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The decay of counterflow turbulence in superfluid ^{4}He

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The letter summarizes recent experiments on thermal counterflow turbulence in superfluid ^{4}He, emphasizing the observation of turbulence in the normal fluid and its effect on the decay process when the heat flux is turned off. We argue that what is observed as turbulence in the normal fluid is a novel form of coupled turbulence in the superfluid and normal components, with injection of energy on the scales of both the (large) channel width and the (small) spacing between quantized vortices. Although an understanding of this coupled turbulence remains challenging, a theory of its decay is developed which accounts for the experimental observations.

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Turbulence in the superfluid ^{4}He has attracted much recent interest [1]. Such a superfluid exhibits two-fluid behaviour, a normal viscous fluid (composed of thermal excitations) coexisting with an inviscid superfluid component, the flow of which must be irrotational; rotational motion is possible in a simply-connected volume only with the formation of topological defects in the form of vortex lines, each of which carries a circulation of $\kappa = h/m$, where h is Planck's constant and m is the mass of a helium atom. Turbulence in the superfluid component must therefore take the form of a tangle of vortex lines. Turbulence can also occur in the normal fluid, where it is similar to that in a classical fluid, except for the possible presence of a force of *mutual friction* between the two fluids arising from the scattering of thermal excitations by the vortex lines. Because it is dominated by quantum effects, turbulence in a superfluid is referred to as *quantum turbulence*. Such turbulence brings to light new and theoretically challenging problems, unknown in its classical counterpart.

This letter is concerned with a particular type of quantum turbulence: that generated by a forced relative motion of the two fluids, such as exists in a heat current; the superfluid component moves towards the source of heat, while the normal fluid, carrying thermal energy, moves away from it, with no net mass flow. That this counterflow can generate turbulence in the superfluid component has long been known [2] and understood [3]. The vortex lines lead to mutual friction, which is observed as an increase in the attenuation of second sound. The mutual friction can lead to the expansion of vortex loops, and reconnections can then lead to a self-sustaining turbulence in the superfluid component. The observation that the vortex-line density (length of line per unit volume) was related to the relative velocity between the two fluids by

$$L \cong \gamma^2 (v_s - v_n)^2, \tag{1}$$

was easily understood. Important refinements of this early theory followed (e.g., [4]), but it was always assumed that the flow was spatially uniform and that the flow of the normal fluid remained laminar, in spite of the fact that experiments always related to flow in a channel of finite cross-section. According to these theories the turbulence in the superfluid component takes the form of a random tangle of vortex lines, so that there is little turbulent energy on length scales significantly larger than the spacing, $\ell = 1/\sqrt{L}$, between vortex lines. Polarization of these lines, leading to the existence of turbulent energy on larger scales, is absent.

However, the suspicion has long existed that this is not the whole story. It seemed likely that flow of the normal fluid would not remain laminar [5, 6], and the observed decay of L was quite different from that pre-

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dicted [2, 7]. In this letter we shall first review, or report for the first time, very recent experimental results that throw light on these problems, and then present a simple theoretical model that accounts for the form of the observed decays.

The development of metastable He_2 excimer molecules as tracers of the motion of the normal fluid [8, 9] has shown that transitions to turbulence in the normal fluid do occur. Heat flow in a channel of square cross-section ($D = 10\,\text{mm}$) displays three regimes of normal fluid flow: laminar with a parabolic velocity profile for heat fluxes $W < W_{c1}$; still laminar but with a distorted velocity profile for $W_{c1} < W < W_{c2}$; and turbulent for $W > W_{c2}$. At $T = 1.65\,\text{K}$ $W_{c1} \sim 30\,\text{mWcm}^{-2}$, and $W_{c2} \approx 60{-}65\,\text{mWcm}^{-2}$ [10]. The second order structure function for the turbulent normal fluid indicates an energy spectrum associated with components of the fluid velocity parallel to W that is proportional to k^{-2}, for k not too close to $2\pi/D$ or to $2\pi/\ell$; the turbulence exists therefore on large scales, but differs from the Kolmogorov form.

