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GAUSSIAN MANIFOLDS IN RANDOM MEDIA

E.B.Kolomeisky

Institute of Crystallography Academy of Sciences of the USSR
117333, Moscow

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A unifying picture of scaling properties of D-dimensional Gaussian manifolds embedded in d-dimensional random media is presented. It is demonstrated, in particular, that for the special case of uncorrelated disorder manifold is stretched for $D > 2d/(4+d)$. The phase transitions between different stretched states and from the Gaussian form to the stretched one are predicted for $2d/(2+d) < D < 2$ and $0 < D < 2d/(4+d)$, respectively.

The properties of various manifolds (e.g. interfaces, membranes, polymers and so on) interacting with random environment are the subjects of great importance in many branches of physics, chemistry and biology. Despite their practical relevance and intrinsic interest the theoretical understanding of these problems are still incomplete.

The aim of the present Letter is to give comprehensive description of the scaling properties of above objects using a Flory-like approach invented by Zhang [1] in different context.

We begin with the D-dimensional generalization of the Edwards model [2] for polymers with no account for the self-avoidance effects. Consider a configuration $h(\rho)$ of a D-dimensional manifold immersed in a d-dimensional random media. It can be described by the partion function

$$Z(h,\rho) = \exp(-F/T) = \int Dh \exp\{-\int d^D\rho(\frac{\Gamma}{2}| \nabla_D\, h(\rho)|^2 + V(h)/T)\} \qquad (1)$$

where F is the free energy, T is the temperature, Γ is the manifold stiffness, $V(h)$ is the random potential with zero mean which properties are specified by its correlation function $< V(h)V(0) >= \Delta^2 R_a(h)$, where Δ^2 is disorder strength, $R_a(h)$ is a function of characteristic width a (for scales $h < a$ random potential is strongly correlated and $R_a(h < a) = \text{const}$). The finiteness of a will play an important role, as seen below. In what follows we suppose that $R_a(h \geq a) = h^\alpha$. The problem formulated involves also another intrinsic scale l_d that follows simply from dimensional arguments. Indeed, for a manifold of linear size ρ and total size h the two terms in the Hamiltonian in (1) scale as $\Gamma\rho^{D-2}h^2$ and $\Delta\rho^D h^{\alpha/2}/T$ from which we can form a dimensionless coupling constant $g(h) = \Delta\rho^D h^{\alpha/2}/T\Gamma\rho^{D-2}h^2$. In the absence of defects manifold is Gaussian one, i.e. $\Gamma\rho^{D-2}h^2 \cong 1$. Substituting this expression in above determination of g we obtain

$$g = (h/l_d)^{[4D+\alpha(2-D)]/2(2-D)}; \qquad l_d \cong (T/\Delta\Gamma^{D/(2-D)})^{2(2-D)/[4D+\alpha(2-D)]} \qquad (2)$$

Note that $g(a)$ controls the dimensionless disorder strength.

For the special case $D = 1$ the partion function (1) obeys to the Schroedinger-like diffusion equation (with ρ being the time) and describes diffusion in a random environment (see [2,3] for review). Therefore, the comparison with some rigorous results will be possible.

We are interested in the scaling dependence of moments $< Z^n >$ and the manifold size $h \cong A\rho^\nu$. The free energy fluctuation ΔF is also of interest. There is a standard assumption that

$$\Delta F/T \cong \Gamma A^2 \rho^{D-2+2\nu}, \tag{2}$$

which follows from a dimensionality argument about the elastic term ($\Gamma\rho^D(h/\rho)^2 \cong \Gamma A^2\rho^{D-2+2\nu}$) in (1). Let us take the partion function (1) to the n-th power and average it over disorder

$$< Z^n >=< \exp(-nF/T) >= \int Dh \exp\{-(\Gamma/2)\int d^D\rho \sum_{i=1}^{n} |\nabla_D h_i|^2 +$$

$$+(\Delta^2/2T^2)\int\int d^D\rho d^D\rho' \sum_{i,j}^{n} R_a(|h_i - h_j|)\} \tag{3}$$

