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Numerical simulation of positive streamer development in thundercloud field enhanced near raindrops

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As the threshold field strength for the breakdown in air significantly exceeds the maximum measured thundercloud strength 3 kV/cm/atm, the problem of lightning initiation remains unclear. According to the popular idea, lightning can be initiated from streamer discharges developed in the enhanced electric field in a vicinity of hydrometeors. To test the idea, we carry out numerical simulations of positive streamer development around charged water drops at air pressure typical at thundercloud altitudes and at different background fields, drop sizes and charges. With real drop sizes and charges, the electric field required for the streamer formation is stronger than the measured fields; therefore, second mechanism is required to amplify the local field.

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1. Introduction. A problem of lightning initiation is not resolved till now despite a long history of research. A two-stage initiation process is commonly accepted (e.g. [1]). Initially, the thundercloud electric field strength is to be locally increased to the breakdown magnitude E_{br}, above which electron avalanches, transforming in streamers, develop. The reduced to the pressure P_g threshold field strength for dry air is $E_{br}/P_g \approx \approx 26\,\mathrm{kV/cm/atm}$ [2], scaling to about 10 kV/cm at 0.4 atm typical at thundercloud altitudes. During the second stage, numerous streamers form the streamer corona, a current of which heats the air and a hot leader channel is developed.

The first stage of lightning initiation was analyzed in [3–13]. Obviously, to initiate lightning, the field strength at least locally must be above the E_{br}/P_g, but the E/P_g magnitudes measured in thunderclouds do not exceed 3–4 kV/cm/atm [14–16]. However, in balloon measurements, field sensors cannot resolve scales below ~ 1 m and a temporal resolution is far from the requirements for detection of the transient fields. Hence, for the streamer initiation, one may look for processes at scales below about 1 m and 1 μs.

The most popular is an idea that the thundercloud field is enhanced in the vicinity of hydrometeors as a result of their polarization or (and) accumulated electric charge [3–6]. A number of numerical simulations have been carried out to study this mechanism.

1D simulations by Solomon et al. [8] demonstrated that streamers could be initiated during collisions of mm drops in the field of 4 kV/cm/atm and from ice needles in the field of 22 or of 18 kV/cm/atm if the needle is charged up to 100 pC.

Sadighi [11] carried out 2D simulations of streamer initiation from uncharged mm-scale hydrometeors: spherical ice hydrometeor and columniform water filament in the field $0.9\,E_{br}$ and $0.8\,E_{br}$, respectively. The discharge is initiated by a plasma cloud. In both cases, a stable streamer discharge develops. Sadighi et al. [12] study streamer formation in the field of $0.3\,E_{br}$ from hydrometeor modeled by an ionized column. The authors conclude that the minimum column length required for the streamer initiation is from 5 to 8 mm with a background electron density of $10^{15}\,\mathrm{m}^{-3}$, which is to be non-uniform; otherwise the streamer branches.

Dubinova et al. [13] presented a 2D model of positive streamers from uncharged ice hydrometeors. The authors assume density $100\,\mathrm{cm}^{-3}$ of seed electrons to start the streamer process; this large density is hypothesized to be a result of a cosmic ray shower. Too weak criterion for the avalanche-to-streamer transition was adopted, namely, a number in the right side of Eq. (8) (see below) was set be 10. The simulations demonstrate that a streamer moving with an average velocity of 10^5 m/s can be generated at fields of $0.15 E_{br}$ for 6 cm long meteors.

In our paper, we carry out 2D numerical simulations to compute magnitudes of thundercloud electric field strength required for the streamer initiation from *charged raindrops of observed sizes and charges* at the mm and ns scales.