In discussing decays, we note first that the decay with time of a random vortex tangle in the absence of any mean flow is predicted to obey the equation [2, 3, 11]

$$d\tilde{L}/dt = -\tilde{L}^2/\tau_1, \quad \text{so that} \quad \tilde{L} = 1/(1 + t/\tau_1), \tag{2}$$

where $\tilde{L} = L(t)/L(0)$, $\chi_2 \sim 1$ is a dimensionless parameter, and $\tau_1 = 2\pi/\chi_2\kappa L(0)$. The forms of decay that are usually observed are shown as the two upper sets of experimental points in Fig. 1. These observations [10] re-

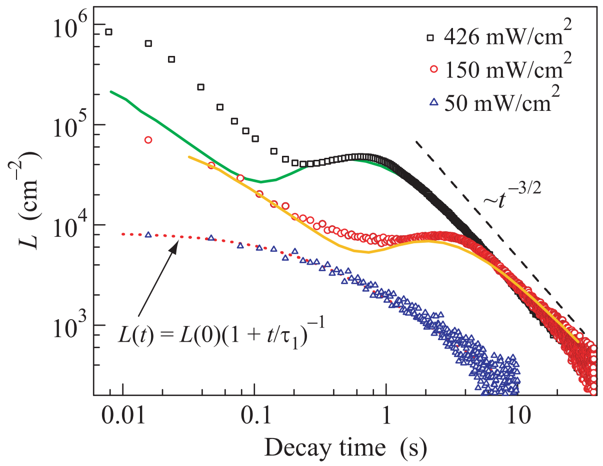

Fig. 1. (Color online) Observed decays of vortex line density for different values of the heat flux in the steady state at 1.65 K [10]; the lines show our theoretical predictions

late to the channel used for the visualization, but similar decays have been observed in other wide channels, most recently by Babuin et al. [12]. The observed decays may obey Eq. (2) at very short times, although the distorting effect of the thermal RC time constant, referred to later, makes it hard to be sure; but subsequently there is a "bump", followed by a $t^{-3/2}$ regime. That the late decay goes as $t^{-3/2}$ was first recognized by Skrbek et al. [7], who realized that it indicated a quasi-classical decay of coupled turbulence in the two fluids, when the largest eddy size is limited by the size of the channel, a type of decay that was studied in quantum grid turbulence [13–15]. It was not clear how such large-scale turbulence might be generated. Various explanations of the bump have not been convincing [11, 16–19]. Recently it was realised that all these anomalous decays related to steady-state heat fluxes greater than W_{c2}; the consequent suspicion that such decays might be associated with a turbulent normal fluid in the steady state led Gao et al. [10] to study decays from steady-state heat currents less than W_{c2}. An example of such a decay is shown for a heat flux of $50\,\text{mW}\cdot\text{cm}^{-2}$ in Fig. 1. We see that this result is consistent with Eq. (2), confirming that the anomalous decays are indeed associated with steady states in which there is large-scale turbulence in the normal fluid. The appearance of large scale turbulence during the decay is then no longer mysterious; it is already present in the steady state.

Visualization of the normal fluid with He_2 excimer molecules is possible not only in the steady state but also during the decay [10]. Fig. 2 shows two results ob-

