Pis'ma v ZhETF, vol. 102, iss. 1, pp. 40 – 44

Interaction-induced merging of Landau levels in an electron system of double quantum wells

In memory of V.F.Gantmakher

A. A. Shashkin[+1)], *V. T. Dolgopolov*[+], *J. W. Clark*[*], *V. R. Shaginyan*[×], *M. V. Zverev*[∘∇], *V. A. Khodel*[*∘]

[+]*Institute of Solid State Physics, 142432 Chernogolovka, Russia*

[*]*McDonnell Center for the Space Science and Department of Physics, Washington University, MO 63130 St. Louis, USA*

[×]*Petersburg Nuclear Physics Institute, National Research Center "Kurchatov Institute", 188300 Gatchina, Russia*

[∘]*National Research Centre "Kurchatov Institute", 123182 Moscow, Russia*

[∇]*Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia*

Submitted 25 May 2015
Resubmitted 27 May 2015

We show that the disappearance of the chemical potential jumps over the range of perpendicular magnetic fields at fixed integer filling factor in a double quantum well with a tunnel barrier is caused by the interaction-induced level merging. The distribution function in the merging regime is special in that the probability to find an electron with energy equal to the chemical potential is different for the two merged levels.

DOI: 10.7868/S0370274X15130081

More than twenty years ago a topological phase transition that is related to the emergence of a flat portion of the single-particle spectrum $\varepsilon(k)$ at the chemical potential was predicted at $T = 0$ in strongly correlated Fermi systems [1–5]. In more vivid terms, this transition is associated with the band flattening or swelling of the Fermi surface (for recent reviews, see Refs. [6–8]). The flattening of the single-particle spectrum means that the probability to find a fermion with energy equal to the chemical potential depends on the fermion momentum k. The swelling of the Fermi surface is preceded by an unleashed increase of the quasiparticle effective mass m at the quantum critical point [9, 10].

The topological phase transition characterized by the unusual form of the distribution function is not the only non-trivial manifestation of fermion interactions in strongly correlated Fermi liquids. Another example is the merging of quantum levels in a Fermi system with discrete spectrum in which case the fillings of the two quantum levels at the chemical potential are different [11]. The merging of the spin- and valley-split Landau levels at the chemical potential has been detected near the quantum critical point in a clean strongly-interacting two-dimensional (2D) electron system in (100) Si metal–oxide–semiconductor field-effect transistors (MOSFETs) [12]. The fact that the merging detected is governed by the effective mass depending on electron density may create the impression that the level merging is a precursor of the swelling of the Fermi surface. As a matter of fact, the two effects are not always related to each other. The diverging effective mass is not necessary for the existence of the effect of level merging.

Here, we show that the disappearance of the chemical potential jumps over the range of perpendicular magnetic fields at fixed integer filling factor in a double quantum well with a tunnel barrier is caused by the interaction-induced level merging. The merging regime corresponds to the special form of the distribution function. In this case the probability to find an electron with energy equal to the chemical potential is different for the two merged levels.

To begin with, we assume that the weakly interacting two-dimensional electron system is subjected to a perpendicular magnetic field B. In the simplest case there are two equidistant ladders of quantum levels for spin up and down directions (see, e.g., Ref. [13]). Both the thermodynamic and kinetic properties of the electron system are determined by the position of the chemical potential relative to the quantum levels, which is in turn determined by the magnetic field and electron density n. The filling factor is equal to $\nu = n/n_0$, where

[1)]e-mail: shashkin@issp.ac.ru

$n_0 = eB/hc$ is the level degeneracy. When ν is fractional, the chemical potential is pinned to the partially filled quantum level. The probability to find an electron at the chemical potential is given by the fractional part of the filling factor and can be varied between zero and one. At integer filling factor there is a jump of the chemical potential. In experiment, the jump manifests itself as a minimum in the longitudinal electrical resistance in the Shubnikov–de Haas effect. The resistance minima in the (B, n) plane correspond to a Landau level fan chart like the one shown in Fig. 1.