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2. Mathematical formulation and simulation approach. The kinetics of electrons (e), positive (p), and negative (n) ions in a streamer discharge is described by the conventional set of equations:

$$\frac{\partial n_e}{\partial t} + \operatorname{div}\mathbf{j}_e = (\alpha_{\rm ion} - \alpha_{\rm att}) \cdot |\mathbf{j}_e| - \beta_{ep} \cdot n_e \cdot n_p + S_{ph},$$

$$\frac{\partial n_p}{\partial t} + \operatorname{div}\mathbf{j}_p = \alpha_{\rm ion} \cdot |\mathbf{j}_e| - \beta_{ep} \cdot n_e \cdot n_p - \beta_{pn} \cdot n_p \cdot n_n + S_{ph},$$

$$\frac{\partial n_n}{\partial t} + \operatorname{div}\mathbf{j}_n = \alpha_{\rm att} \cdot |\mathbf{j}_e| - \beta_{pn} \cdot n_p \cdot n_n \qquad (1)$$

with the boundary condition on the ellipsoid surface, G_{dr}:

$$\mathbf{J}_{dr}(z,r) = \mathbf{J}_{\rm dis}(z,r), \quad (z,r) \in G_{dr}. \qquad (2)$$

Here n_e, n_p, n_n are the particles number densities, $\mathbf{j}_e = -\mu_e \mathbf{E} - D_e \boldsymbol{\nabla} n_e$, $\mathbf{j}_p = \mu_p \mathbf{E}$, $\mathbf{j}_n = -\mu_n \mathbf{E}$ are the flux densities, $\mathbf{J}_{\rm dis} = e \cdot (\mathbf{j}_p - \mathbf{j}_n - \mathbf{j}_e)$ is the discharge current density on the ellipsoid surface, e is the elementary charge, μ_e and $\mu_{p,n}$ are the electron and ion mobilities, D_e is the electron diffusion coefficient, $\alpha_{\rm ion}$ is the coefficient of ionization by electron impacts, $\alpha_{\rm att}$ is the coefficient of electron attachment to oxygen molecules, β_{ep} and β_{pn} are the coefficients of recombination of electrons and positive ions and positive and negative ions, S_{ph} is a source of the photoionization. Though the latter is weak in comparison with the impact ionization but it permits anti-force streamer propagation. The S_{ph}, μ_e, $\mu_{p,n}$, and D_e as functions of the E/P are determined using BOLSIG+ [17], the other coefficients are as in [18]. The S_{ph} is computed using a model by Bourdon et al. [19].

We investigate a formation of positive streamers around water drops with a positive charge Q_{dr} in an external electric field. The downward falling drop can be modeled by rotation ellipsoid with a minor semi-axis, R_{dr}, and major semi-axis, $L_{dr}/2$, aligned with an external uniform electric field $\mathbf{E}_{\rm ext}(z,r) = E_{\rm ext} \cdot \mathbf{e}_z$. The vector $\mathbf{e}_z$ is downward directed because in typical thunderclouds the positive charge is located in the cloud top and the negative charge in the bottom. The problem is solved in cylindrical coordinates (z,r) with downwards directed axis z.

Because of the drop polarization, the positive charge is concentrated at the ellipsoid bottom and the negative at the top. The charge motion in the drop is described by equations:

$$\frac{\partial \rho_{dr}}{\partial t} + \operatorname{div}\mathbf{J}_{dr} = 0; \quad \mathbf{J}_{dr} = \sigma_{dr}\mathbf{E} \qquad (3)$$

with boundary conditions on the border of the computation domain, $G_{\rm sim}$:

$$\mathbf{j}_e(z,r) = \mathbf{j}_p(z,r) = \mathbf{j}_n(z,r) = 0, \quad (z,r) \in G_{\rm sim}. \qquad (4)$$

Here $\mathbf{J}_{dr}$, ρ_{dr}, σ_{dr} are the current density, charge density, and electrical conductivity in the drop. The raindrop conductivity is within 0.5 and $10\,\mathrm{mS\,m^{-1}}$ [20]; we use:

$$\sigma_{dr}(z,r) = \begin{cases} 10\,\mathrm{mS/m}, & (z,r) \in D_{dr}, \\ 0, & (z,r) \notin D_{dr}. \end{cases} \qquad (5)$$

Here D_{dr} is the space domain occupied by the ellipsoid (drop).