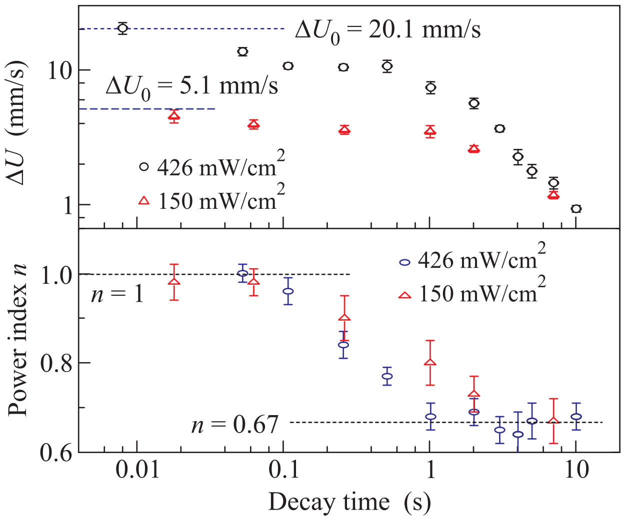

Fig. 2. (Color online) Decays at $T = 1.65\,\text{K}$ observed by visualization. The upper graph shows how the root mean square velocity fluctuations in the turbulent normal fluid decay with time. The lower graph shows the time dependence of the index n in the second order transverse structure function, $S_2(r) \sim r^n$. The corresponding energy spectra have the form $E(k) \sim k^{-x}$, $x = n + 1$ [20]

tained at 1.65 K; they are similar to the published results at 1.83 K [8]. The upper graph shows how the intensity of the turbulence (root mean square velocity fluctuation) decays in time[2)]. The lower graph relates to the change with time of the turbulent energy spectrum $E(k) \propto k^{-n-1}$, and it shows how n changes gradually from unity to 2/3, corresponding to a gradual change to a Kolmogorov-41 spectrum [20], thus confirming that the observed $t^{-3/2}$ decay in line density is due to the onset of this spectrum. This change occurs more rapidly as the steady-state heat flux is increased.

Immediately after the heater is turned off, the heat flux decays in a time determined by a RC time constant formed from the product of the thermal resistance and thermal capacity of the helium [2, 21]; this has a serious effect only for the highest heat fluxes, as we shall see later. In a very short time after this thermal decay, motion of the two fluids becomes completely coupled by mutual friction on scales larger than ℓ [14]. If the large-scale turbulence in the steady state were to involve the normal fluid only, we would expect the root mean square turbulent velocity to fall by a factor ρ_n/ρ (~ 0.2 at 1.65 K), where ρ_n and ρ are the densities of the normal fluid and the whole fluid respectively. It is evident from Fig. 2 that in reality the fall at short times is by a factor of not less than about 0.8. Thus even in the steady state the large-scale turbulent motion observed in the normal fluid must involve to a large extent the coupled motion of both fluids; reference to simply normal-fluid turbulence is misleading.

We turn now to a **theoretical discussion**. We begin by commenting on the steady state, which is very interesting, but which remains hard to understand in detail. However, we shall show that, given the experimental observations about the steady state, a plausible explanation of the mysterious decay process can be developed.

In classical high Reynolds number (Re) homogeneous turbulence, turbulent energy is injected at a large length scale and flows at a rate ϵ in a Richardson cascade to smaller and smaller scales until it is dissipated by viscosity in eddies for which $\mathrm{Re} \simeq 1$; the inertial (energy-conserving) range has an energy spectrum of approximately the Kolmogorov-41 form [20]

$$E(k) = C\epsilon^{2/3}k^{-5/3}. \qquad (3)$$

In steady-state counterflow turbulence, energy is always injected on the scale, ℓ, by the mechanism identified by Schwarz [3]. For heat fluxes exceeding W_{c2}, energy must be injected also at a large scale; this large scale must be the width, D, of the channel, since this is the only large scale in the problem. Energy injected at the scale ℓ is confined, mostly, to the superfluid component and is dissipated on a similar scale by mutual friction; it gives rise to a "quantum peak" in the energy spectrum at $k \gtrsim 2\pi/\ell$. Energy injected at scale D must involve to a large degree both fluids, as we have shown, and experiment tells us that it gives rise to a second component of the energy spectrum $E_2(k) \propto k^{-2}$. This energy will flow in a cascade to smaller scales, but this flow will not be free from dissipation, as we now explain.