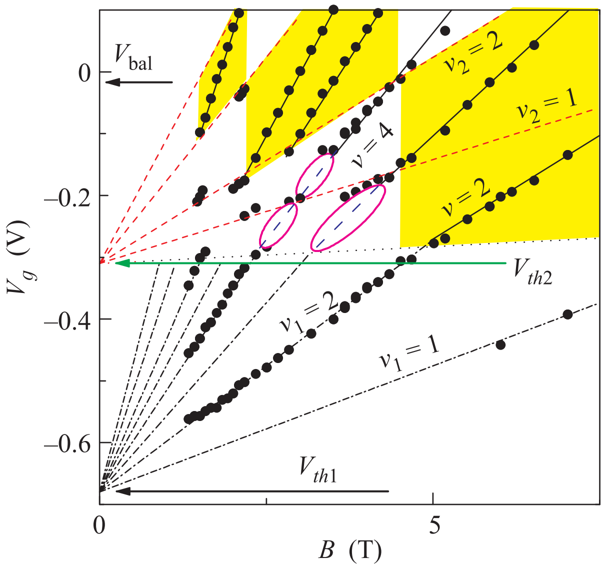

Fig. 1. Landau level fan chart for the double quantum well shown in Fig. 2. Positions of the longitudinal resistance minima in the (B, V_g) plane are marked by the dots. The filling factor ν for the double layer electron system as well as the filling factor ν_1 (ν_2) for the back (front) layer are indicated. Over the shaded areas, the merging of quantum levels in perpendicular magnetic fields is impossible. In the regions marked by the ovals, no resistance minima are observed in a perpendicular magnetic field, whereas these appear in a tilted magnetic field

If the magnetic field is tilted by an angle β, the spacing between the quantum levels in each of the spin ladders is equal to $\hbar\omega_c = \hbar eB\cos(\beta)/mc$, and the shift between the ladders equals $g\mu_B B$, where the Lande factor g is assumed to be isotropic, $\mu_B = e\hbar/2m_e c$ is the Bohr magneton, and m_e is the free electron mass. Increasing the tilt angle leads to crossing the quantum levels of the two ladders. The crossing happens for the first time at an angle β_1 that satisfies the condition

$$\cos(\beta_1) = \frac{gm}{2m_e}. \tag{1}$$

At $\beta = \beta_1$, the chemical potential jumps at even filling factors and the corresponding fan chart lines will disappear. This effect is well known and is used for the experimental determination of the product gm [14–16]. Note that in experiment, the chemical potential jumps should be absent at tilt angles in the vicinity of β_1, depending on the sample quality and temperature.

We now take into account the interaction between the electrons of neighboring quantum levels and increase the tilt angle in the vicinity of β_1. Tentatively, the quantum level filled before crossing should have got emptied with increasing β. However, if the single-particle energy of electrons on the emptying level decreases due to the electron interaction, both levels remain pinned to the chemical potential over a wide range of angles $\Delta\beta_1$ that is determined by the interaction strength. The probability to find an electron at the chemical potential is different for opposite spin orientations, being dependent on the external parameter which is the tilt angle. Such a behavior is known as the merging of quantum levels.

In the above hypothetical consideration the crossing or merging of quantum levels is controlled by the tilt angle of the magnetic field. In the experiments on a strongly-interacting 2D electron system in (100) Si MOSFETs, the disappearance of the longitudinal resistance minima is analyzed when changing both the perpendicular magnetic field and electron density at fixed filling factor $\nu = 4(i + 1)$, where i is an integer. In this case the level merging occurs near the quantum critical point, as controlled by the effective mass depending on electron density [12]. One might think that the level merging is a precursor of the Fermi surface swelling. In fact, the two effects are not necessarily related to each other. Below, we demonstrate that the effect of level merging occurs in a bilayer 2D electron system with a tunnel barrier between the electron layers. Note that although the Shubnikov–de Haas effect in similar double layer electron systems was investigated in a number of publications [17–21], only the level crossing was observed in Refs. [17, 20, 21].