The equations for the self-consistent electric field are as follows:

$$\operatorname{div}(\varepsilon \boldsymbol{\nabla} \varphi_{\rm int}) = -\frac{\rho_{dr}}{\varepsilon_0} - \frac{\rho_{\rm dis}}{\varepsilon_0}; \quad \mathbf{E} = -\boldsymbol{\nabla}\varphi_{\rm int} + \mathbf{E}_{\rm ext}, \qquad (6)$$

where $\varphi_{\rm int}$ is the electric potential of the space charge field, $\rho_{\rm dis} = e \cdot (n_p - n_n - n_e)$ is the charge density in the avalanche/streamer, ε_0 is the dielectric permittivity of the free space and ε is the medium dielectric permittivity: $\varepsilon(z,r) = 80$ if $(z,r) \in D_{dr}$ and $\varepsilon(z,r) = 1$ if $(z,r) \notin D_{dr}$.

The potential $\varphi_{\rm int}$ on the border of the simulation region is determined from Poisson's equation:

$$\varphi_{\rm int}(z,r) = \frac{1}{4\pi\varepsilon_0} \times \qquad (7)$$

$$\times \int\limits_{D_{\rm sim}} \frac{[\rho_{dr}(z',r') + \rho_{\rm dis}(z',r')] \cdot 2\pi r' dr' dz'}{\sqrt{(z-z') + (r-r')}}, \ (z,r) \in G_{\rm sim}.$$

The avalanche-to-streamer transition occurs when the avalanche space charge field becomes comparable to the background field $E_{\rm ext}$ [2]. As in our case, the field is strongly inhomogeneous this criterion can be written as follows:

$$\int\limits_{L_{dr}/2} \{\alpha_{\rm ion}[E(z), P_g] - \alpha_{\rm att}[E(z), P_g)]\} dz = 20, \qquad (8)$$

where z_{in} is the coordinate at which $\alpha_{\rm ion} = \alpha_{\rm att}$.

The minimum value of the field intensity at which a cathode-directed (positive) streamer can develop in air at STP is 4.65 kV/cm [21]. We therefore chose to simulate the reduced field intensities $E_{\rm ext}/P_g = 5$, 10, 15 and 20 kV/cm/atm at $P_g = 0.4$ atm (altitude of 8 km).

At fixed drop dimensions (R_{dr}, L_{dr}) and in a given $E_{\rm ext}$, the drop charge, Q_{dr}, is the unique parameter defining the field distribution. To find the required Q_{dr}, for which the condition (8) will be satisfied, the drop polarization in the external field was found simulating Eqs. (3) and (6) with the initial conditions:

$$\rho_{dr}(z,r,0) = Q_{dr}/V_{dr}, \quad \mathbf{E}(z,r,0) = \mathbf{E}_{\rm ext}, \qquad (9)$$

where V_{dr} is the drop volume. The simulations were run until the stationary field distribution. The various configurations for which condition (9) is satisfied are shown in Table 1. The Q_{dr} values are in the range of measured charge on particles with 1–3 mm diameter inside thunderclouds, which are between 10 and 200 pC with a few particles having charges from 200 to 400 pC [22].

Table 1. Drop charge Q_{dr}, ensuring validity of Eq. (9), and streamer velocity $v_f^{*)}$

R_{dr}, mm	L_{dr}, mm	E_{ext}/P_g, kV/cm/atm	Q_{dr}, pC	v_f, m/s
0.75	1.5	5	400	$< 1.2 \cdot 10^4$
		10	355	$1.4 \cdot 10^5$
		15	315	$2.8 \cdot 10^5$
0.5	1.0	5	220	$< 6.9 \cdot 10^3$
		10	200	$1.1 \cdot 10^5$
		15	178	$2.2 \cdot 10^5$
0.25	0.5	5	80	$< 1.4 \cdot 10^4$
		10	75	$6.8 \cdot 10^4$
		15	69	$1.3 \cdot 10^5$
0.75	3.0	5	485	$< 1.3 \cdot 10^4$
		10	405	$1.5 \cdot 10^5$
		15	340	$2.8 \cdot 10^5$
0.5	2.0	5	270	$< 7.7 \cdot 10^3$
		10	235	$8.6 \cdot 10^4$
		15	200	$2.1 \cdot 10^5$
0.25	1.0	5	103	$< 3.0 \cdot 10^3$
		10	93	$8.4 \cdot 10^4$
		15	84	$1.4 \cdot 10^5$