In the absence of a steady counterflow, energy injected at $D \gg \ell$, will, because of coupling by mutual friction, be divided between the two fluids in such a way that the two velocity fields are essentially identical. There will be a quasi-classical, inertial range, until either viscous dissipation occurs in the normal component or quantum-dominated flow of the superfluid component on the scale ℓ leads to a breakdown of the coupling and so to dissipation by mutual friction (in practice both processes set in on a scale of order ℓ) (see, e.g., Ref. [14]). In counterflow, coupling on all length scales must, to some extent, break down, because similar eddies in the two components are continually pulled apart, and this leads to dissipation on all length scales. This effect was mentioned in [22], which referred to unpublished calculations with toy eddies that indicated that the corresponding dissipation would decrease with increasing size of eddy. Recent theory [23] extends and confirms this conclusion.

The quantitative effect of dissipation by mutual friction on all length scales can be seen as follows. If we write an energy spectrum (3) in the form

$$E(k) = C\,[\epsilon(k)]^{2/3}k^{-5/3}, \qquad (4)$$

we can interpret $\epsilon(k)$ as a k-dependent energy flux. Although we do not yet understand why the observed steady-state spectrum goes as k^{-2}, we see that such a spectrum implies that $\epsilon(k) \propto k^{-1/2}$, so that the energy flux reaching the scale ℓ, instead of being equal to the energy input at $k \sim 2\pi/D$, as in an inertial regime, has fallen by a factor $(\ell/D)^{1/2}$, which is in practice significantly less than unity.

Very soon after the heat current is switched off, the two fluids become more strongly coupled on scales larger than ℓ, and dissipation ceases on these large scales. This establishment of strong coupling has little immediate effect on the energy spectrum of the large scale turbulence, as we see in Fig. 2. However, there is a gradual evolution to a Kolmogorov-41 spectrum in a time that

[2)] This measurement relates only to components of the velocity parallel to the original heat flux; if the turbulence is isotropic, the total mean square velocity fluctuation is a factor of $\sqrt{3}$ larger.

we expect to be of order the turnover time of the largest eddies, as can be verified by studying the evolution with the Leith differential equation [24] or with the Sabrashell model of superfluid turbulence [25]. The observed evolution of the spectrum shown in Fig. 2 for two values of the steady-state heat flux is consistent with this expectation. As long as the k^{-2} spectrum persists the flux of energy from large scales to the scale ℓ remains small. As the Kolmogorov spectrum becomes established, the flux of energy into scales of order ℓ rises to a value equal to $\epsilon(2\pi/D)$, which can, according to Kolmogorov scaling, be estimated as $U^3/2D$, where U is the velocity characteristic of the largest (coupled) eddies. We see that U must decay with time according to the equation

$$dU^2/dt = -U^3/D, \tag{5}$$

so that

$$U(t) = \frac{U(0)}{(1+t/\tau_2)}, \quad \text{and} \quad \tau_2 = \frac{2\,D}{U(0)}, \tag{6}$$

is the initial turnover time of the largest eddies. Thus the energy flux into a scale approaching ℓ is

$$\epsilon_2(t) = \frac{[U(0)]^3 F(t)}{2D\,(1+t/\tau_2)^3}, \tag{7}$$

where the function $F(t)$ rises smoothly from essentially zero to unity as t increases through the value $D/U(0)$.

The energy in the quantum peak arising from motion in the superfluid component is

$$E_Q = (\rho_s/\rho)\kappa^2\Lambda L\,, \tag{8}$$

per unit mass of helium, where $\Lambda = (1/4\pi)\ln(\ell/\xi)$, and ξ is the vortex core parameter. This energy is being dissipated by mutual friction at a rate that is easily derived from Eq. (2). At the same time there is an input of energy to E_Q from decay of the large scale eddies, given by $\beta\epsilon_2(t)$, where the factor β allows for the fact that, strictly speaking, some of the energy flux $\epsilon_2(t)$ is dissipated in the normal fluid on scales $\gtrsim \ell$. It follows that

$$\frac{d\tilde{L}}{d\tilde{t}} = -\frac{\nu'}{\kappa}\Big(\frac{\kappa L(0)\tau_2}{\Lambda}\Big)\tilde{L}^2 + \frac{\beta\rho}{\rho_s}\Big(\frac{U^2(0)}{\kappa^2\Lambda L(0)}\Big)(1+\tilde{t})^{-3}F(\tilde{t}), \tag{9}$$

where $\tilde{t} = t/\tau_2$, and $\nu' = \chi_2\kappa\Lambda/2\pi$ can be shown to be the effective kinematic viscosity that governs dissipation on the scale ℓ in quantum turbulence [10].

