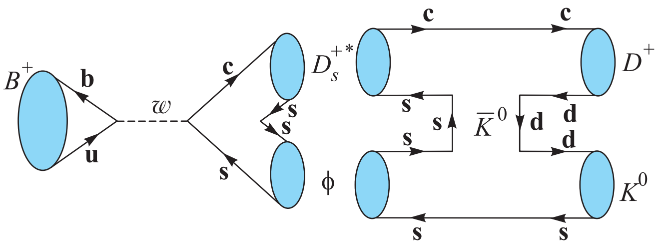

Fig. 8. Quark level diagram for $B^+ \to D_s^{+*}\phi$

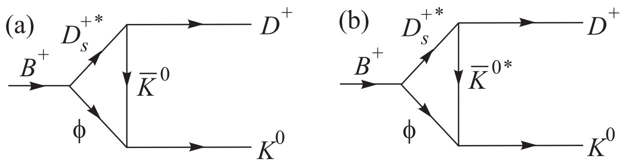

Fig. 9. HLL diagrams for long distance t-channel contribution to $B^+ \to D^+K^0$

the hadronic level diagrams are shown in Fig. 9. The amplitude of the mode $B^0 \to D_s^{+*}(\varepsilon_1, p_1)\phi(\varepsilon_1, p_2) \to \to D^+(p_3)K^0(p_4)$ via the exchange of $\bar{K}^0$ are given by

$$\mathrm{Abs(9a)} =$$
$$= \frac{1}{2}\int_{-1}^{1} \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B - p_1 - p_2) \times$$
$$\times (\varepsilon_1 q)(-ig_{D_s^*KD})(-ig_{KK\phi})(2\varepsilon_2 p_4) \times$$
$$\times M(B^+ \to D_s^{+*}\phi)\frac{F^2(q^2, m_K^2)}{T_1} =$$
$$= \frac{iG}{8\sqrt{2}\pi m_B} g_{D_s^*KD}\, g_{KK\phi}\int_{-1}^{1}|p_1|d(\cos\theta) \times$$
$$\times f_B f_{D_s^*} f_\phi [b_1 V_{cb}V_{us}^* + b_2 V_{ub}V_{cs}^*]\frac{F^2(q^2, m_K^2)}{T_1}H_1. \quad (31)$$

Here

$$H_1 = (\varepsilon_1 q)(\varepsilon_2 p_4) =$$
$$= -\left(\frac{E_3|p_1| - E_1|p_3|\cos\theta}{m_B|p_1|}\right) - \left(\frac{E_4|p_2| - E_2|p_4|\cos\theta}{m_B|p_2|}\right),$$
$$T_1 = (p_1 - p_3)^2 - m_K^2 = p_1^2 + p_3^2 - 2p_1^0p_3^0 + 2\mathbf{p}_1\mathbf{p}_3 - m_K^2,$$
$$q^2 = m_1^2 + m_3^2 - 2E_1E_3 + 2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta =$$
$$= m^2_{D_s^*} + m_D^2 - 2p_1^0p_3^0 + 2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta, \quad (32)$$

and

$$\mathrm{Abs(9b)} =$$
$$= \frac{1}{2}\int_{-1}^{1} \frac{d^3\mathbf{p}_1}{2E_1(2\pi)^3}\frac{d^3\mathbf{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B - p_1 - p_2) \times$$
$$\times M(B^+ \to D_s^{+*}\phi)(-ig_{D_s^*K^*D})(-ig_{K^*K\phi}) \times$$
$$\times \varepsilon_{\mu\nu\alpha\beta}\varepsilon_{\rho\sigma\lambda\eta}\varepsilon_1^\mu\varepsilon_{K^*}^\nu p_3^\alpha p_1^\beta \varepsilon_2^\rho \varepsilon_{K^*}^\sigma p_4^\lambda p_2^\eta \frac{F^2(q^2, m^2_{K^*})}{T_2}. \quad (33)$$

Here

$$T_2 = (p_1-p_3)^2 - m^2_{K^*} = p_1^2+p_3^2-2p_1^0p_3^0+2\mathbf{p}_1\mathbf{p}_3-m^2_{K^*},$$
$$q^2 = m_1^2 + m_3^2 - 2E_1E_3 + 2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta =$$
$$= m^2_{D_s^*} + m_D^2 - 2p_1^0p_3^0 + 2|\mathbf{p}_1||\mathbf{p}_3|\cos\theta. \quad (34)$$

The dispersion relation is

$$\mathrm{Dis\,9}(m_B^2) = \frac{1}{2}\int_s^\infty \frac{\mathrm{Abs\,9a}(s') + \mathrm{Abs\,9b}(s')}{s' - m_B^2}. \quad (35)$$

The decay amplitude of $B^+ \to D^+K^0$ via the HLL diagrams is

$$\mathrm{A}(B^+ \to D^+K^0) = \mathrm{Abs(5a)} + \mathrm{Abs(5b)} + \mathrm{Abs(5c)} +$$
$$+ \mathrm{Abs(7a)} + \mathrm{Abs(7b)} + \mathrm{Abs(7c)} + \mathrm{Abs(9a)} +$$
$$+ \mathrm{Abs(9b)} + \mathrm{Dis\,7}(m_B^2) + \mathrm{Dis\,9}(m_B^2). \quad (36)$$