The samples used contain a parabolic quantum well grown on a GaAs substrate, as shown schematically in Fig. 2. The width of the parabolic part of the well, limited by vertical walls, is about 760 Å. At the center of the well there is a narrow tunnel barrier that consists of three monolayers of $Al_xGa_{1-x}As$ ($x = 0.3$). The symmetrically doped structure is capped by 600 Å AlGaAs and 40 Å GaAs layers over which a metallic gate is evaporated. The presence of the tunnel barrier leads to a splitting of each subband bottom caused by quantization in the z direction. At the point of the symmetric electron density distribution, the splitting energy is

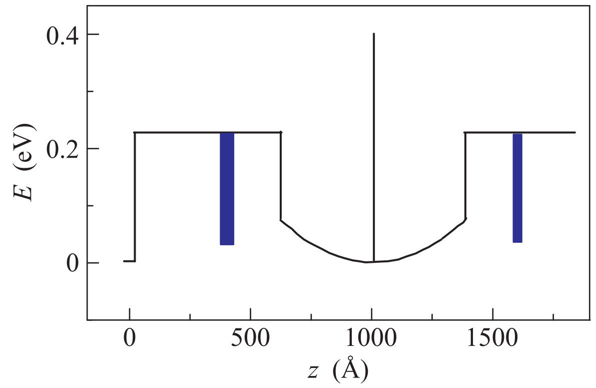

Fig. 2. (Color online) Schematic diagram of the bottom of the conduction band for the AlGaAs double quantum well in the absence of electrons. The parabolic part of the well is grown when varying the Al content from zero at the center to 0.1 on the edge of the well. The tunnel barrier at the center is created by three monolayers of $Al_xGa_{1-x}As$ ($x = 0.3$). The thick blue lines correspond to the silicon doped layers

equal to 1.3 meV. The structure of the quantum levels in the bilayer 2D electron system in perpendicular magnetic fields is similar to that in the 2D electron system in (100) Si MOSFETs, where the spin and valley splittings are present, with a distinction that the spin splitting Δ_z in accessible magnetic fields is the smallest (Fig. 3a).

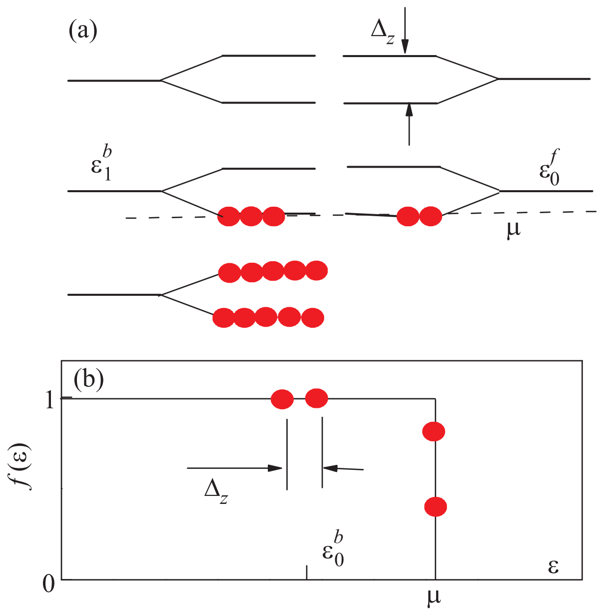

Fig. 3. (a) – Layout and filling of the quantum levels in the bilayer electron system in the merging regime at filling factor $\nu = 3$. (b) – The distribution function of the electrons in the merging regime at $\nu = 3$

Applying a voltage V_g between the gate and the contact to the quantum well makes it possible to tune the electron density. The electrons appear in the back part of the quantum well when the gate voltage is above $V_{th1} \approx -0.7$ V and occupy one subband up to $V_g = V_{th2} \approx -0.3$ V (Fig. 1). At $V_g > V_{th2}$, the electrons appear in the front part of the well and fill the second subband up to the balance point $V_g = V_{\mathrm{bal}} \approx 0$. This behavior is typical of the so-called "soft" double layer electron systems [18, 19, 22].

We focus on the range of gate voltages between V_{th2} and V_{bal} where the electrons in perpendicular magnetic fields occupy the two quantum ladders. Positions of the quantum levels are determined by the magnetic field and gate voltage. Over the shaded areas in Fig. 1, the gaps in the single-particle spectrum and the chemical potential jumps are protected by quantum effects [18]. In the remaining areas, the merging of quantum levels is in principle possible.