*) The sign "<" means that v_f not reach a stationary value.

Eqs. (1), (3), and (6) are solved using a homogeneous square spatial mesh with steps $\Delta z = \Delta r = 2.5 \cdot 10^{-6}$, $5 \cdot 10^{-6}$, and $5 \cdot 10^{-6}$ m for $R_{dr} = 0.25$, 0.5, and 0.75 mm, respectively, and $\Delta t = 0.2$ ps. The stationary electric field around the charged ellipsoid at $t = 0$ is determined as described in the previous section. The time step used here is $\Delta t = 0.1\tau_m$, where $\tau_m = (\varepsilon_{\text{water}}\varepsilon_0)/\sigma_{dr} \approx 70$ ns is the Maxwellian electric field relaxation time.

In the atmosphere, cosmic rays and radon decay daughters permanently produce free electrons. To start an electron avalanche at least one free electron is required, however, the initial conditions $n_e(z, r, 0) = n_p(z, r, 0) = \delta(z - z_{in})$, fitting one-electron initiation, lead to too large electron diffusion. To avoid this difficulty we set $N_e^0(r, z, t) = N_e^0(0, z_{st}, 0)$ with z_{st} computed from the equation:

$$\int_{L_{dr}/2}^{z_{st}} \{\alpha_{\text{ion}}[E(z), P_g] - \alpha_{\text{att}}[E(z), P_g]\} dz = 20 - \ln(N_e^0). \tag{10}$$

This condition allows simply not following the earliest stage of the avalanche. As a result, the initial conditions for (1) are as follows:

$$\begin{gathered} n_e(z, r, 0) = n_e^0 \cdot \delta(z - z_{st}), \\ n_p(z, r, 0) = n_e(z, r, 0), \\ n_n(z, r, 0) = 0, \end{gathered} \tag{11}$$

with $N_e^0(0, z_{st}, 0) = 100$; larger values of N_e^0 do not significantly alter the results. Simulations are run until $t_{\text{run}} = 40$ ns or until the streamer front reaches the boundary S_{sim}.

3. Results. We define the streamer front position as the location of the electric field maximum. In all the simulated configurations, the streamer front passed $z = z_{in}$, i.e. entered into the domain where at the initial moment of time $\alpha_{\text{ion}} < \alpha_{\text{att}}$. The distributions of the field intensity and electron number density are typical for streamers as illustrated in Fig. 1 for one configuration. It is seen, that the radius of the streamer channel is growing, reaching ≈ 0.25 mm. The plot is typical for all configurations in Table 1.

As the streamer propagates, the negative charge, flowing from the streamer channel to the drop, reduces the positive drop charge and the field around the drop. The question is if a streamer will continue propagating after the positive drop charge is annihilated or after the streamer front leaves the domain of enhanced field. In Fig. 2, it is seen that the positive drop charge disappears at $t \approx 15$ ns; afterwards the drop very slowly accumulates negative charge. The field intensity at the streamer front achieves a stationary magnitude $E_f \approx 62$ kV/cm at $t \approx 12$ ns. The electron number N_e grows linearly after $t \approx 8$ ns. The streamer velocity initially decreases and approaches a stationary magnitude $v_f \approx 1.1 \cdot 10^5$ m/s towards the end of the run, which is consistent with the minimum value of the streamer