5. Numerical results. Numerical values of effective coefficients a_i for $\bar{b} \to \bar{d}$ transition at $N_c = 3$ are given by [13]

$$a_1 = 1.05, \qquad a_2 = 0.053,$$
$$a_3 = 0.0048, \qquad a_4 = -0.046 - 0.012i,$$
$$a_5 = -0.0045, \qquad a_6 = -0.059 - 0.012i,$$
$$a_7 = 0.00003 - 0.00018i, \qquad a_8 = 0.0004 - 0.00006i,$$
$$a_9 = -0.009 - 0.00018i, \qquad a_{10} = -0.0014 - 0.00006i. \quad (37)$$

The relevant input parameters used as follows:

$$\begin{array}{lll}
m_b = 4.2 \pm 0.12\,\text{GeV}, & m_u = 1.7 \sim 3.1\,\text{MeV}, & m_d = 4.1 \sim 5.7\,\text{MeV}, \\
m_B = 5279 \pm 0.3\,\text{GeV}, & m_D = 187 \pm 0.2\,\text{MeV}, & m_{D^*} = 2010.2 \pm 0.17\,\text{MeV}, \\
m_K = 493.6 \pm 0.016\,\text{GeV}, & m_{K^*} = 891 \pm 0.26\,\text{MeV}, & m_{D_s} = 197 \pm 0.34\,\text{MeV}, \\
m_{D_s^*} = 2010 \pm 0.17\,\text{GeV}, & m_\pi = 139.5\,\text{MeV}, & m_\rho = 775.4 \pm 0.34\,\text{MeV}, \\
f_B = 176 \pm 42\,\text{GeV}, & f_D = 222.6 \pm 19.5\,\text{MeV}, & f_{D^*} = 230 \pm 20\,\text{MeV}, \\
f_{K*} = 217 \pm 5\,\text{GeV}, & f_\pi = 130.7 \pm 0.46\,\text{MeV}, & f_\rho = 211\,\text{MeV}, \\
V_{ub} = 0.0043 \pm 0.0003\,\text{GeV}, & f_{ud} = 0.974 \pm 0.0002\,\text{MeV}, & f_{us} = 0.2257 \pm 0.002\,\text{MeV}, \\
V_{cs} = 0.9745 \pm 0.11\,\text{GeV}, & V_{cd} = 0.230 \pm 0.011\,\text{MeV}, & V_{cb} = 0.0416 \pm 0.0006\,\text{MeV [9]}, \\
A_1^{B\rho} = A_2^{B\rho} = 0.26 & F_0^{B\pi} = 0.3\ [6, 13], & \phi = -55^\circ\ (\text{PP}), \\
\rho = 0.5 & \Lambda_{\text{QCD}} = 0.225\,\text{GeV}, & G_F = 1.166 \cdot 10^{-5}\ [13, 14], \\
g_{\phi KK} = 5.77\ [7], & g_{\phi K^* K} = 6.48\ [10], & g_{K^* DD_s} = 2.59\ [1], \\
g_{KD^* D_s^*} = 9.23\ [11], & g_{\pi K^* K} = 4.6\ [1], & g_{\rho KK} = 5.55, \\
g_{\rho K^* K} = 6.48\ [12], & g_{\pi D^* D} = 12.5\ [3], & g_{DD\rho} = 2.52, \\
g_{\rho D^* D} = 2.82\ [8]. & &
\end{array} \tag{38}$$

By using the input parameters and according to the QCDF method of $B^+ \to D^+ K^0$ decay, we get

$$BR(B^+ \to D^+ K^0) = (0.16 \pm 0.04) \cdot 10^{-6}. \tag{39}$$

We note that our estimate of branching ratio $B^+ \to D^+ K^0$ of decay according to QCDF method seems less than the experimental result. Before calculating the $B^+ \to D^+ K^0$ decay amplitude via FSI, we have to compute the intermediate state amplitude.

We are able to calculate the branching ratio of $B^+ \to D^+ K^0$ decay with different values of η which are shown in Table.

The branching ratio of $B^+ \to D^+ K^0$ decay with $\eta = 2 \sim 2.4$ and experimental data (in units of 10^{-6})

η	2	2.1	2.2	2.3	2.4	EXP
BR	1.68 ± 0.04	1.92 ± 0.05	2.20 ± 0.07	2.49 ± 0.08	2.80 ± 0.09	< 2.9

6. Conclusion. In this work, we have calculated the contribution of the t-channel FSI, that is, inelastic rescattering processes to the branching ratio of $B^+ \to D^+ K^0$ decay. For evaluating the FSI effects, we have only considered the absorptive part of the HLL because both hadrons which produced via the weak interaction are on their mass shells. We have calculated the branching ratio of $B^+ \to D^+ K^0$ decay by using QCDF and FSI. The experimental result of this decay is less than $2.9 \cdot 10^{-6}$. According to QCDF and FSI, our results are $\text{BR}(B^+ \to D^+ K^0) = (0.16 \pm 0.04) \cdot 10^{-6}$ and $(2.8 \pm 0.09) \cdot 10^{-6}$, respectively.

We have introduced the phenomenological parameter η that its value in the form factor is expected to be of the order of unity and can be determined from the measured rates. For a given exchanged particle, we have used $\eta = 2 \sim 2.4$. If $\eta = 2.4$ is selected, the branching ratio of the $B^+ \to D^+ K^0$ decay approaches to the experimental bound.

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