We now consider the filling factor $\nu = 3$. At $V_g = V_{th2}$, the magnetic field is equal to $B_\nu = n_1(V_{th2})hc/3e$, where $n_1(V_{th2})$ is the electron density in the back layer. The energy $\varepsilon_0 - g\mu_{\mathrm{B}}B_\nu/2$ of the spin up level of the front layer ladder is the same as the energy $\varepsilon_1 - g\mu_{\mathrm{B}}B_\nu/2$ of the spin up level of the back layer ladder (Fig. 3a). Since far from the balance point, the electron density in the back layer remains practically unchanged with increasing V_g above V_{th2} (see, e.g., Fig. 1b of Ref. [17]), the electron density in the front layer in a magnetic field $B = B_\nu + \Delta B$ along the dashed line at $\nu = 3$ restricted by the oval in Fig. 1 is equal to $n_2 \simeq \Delta n = 3e\Delta B/hc$. To balance the change in the cyclotron energy $\hbar e\Delta B/mc$ and have both levels pinned to the chemical potential μ, it is necessary to transfer a small amount of electrons between the levels

$$\delta n_2 = -\delta n_1 > 0, \tag{2}$$

which gives rise to a shift of the single-particle levels

$$\begin{aligned} \delta\varepsilon_1^b &= \Gamma_1^{bb}\delta n_1 + \Gamma_1^{bf}\delta n_2, \\ \delta\varepsilon_0^f &= \Gamma_0^{fb}\delta n_1 + \Gamma_0^{ff}\delta n_2, \end{aligned} \tag{3}$$

where $\Gamma_j^{\lambda\sigma}$ is the electron interaction amplitude, the index "b" ("f") refers to the back (front) layer, and $j = 0, 1$ is the Landau level number. Both levels are pinned to the chemical potential under the condition $\delta\varepsilon_0 - \delta\varepsilon_1 = \hbar e\Delta B/mc$, which yields

$$\delta n_2 = \frac{\hbar e\Delta B}{mc\Gamma}, \tag{4}$$

where $\Gamma = \Gamma_1^{bb} + \Gamma_0^{ff} - \Gamma_1^{bf} - \Gamma_0^{fb}$. In the plate-capacitor approximation, we get

$$\Gamma \simeq \frac{4\pi e^2 a}{\epsilon}, \tag{5}$$

where a is the distance between the weight centers of the electron density distributions in the z direction in both subbands. The level merging holds for the filling factor

$$\nu_2 = \frac{(n_2 + \delta n_2)hc}{eB} < 1. \tag{6}$$

We stress that in the wide range of magnetic fields at fixed filling factor ν, the probability to find an electron with energy equal to the chemical potential is different for the two merged levels, as shown in Fig. 3b.

It is easy to verify that the pinning of the levels ε_0^f and ε_1^b to the chemical potential is energetically favorable. The variation of the system energy δE caused by electron density redistribution is given by the Fermi liquid relation

$$\delta E = (\varepsilon_1^b - g\mu_{\mathrm{B}} B/2)\delta n_1 + (\varepsilon_0^f - g\mu_{\mathrm{B}} B/2)\delta n_2 + \\ + \frac{1}{2}(\Gamma_1^{bb}\delta n_1^2 + \Gamma_0^{ff}\delta n_2^2 + \Gamma_1^{bf}\delta n_1 \delta n_2 + \Gamma_0^{fb}\delta n_2 \delta n_1). \tag{7}$$

Using Eq. (2), one gets

$$\delta E = -\frac{\hbar e \Delta B}{mc}\delta n_2 + \frac{1}{2}\Gamma \delta n_2^2. \tag{8}$$

The redistribution (4) yields the negative shift in the energy of the system

$$\delta E = -\frac{1}{2}\frac{\hbar e \Delta B}{mc}\delta n_2. \tag{9}$$

Let the variation $\delta' n_2$ be different from δn_2 of Eq. (4). Substituting $\delta' n_2$ for δn_2 in Eq. (8), one obtains the energy shift

$$\delta' E = \delta E + \frac{1}{2}\Gamma(\delta' n_2 - \delta n_2)^2. \tag{10}$$

Obviously, the redistribution (4) corresponding to the merging of the levels ε_0^f and ε_1^b provides the maximum gain in the system energy.