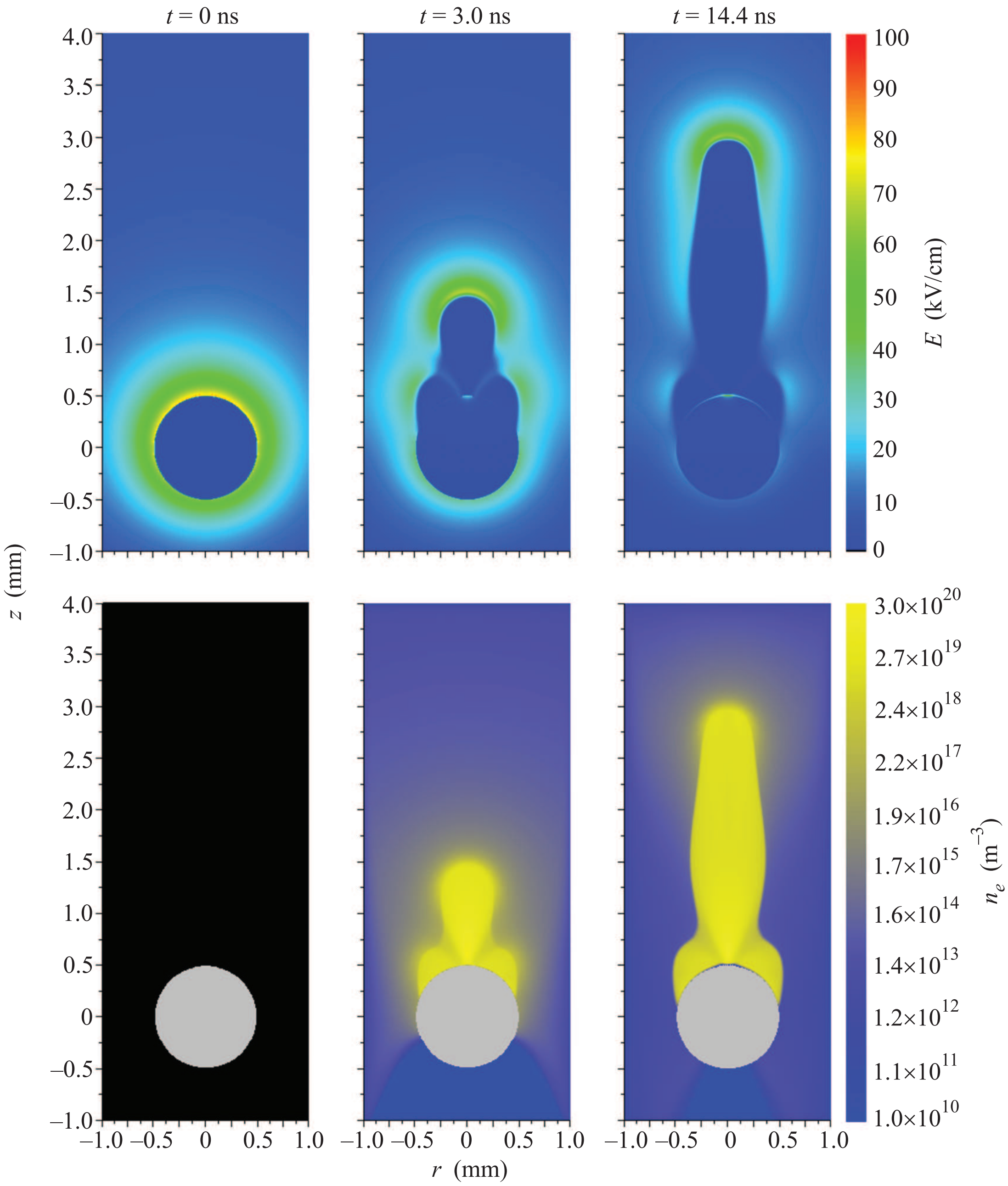

Fig. 1. (Color online) Two-dimensional distribution of the field intensity (top) and electron number density (bottom) at various moments of time; $R_{dr} = 0.5$ mm, $L_{dr} = 1.0$ mm, $E_{\text{ext}}/P_g = 10$ kV/cm/atm.

speed at STP $v_{st,\min} \approx (1.5{-}2) \cdot 10^5$ m/s; streamers with smaller speeds were never observed [21]. Noting that our simulations were carried out for $P_g = 0.4$ atm, and in view of some uncertainty in $v_{st,\min}$, we conclude that for the configuration shown in Fig. 1, the streamer will continue propagating.