A discussion of the precise form of $F(t)$ is outside the scope of this paper. The results are not very sensitive to this precise form, provided that it serves in effect to delay the transfer of energy from scale D to scale ℓ by a time equal to the turnover time $D/U(0)$. We shall put

$$F(\tilde{t}) = [1 - \exp(-2\tilde{t})]^2\,, \tag{10}$$

a form that has the required property. We shall put $\Lambda = 1$, which is good enough for our purposes. A rigorous evaluation of β requires a more detailed theory than is presently available, and we shall therefore follow Ref. [14] and put $\beta = \rho_s/\rho$. Eq. (9) can then be solved by numerical integration.

To compare the results with experiment we need values of $L(0)$, $U(0)$ and ν'/κ. $L(0)$ is taken directly from experiment, $U(0)$ is obtained from the observed fluctuations in the fluid velocity, and $\nu'/\kappa = 0.48$ has been obtained by equating the directly measured rate of decay of the total turbulent energy to $\nu'\kappa^2L^2$ [10]. With the suggested form of $F(t)$ the predicted decay $L(t)$ is then obtained without adjustable parameters and compared with experiment in Fig. 1.

We see that for a steady-state heat flux of $150\,\mathrm{mW}\cdot\mathrm{cm}^{-2}$ the agreement is satisfactory, especially in view of the simplicity of the model. We find similar agreement for other relatively small steady-state heat fluxes and at other temperatures. For the larger heat flux of $426\,\mathrm{mW}\cdot\mathrm{cm}^{-2}$ the predicted bump appears at a little too early a time, and, more seriously, the predicted line densities at times less than 100 ms are too small.

However, these discrepancies can be attributed to an effect mentioned earlier: that the heat flux in the experimental channel does not fall to zero immediately after the heater is turned off, but decays in a time given by an effective thermal RC time constant. Thus the $L(t)$ decay is effectively delayed by this time constant. In the present context this effect is serious only at the largest heat flux, where the (non-linear) thermal resistivity is highest, and we are confident that it is responsible for the discrepancies to which we have drawn attention. For quantitative comparison of our theory with experiment we have used only the recent measurements of Gao et al. [10], because it is only for these measurements that we have corresponding visualization data.

No bump is predicted if the factor $F(t)$ is omitted; the decay is then monotonic and changes smoothly from the form predicted by Eq. (2) to the form $t^{-3/2}$ as the decay proceeds. This situation would obtain if the large-scale turbulence were to have a Kolmogorov energy spectrum at all times; initially the decay would then obey Eq. (2) because the line density would at first be larger than is necessary to dissipate the flux of energy in the Richardson cascade. We see then that the physical origin of the bump lies in the fact that energy from the decaying large-scale eddies reaches the dissipation scale of order ℓ at a time that is delayed, through the factor $F(t)$, by the turnover time of the large-scale eddies. The time at which the bump appears is therefore determined by this turnover time, and this leads to the bump appearing at a time that decreases with increasing steady-state heat flux, as is observed.

Thermal counterflow is a special case of a more general type of counterflow in which there is also a net mass flow. Another special case, in which the mean velocity of the normal fluid is zero, has been studied by Babuin et al. [12, 26]. For a given relative velocity, $v_s - v_n$, the vortex line density in the steady state is hardly changed. Furthermore, within the range of initial velocities that was studied, the decays still exhibited the forms of Eq. (2) and $t^{-3/2}$ at small and large times respectively, but the bump was absent except at the largest initial rates of flow and was replaced by only a mild point of inflexion. No visualization is yet available for this type of flow, but the observations can be accounted for within the framework of our theory, if we assume that the large-scale turbulence is still present, but is much weaker than it is in the case of thermal counterflow.

