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Condensation of Fermion Zero Modes in the Vortex

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The spectrum of the low-energy fermionic bound states in the core of the symmetric vortex with winding number $m = \pm 1$ in the isotropic model of s-wave superconductor was obtained in a microscopic (BCS) theory by Caroli, de Gennes and Matricon [1]:

$$E_n = \left(n + \frac{1}{2}\right)\omega_0(p_z). \tag{1}$$

Here p_z is the momentum of the bound states along the vortex line, and n is related to the angular momentum quantum number L_z. This spectrum is two-fold degenerate due to spin degrees of freedom. The level spacing – the so called minigap – is small compared to the energy gap of the quasiparticles outside the core, $\omega_0 \sim \Delta^2/E_F \ll \Delta$.

For the chiral superfluid/superconductor with an odd winding number of the phase of the gap function in momentum space (i.e., $\Delta(\mathbf{p}) \propto (p_x + ip_y)^N$ with odd N), the spectrum of fermions in the symmetric vortex is modified. For the most symmetric vortex in the Weyl superfluid ^{3}He-A one has [2, 3]:

$$E_n = n\omega_0(p_z). \tag{2}$$

The spectrum contains the zero energy states at $n = 0$. In the two-dimensional case the $n = 0$ levels represent two Majorana modes [4, 5]. The 2D half-quantum vortex, which is the vortex in one spin component, contains single Majorana mode. In the 3D case the Eq. (2) at $n = 0$ describes the flat band [6]: all the states in the interval $-p_0 < p_z < p_0$ have zero energy, where $p_0\hat{\mathbf{z}}$ and $-p_0\hat{\mathbf{z}}$ mark the positions of two Weyl points in the bulk material [7].

Here we consider vortices, in which the minigap $\omega_0(p_z)$ vanishes at $p_z = 0$. Examples are provided by half-quantum vortices [8] in the recently discovered [9] non-chiral ($N = 0$) spin-triplet polar phase of superfluid ^{3}He, and by vortices in chiral ($N = 1$) spin-singlet superconductors with $(d_{xz} + id_{yz})$ pairing [10] (such pairing has been suggested in the heavy-fermion compound URu_2Si_2 [11, 12]). For small $p_z \ll p_F$ the minigap in these phases has the following form:

$$\omega_0(p_z) = \omega_{00}\frac{p_z^2}{p_F^2}\ln\frac{p_F^2}{p_z^2}, \quad \omega_{00} \sim \frac{\Delta_0^2}{E_F}, \tag{3}$$

where ω_{00} has an order of the minigap in the conventional s-wave superconductors. The spectrum is shown in Fig. 1 for vortex in a polar phase (Fig. 1a) and in $(d_{xz} + id_{yz})$-wave superconductor, see Fig. 1b. All the branches with different n touch the zero energy level. It looks as the flat band in terms of n for $p_z = 0$. In addition the vortex in chiral superconductor has a flat band in terms of p_z at $n = 0$. The effect of squeezing of all energy levels n towards the zero energy at $p_z \to 0$ can be called the condensation of Andreev–Majorana fermions in the vortex. It leads to the non-analytic behavior of

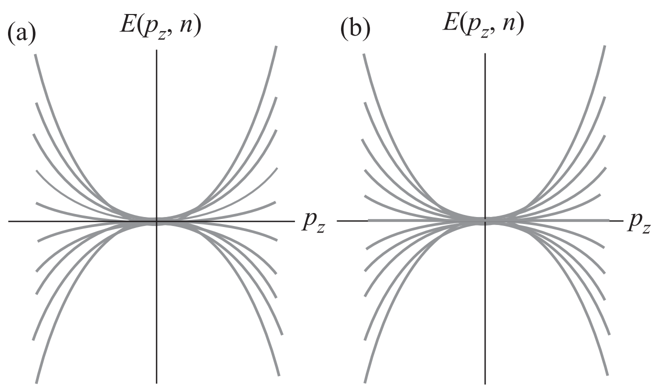

Fig. 1. Illustration of the spectrum of fermion zero modes at $|p_z| \ll p_F$ on vortices in the polar phase of superfluid ^{3}He (a) and in the chiral $(d_{xz}+id_{yz})$-wave superconductor (b). The branches with different n approach zero-energy level at $p_z \to 0$. In addition, the vortex in $(d_{xz}+id_{yz})$-wave superconductor contains the flat band at $n = 0$ [10]

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the density of states as a function of magnetic field in superconductor or of rotation velocity in superfluid.

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