It describes n identical manifolds, or replicas, with mutual interaction $R_a(h)$. In what follows we shall calculate $< Z^n >$ with exponential accuracy omitting all the numerical factors. However, before actually tackling the above path integral let us demonstrate following Zhang [1] how to obtain exponent ν and prefactor A. Suppose (3) is of the form $\exp(Bn^\beta\rho^\gamma)$, i.e.

$$\int dFP(F)\exp(-nF/T) =< \exp(-nF/T) >= \exp(Bn^\beta\rho^\gamma) \tag{4}$$

then (4) can be interpreted as the Laplace transform of the probability distribution density $P(F)$ of the free energy of a single manifold in random media. Inverting (4) permits us to find the free energy distribution density

$$P(F) \cong \exp[-(\Delta F/T(\rho^\gamma B)^{1/\beta})^{\beta/(\beta-1)}],$$

where we put $< F >= 0$. The knowledge of $P(F)$ enables to determine the free energy fluctuation $\Delta F/T \cong (B\rho^\gamma)^{1/\beta}$. Comparing it with (2) we finally obtain

$$A^2 \cong B^{1/\beta}/\Gamma; \quad \nu = (2-D)/2 + \gamma/2\beta \tag{5}$$

The last identity was first obtained by Zhang [1]. From (3) we have estimate for the free energy of n replicas:

$$F_n/T \cong n(\Gamma\rho^{D-2}h^2 + \rho^D/\Gamma^{D/(2-D)}h^{2D/(2-D)}) - n^2\Delta^2\rho^{2D}h^\alpha/T^2, h \geq a \tag{6}$$

The first term is the elastic energy of distorted manifold, the second one (that is relevant for $0 < D < 2$ only) is an entropic repulsion among replicas confined into a well of characteristic size $h^{2,4}$. The third term is defect-induced interaction among replicas. For $\Delta^2 = 0$ minimization of (6) with respect to h leads naturally to the Gaussian manifold

$$h_G^2(\rho) \cong \rho^{2-D}/\Gamma. \tag{7}$$

For $\alpha > 0$ the third term in (6) corresponds to repulsion between replicas. In this situation minimum of (6) for $\rho \to \infty$ is determined by the first and the third terms: $h \cong (n\Delta^2\rho^{D+2}/T^2\Gamma)^{1/(2-\alpha)}$. This solution is valid in the range $0 \leq \alpha < 2$. Substituting it into (6) one obtains with the help of (5)

$$\ln < Z^n > \cong (\Delta^4/T^4\Gamma^\alpha)^{1/(2-\alpha)} n^{(4-\alpha)/(2-\alpha)} \rho^{(4D-D\alpha+2\alpha)/(2-\alpha)}, \tag{8}$$

$$h \cong (\Delta/\Gamma T)^{2/(4-\alpha)} \rho^{4/(4-\alpha)}. \tag{9}$$

However, (9) is not asymptotic. Indeed, for α positive $\nu > 1$, i.e. the Hamiltonian in (1) is unstable with respect to adding infinitely many gradient terms. However, this change in the partion function (1) appears only at $h \cong \rho$.

For α negative it is useful to make change $\alpha = -d$ to cover the case of δ -correlated disorder. If $2D/(2-D) > d$ and $0 < D < 2$, or $2d/(2+d) < D < 2$ Eq.(6) has a *single* minimum $h \cong max\{(T^2/\Delta^2 n\rho^D\Gamma^{D/(2-D)})^{(2-D)/[2D-d(2-D)]}, a\}$. If

$$(T^2/\Delta^2 n\rho^D\Gamma^{D/(2-D)})^{(2-D)/[2D-d(2-D)]} > a \tag{10}$$

straightforward algebra gives rise to results

$$\ln < Z^n > \cong \{(\Delta/T)^{4D}\Gamma^{Dd}(n\rho^D)^{4D-d(2-D)}\}^{1/[2D-d(2-D)]}, \tag{11}$$