For the sake of simplicity, we have used in our argumentation the case of filling factor $\nu = 3$. Still, the same arguments are valid for higher filling factors.

The occurrence of the merging of quantum levels in the experiment is confirmed by using tilted magnetic fields. With tilting magnetic field the magnetoresistance minima and chemical potential jumps arise [19] particularly along the dashed lines at $\nu = 3$ and 4 indicated by the ovals in Fig. 1. The appearance of the chemical potential jumps in the double layer electron system in tilted magnetic fields signals that the quantum levels are narrow enough.

As has been mentioned above, the chemical potential jumps can be protected by quantum effects. In general, a transfer of electrons between the quantum levels of different subbands leads to mixing the wave functions of the subbands and opening an energy gap if the non-diagonal matrix elements are not equal to zero [18]. This is realized over the shaded areas in Fig. 1. In contrast, in the merging regions at $\nu = 3$ and 4 indicated by the ovals in Fig. 1, the non-diagonal matrix elements in perpendicular magnetic fields equal zero because of orthogonality of the in-plane part of the wave functions in the bilayer electron system. Tilting the magnetic field breaks the orthogonality of the wave functions of the neighboring quantum levels, and the energy gap emerges [19, 21].

In a number of publications [17, 20, 21], the so-called ring-like structures were revealed by transport measurements in double layer or double subband electron systems. The ring-like structures corresponding to the maximum resistance are topologically equivalent to the regions in between the data points in Fig. 1 with a distinction that at fixed integer filling factor ν, the latter regions are considerably wider, as expected.

It is easy to formulate the condition when the merging of quantum levels is observed. For the level merging to occur in a wide range of magnetic fields, the value δn_2 of Eq. (4) must be small compared to n_2, which gives an estimate for Γ

$$\Gamma \gg \frac{2\pi\hbar^2}{3m}. \tag{11}$$

Using the evaluation of Eq. (5), we arrive at the condition of the level merging

$$\alpha = \frac{a}{a_{\mathrm{B}}} \gg 1, \tag{12}$$

where a_{B} is the effective Bohr radius. It is the relation that describes the softness of a bilayer electron system, i.e., the sensitivity of the subband spacing to intersubband electron transfer.

It is worth noting that in experiments, the quantum level width and temperature are always finite. The merging at finite temperatures implies that the single-particle energies of the merged levels have an energy difference $\lesssim k_{\mathrm{B}}T$ so that the levels converge with decreasing temperature. On the other hand, the value δn_2 is restricted from below: $S\delta n_2 \geq 1$, where S is the sample area. The finite value of δn_2 means that the energy difference between the levels at filling factor $\nu = 3$ exceeds Γ/S. This determines the characteristic temperature

$$T^* \simeq \frac{\Gamma}{k_{\mathrm{B}}S}. \tag{13}$$

Using the realistic values $a = 300\,\text{Å}$ and $S = 10 \times 10\,\mu\text{m}^2$, one estimates $T^* \sim 5\,\text{mK}$. While at $T \gg T^*$ the electron system with the merged levels is characterized by the conventional Fermi distribution, in the opposite limit $T \ll T^*$ (and the small level width compared to $k_\text{B}T^*$) the distribution function has non-Fermi liquid form.

For the non-Fermi distribution function, the finite entropy persisting down to very low temperatures threatens violation of the Nernst theorem. Conventionally, it is supposed that as $T \to 0$, shedding the paradoxical entropy excess occurs by virtue of one of the possible mechanisms discussed in Ref. [7]. For example, in case of Cooper-pair formation, the gap $\Delta(T)$ in the spectrum ensures vanishing entropy $S(T \to 0) \propto \exp[-\Delta(0)/k_\text{B}T]$. Assuming that the magnetic field is weak enough to save the superconductivity, the critical temperature T_c of the possible Cooper transition for our case can be evaluated at $T_c \sim T^*/\ln(\alpha)$.