The final streamer velocity for all the simulated configurations is given in Table 1. It is seen that for all configurations with $E_{\text{ext}}/P_g = 5$ kV/cm/atm the velocity v_f is less than $v_{st,\min}$. In addition, we find that at some time, the number of electrons begins to decrease (not shown). This means that streamers are not initiated at this E_{ext}/P_g. The conclusion follows from our model that stable propagation of streamers requires higher fields. Adopting the criterion that $v_f > v_{st,\min}$ we find that streamer initiation from drops with $R_{dr} \geq 0.5$ mm and $R_{dr} \geq 0.25$ mm is possible in fields $E_{\text{ext}}/P_g \geq 10$ and 15 kV/cm/atm, respectively.

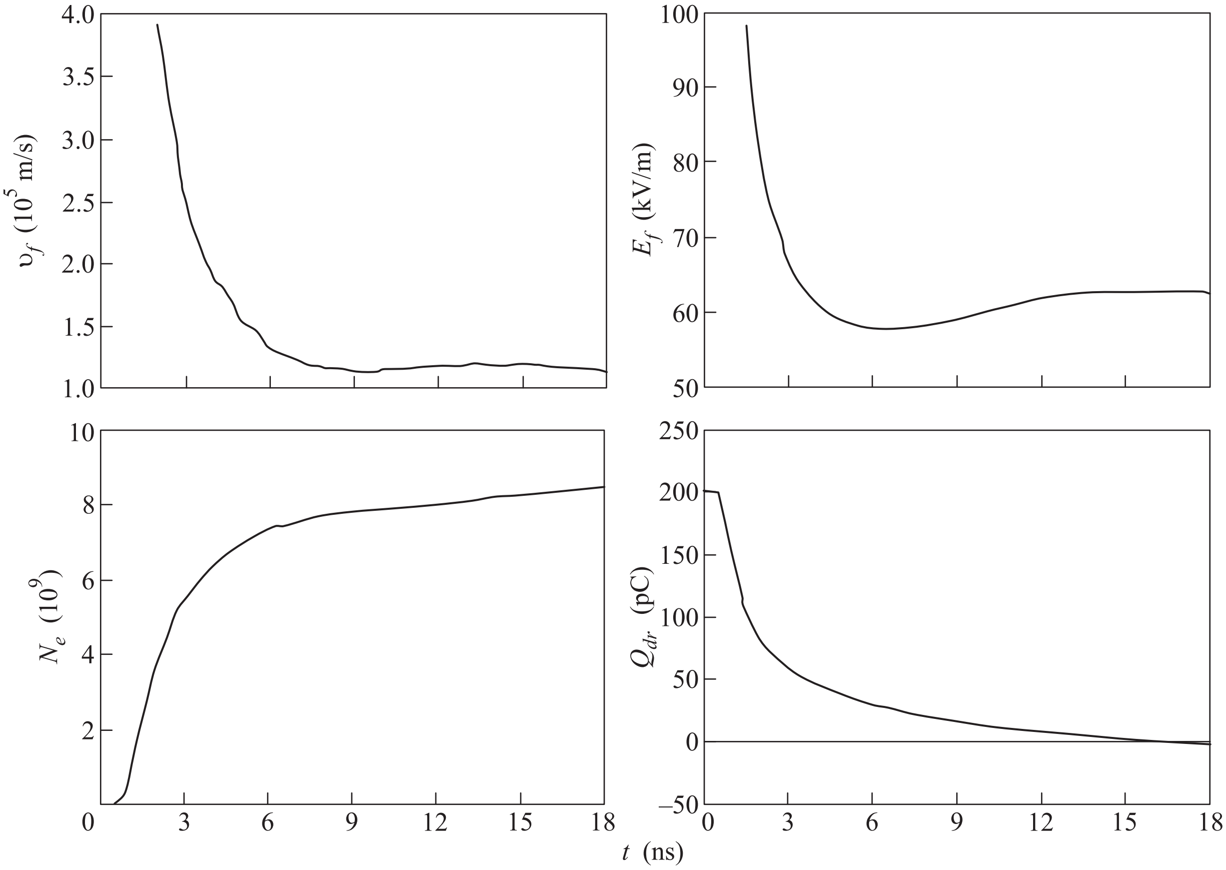

Fig. 2. Time dependencies of the streamer velocity v_f, the field intensity at the streamer front E_f, the number of electrons in the discharge N_e, and the drop charge Q_{dr}; $R_{dr} = 0.5$ mm, $L_{dr} = 1.0$ mm, $E_{\text{ext}}/P_g = 10$ kV/cm/atm

4. Discussions and conclusions. In contrast to the most published studies, focused on uncharged hydrometeors as a source of positive streamer origin, we simulated streamer initiation in a self-consistent electric field in a vicinity of a *charged* raindrop. Unlike [12], we did not require enhanced pre-ionization and started the simulation with a condition fitting one-electron initiation. We used quite realistic drop sizes and charge unlike in [13]. We found that avalanche-to-streamer transition is possible if the drop charge is in the range $Q_{dr} = (63-485)$ pC. For drops with sizes $R_{dr} \geq 0.5$ mm and $R_{dr} \geq 0.25$ mm streamers can develop in fields with reduced strength magnitudes $E_{\text{ext}}/P_g \geq 10$ and 15 kV/cm/atm, respectively, corresponding to $E_{\text{ext}} \approx \approx 0.31E_{br}$ and $\approx 0.47E_{br}$ ($E_{br} = 32$ kV/cm) at 1 atm.