In **summary**, we have outlined the results of recent important experiments on thermal counterflow turbulence in superfluid ^{4}He, and we have presented a theoretical model that accounts in a reasonably convincing way for the observed decay of this type of quantum turbulence. We cannot yet account for all aspects of the steady state, relating particularly to the generation and characteristics of large-scale turbulence, leaving us with challenging problems for the future. But our model for the decay does serve to solve the long-standing problem presented by the anomalous forms of decay and to illustrate interesting and unique features of an important form of quantum turbulence.

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1. C. F. Barenghi, L. Skrbek, and K. R. Sreenivasan, Proc. Natl. Acad. Sci. U.S.A. **111**, 4647 (2014).
2. W. F. Vinen, Proc. R. Soc. **240**, 114 (1957); Proc. R. Soc. **240**, 128 (1957); Proc. R. Soc. **242**, 493 (1957); Proc. R. Soc. **243**, 400 (1958).
3. K. W. Schwarz, Phys. Rev. B **38**, 2398 (1988).
4. H. Adachi, S. Fujiyama, and M. Tsubota, Phys. Rev. B **81**, 104511 (2010).
5. D. J. Melotte and C. F. Barenghi, Phys. Rev. Lett. **80**, 4181 (1998).
6. A. W. Baggaley and S. Laizet, Phys. Fluids **25**, 115101 (2013).
7. L. Skrbek, A. V. Gordeev, and F. Soukup, Phys. Rev. E **67**, 047302 (2003).
8. A. Marakov, J. Gao, W. Guo, S. W. Van Sciver, G. G. Ihas, D. N. McKinsey, and W. F. Vinen, Phys. Rev. B **91**, 094503 (2015).
9. J. Gao, A. Marakov, W. Guo, B. T. Pawlowski, S. W. Van Sciver, G. G. Ihas, D. N. McKinsey, and W. F. Vinen, Rev. Sci. Instrum., **86**, 093904 (2015).
10. J. Gao, W. Guo, and W. F. Vinen, to be published.
11. Y. Mineda, M. Tsubota, and W. F. Vinen, J. Low Temp. Phys. **171**, 511 (2013).
12. S. Babuin, E. Varga, W. F. Vinen, and L. Skrbek, Phys. Rev. B **92**, 184503 (2015).
13. S. R. Stalp, L. Skrbek, and R. J. Donnelly, Phys. Rev. Lett. **82**, 4831 (1999).
14. W. F. Vinen and J. J. Niemela, J. Low Temp. Phys. **128**, 167 (2002).
15. L. Skrbek and K. R. Sreenivasan, Phys. Fluids **24**, 011301 (2012).
16. K. W. Schwarz and J. R. Rozen, Phys. Rev. Lett. **66**, 1898 (1991).
17. K. W. Schwarz and J. R. Rozen, Phys. Rev. B **44**, 7563 (1991).
18. C. F. Barenghi and L. Skrbek, J. Low Temp. Phys. **146**, 5 (2007).
19. C. F. Barenghi, A. V. Gordeev, and L. Skrbek, Phys. Rev. E **74**, 026309 (2006).
20. U. Frisch, *Turbulence: the Legacy of A. N. Kolmogorov*, Cambridge University Press (1995).
21. A. V. Gordeev, T. V. Chagovets, F. Soukup, and L. Skrbek, J. Low Temp. Phys. **138**, 649 (2005).
22. W. F. Vinen, J. Low Temp. Phys. **175**. 305 (2014).
23. D. Khomenko, V. S. L'vov, A. Pomyalov, and I. Procaccia, Phys. Rev. B **93**, 014516 (2016).
24. C. E. Leith, Phys. Fluids **10**, 1409 (1967).
25. L. Boue, V. S. L'vov, Y. Nagar, S. V. Nazarenko, A. Pomyalov, and I. Procaccia, Phys. Rev. B **91**, 144501 (2015).
26. S. Babuin, M. Stammeier, E. Varga, M. Rotter, and L. Skrbek, Phys. Rev. B **86**, 134515 (2012).