In summary, we have shown that the merging of quantum levels is not necessarily a precursor of the Fermi surface swelling. We demonstrate the occurrence of the level merging in a soft double layer electron system in perpendicular magnetic fields. The distribution function in the merging regime has non-Fermi liquid form when the probability to find an electron with energy equal to the chemical potential is different for the two merged levels.

This work was supported in part by RFBR (projects # 15-02-03537, 13-02-00095, 15-02-06261, and 14-02-00044), RAS, the Russian Ministry of Sciences (project # NS-932.2014.2), RCSF (project # 14-12-00450), the U.S. DOE, Division of Chemical Sciences, the Office of Basic Energy Sciences, the Office of Energy Research, AFOSR, and the McDonnell Center for the Space Sciences.

1. V. A. Khodel and V. R. Shaginyan, JETP Lett. **51**, 553 (1990).
2. G. E. Volovik, JETP Lett. **53**, 222 (1991).
3. P. Nozi'eres, J. Phys. I France **2**, 443 (1992).
4. V. A. Khodel, J. W. Clark, and M. V. Zverev, Phys. Rev. B **78**, 075120 (2008).
5. D. Yudin, D. Hirschmeier, H. Hafermann, O. Eriksson, A. I. Lichtenstein, and M. I. Katsnelson, Phys. Rev. Lett. **112**, 070403 (2014).
6. M. Ya. Amusia, K. G. Popov, V. R. Shaginyan, and V. A. Stephanovich, *Theory of Heavy Fermion Compounds*, Springer, Berlin (2014).
7. J. W. Clark, M. V. Zverev, and V. A. Khodel, Ann. Phys. **327**, 3063 (2012).
8. M. V. Zverev, V. A. Khodel, and S. S. Pankratov, JETP Lett. **96**, 192 (2012).
9. A. A. Shashkin, S. V. Kravchenko, V. T. Dolgopolov, and T. M. Klapwijk, Phys. Rev. B **66**, 073303 (2002).
10. V. T. Dolgopolov, Pis'ma v ZhETF **101**, 300 (2015).
11. V. A. Khodel, J. W. Clark, H. Li, and M. V. Zverev, Phys. Rev. Lett. **98**, 216404 (2007).
12. A. A. Shashkin, V. T. Dolgopolov, J. W. Clark, V. R. Shaginyan, M. V. Zverev, and V. A. Khodel, Phys. Rev. Lett. **112**, 186402 (2014).
13. V. T. Dolgopolov, Phys. Usp. **57**, 105 (2014).
14. T. Ando, A. B. Fowler, and F. Stern, Rev. Mod. Phys. **54**, 437 (1982).
15. S. V. Kravchenko, A. A. Shashkin, D. A. Bloore, and T. M. Klapwijk, Sol. State Com. **116**, 495 (2000).
16. V. M. Pudalov, M. E. Gershenson, H. Kojima, N. Butch, E. M. Dizhur, G. Brunthaler, A. Prinz, and G. Bauer, Phys. Rev. Lett. **88**, 196404 (2002).
17. A. G. Davies, C. H. W. Barnes, K. R. Zolleis, J. T. Nicholls, M. Y. Simmons, and D. A. Ritchie, Phys. Rev. B **54**, R17331 (1996).
18. V. T. Dolgopolov, A. A. Shashkin, E. V. Deviatov, F. Hastreiter, M. Hartung, A. Wixforth, K. L. Campman, and A. C. Gossard, Phys. Rev. B **59**, 13235 (1999).
19. E. V. Deviatov, V. S. Khrapai, A. A. Shashkin, V. T. Dolgopolov, F. Hastreiter, A. Wixforth, K. L. Campman, and A. C. Gossard, JETP Lett. **71**, 496 (2000).
20. X. C. Zhang, G. D. Scott, and H. W. Jiang, Phys. Rev. Lett. **98**, 246802 (2007).
21. C. A. Duarte, G. M. Gusev, A. A. Quivy, T. E. Lamas, A. K. Bakarov, and J. C. Portal, Phys. Rev. B **76**, 075346 (2007).
22. A. Gold, Z. Phys. B **95**, 341 (1994).