Another proposed process for thundercloud field intensification is connected with the Relativistic Runaway Electron Avalanche (RREA) seeded by cosmic ray air showers [7]. Numerical simulations [10], however, demonstrate that the achievable field is of 8.5 kV/cm/atm and that the external field must be of 4 kV/cm/atm over a length of 2 km, which surpasses a fundamental limit of the field size and extension by Dwyer [23]. The stationary background cosmic radiation can also be a source of RREAs [10]. An RREA may lead to a formation of a conducting channel sprouting downwards from the upper negative cloud region. During the channel development, the field at its front is amplified and for a time of 10 s reaches values close to the E_{br} in a region of 10 m [10]. This requires rather large rates of the cloud electrification 2–13 C/s. With more realistic rates 0.3–0.6 C/s the maximum value of the field strength is 6–16 kV/cm/atm [10, 18].

To avoid difficulty of requirement of too high E_{ext}/P_g magnitudes, our proposition is that streamers in thunderclouds originate near hydrometeors in local regions where the ambient thundercloud field is amplified either by the RREA or by cloud charge fluctuations arising from a local assemblage of charged hydrometeors.

As in the earlier publications [10–13], we have simulated the important "smallest" element seeding the streamer corona. However, the problem remains that the scale is so small and the required fields are larger than observed fields, that a second stage of larger scale is required to amplify the field in larger volume. The scale of the second stage must also satisfy the limitation imposed by the spatial and temporal resolution of instruments measuring the intracloud electric field, otherwise, this stage would be detected. Collective effects of many hydrometeors and spatial variations in a cloud space charge density up to instrumental spatial resolution about 0.1–1 m could be causal to the development of enhanced fields in regions that larger than those of individual hydrometeors [23].

Thus, the electric field required for the streamer formation is larger than the measured fields. However, an opportunity of detecting stronger fields is limited to spatial and temporal resolution of instruments. The results of simulations suggest, therefore, that a second mechanism must operate to amplify the local field.

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