$$h \cong \{(\Delta/T)^{2D}\Gamma^{d-2D}\}^{1/[4D-d(2-D)]}\rho \tag{12}$$

Note, that (11) coincides with available exact solution [5] for the special case $a = 0$, $d = D = n = 1$. If (10) is broken we obtain

$$\ln < Z^n > \cong \Delta^2 n^2 \rho^{2D}/T^2 a^d, \tag{13}$$

$$h \cong (\Delta/T\Gamma a^{d/2})^{1/2}\rho. \tag{14}$$

For $D = 1$ Eq.(13) was first obtained by Zeldovich et al [6].

Provided that $0 < D < 2d/(2+d)$ Eq.(6) has two minima $h_1 = h_G(\rho)$ (7) and $h_2 \cong a$. Substituting (7) into (6) we obtain

$$F_n/T \cong n - \Delta^2\Gamma^{d/2}T^{-2}n^2\rho^{2D-d(2-D)/2}$$

It is clear that the results depend on the sign of $4D-d(2-D)$. For $2d/(4+d) < D < 2d/(2+d)$ one finds

$$\ln < Z^n > \cong \Delta^2\Gamma^{d/2}T^{-2}n^2\rho^{2D-d(2-D)/2} \tag{15}$$

$$h \cong (\Delta\Gamma^{d/4-1}/T)^{1/2}\rho^{1-d(2-D)/8}. \tag{16}$$

Comparing (15) and (13) one concludes that above solution is stable only for scales $\rho < (\Gamma a^2)^{1/(2-D)}$. In the case of opposite inequality one goes back to (13), (14).

The solution (13), (14) is also stable for $0 < D < 2d/(4+d)$ if the inequality (10) takes place. Provided that (10) is broken the Gaussian manifold solution (7) is stable. For $D > 2$ expression (6) contains the only minimum $h \cong a$ and one goes back to results (13), (14). Let us discuss now how above replica results manifest themselves in the original random system, i.e. for $n \to 0$. The situation for $-d = \alpha > 0$, $D > 2$ and $2d/(4+d) < D < 2d/(2+d)$ is clear: the manifold is stretched according to (9) and (14) (with the crossover regime (16), (17) for $2d/(4+d) < D < 2d/(2+d)$), respectively. Other cases $0 < D < 2d/(4+d)$ and $2d/(2+d) < D < 2$ are connected with the inequality (10) that cannot be continued to $n = 0$ in a straightforward way. However, this problem has a simple solution. Indeed, the probability distribution function $P(F)$ in (4) is obtained for large ρ. Therefore, (4) in fact imposes a relation between the saddle- point value of n and ρ: for $n \to 0$, $\rho \to \infty$, one needs $Bn^{\beta}\rho^{\gamma} \cong 1$. The results which follow from this rule and (7), (10-14) are convenient to be represented in the terms of $g(a)$ (2)(for $D = 1$ similar results were obtained by Nattermann and Renz [3]).

For $2d/(2+d) < D < 2$ one predicts the phase transition between different stretched states: from (11), (12) (for $g(a) < 1$) to (13), (14) (for $g(a) > 1$).

For $0 < D < 2d/(4+d)$ the phase transition from the Gaussian manifold (7) (for $g(a) < 1$) to the stretched one (13), (14) (for $g(a) > 1$) takes place.

In conclusion, let us discuss the range of validity of the results obtained. First, the path integral (3) was estimated using the saddle-point approximation. Such approach differs from the rigorous solution only in sub-leading-order terms for large ρ. Moreover,this estimate was fulfilled with the help of a Flory-like arguments [1-3]. However such an approximation is essentially incontrolled one [2]. Therefore, an independent treatment is necessary to confirm (or reject) the Flory-type results. We believe that our method gives the correct (if not exact) values of exponents since it reproduces both the exact result [5] for $a = 0$, $d = D = n = 1$ and the results obtained independently for $D = 1$ [6].

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