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EXISTENCE OF AN EFFECTIVE MAGNETIC FIELD IN DISORDERED MANY-VALLEY SEMICONDUCTORS

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An internal effective magnetic field is shown to affect an electron when moving in a valley of a silicon-like crystal in the presence of a random static potential. The phase relaxation due to the scattering by a randomly inhomogeneous effective magnetic field is considered.

Suppose the electron spectrum in crystal to have several degenerate extrema being located inside the Brillouin zone (not on its boundary). Let the intervalley scattering by a random static potential be weak compared to the intravalley one. In this Letter, we should like to call attention to the existence of an effective magnetic field with which an electron interacts when diffusing inside one of the valleys. It is convenient to elucidate the gist by considering inhomogeneities smooth on the scale of a wavelength. Then, the point is that the position in $\vec{k}$-space of the band extremum is not the same in different regions of crystal. Really, provided the wave vector $\vec{k}_0$ defining the bottom of the valley is dependent on $\vec{r}$, the function $(-\hbar c/e)\vec{k}_0(\vec{r})$ plays the role of a vector potential, while $(-\hbar c/e)\mathrm{rot}\vec{k}_0(\vec{r})$ of a magnetic field. Notice the idea of existing of an effective magnetic field due to the spatial variation of $\vec{k}_0$ has been stated by Kroemer [1] yet many years ago as applied to a rather similar object, a solid solution with smoothly varying chemical composition.

Thus, corresponding to each valley is its own magnetic field. By time symmetry, the fields of valleys which differ in $\vec{k}_0$ sign are opposite in direction. We consider the action of an effective magnetic field caused by a random electric field or a random lattice deformation and fluctuating in space on a scale much less the mean free path. Typically, the time of inter- valley transitions in degenerate semiconductors with multiply connected Fermi surface at low temperatures is far larger than the momentum relaxation time, it is assumed in what follows.

For definiteness, we mean a crystal of silicon whose conduction band consists of six valleys being located on axes of [100] type, value of k_0 being $\simeq 0,85\cdot 2\pi/a$, a denotes a lattice constant. Existence of an effective magnetic field implies the linear in the wave vector $\vec{k}$, counted off the valley bottom, term $H_{\text{м}}$ in the band Hamiltonian

$$H_{\text{м}} = \frac{\hbar}{2}(\vec{k}\vec{v}(\vec{r}) + \vec{v}(\vec{r})\vec{k}).$$

The relationship between the velocity $\vec{v}$ and the electric field $\vec{E}$, not too sharply inhomogeneous, may be established by making use of $\vec{k}\vec{p}$ theory to fourth order. It is given by

$$\vec{v}(\vec{r}) = \alpha\frac{\vec{k}_0}{k_0}\left(\frac{\partial\vec{E}}{\partial\vec{r}}\right) + \left(\frac{\vec{k}_0}{k_0}\frac{\partial}{\partial\vec{r}}\right)\hat{\beta}\vec{E} \tag{1}$$

with three constants of an "atomic" scale: α and independent components β_1 and β_2 of a tensor $\hat{\beta}$ of rank two[1)]. The first term in (1) is available only in regions with nonzero charge density. By choosing z-axis along $\vec{k}_0$

$$v_z = \alpha\left(\frac{\partial\vec{E}}{\partial\vec{r}}\right) + \beta_1\frac{\partial E_z}{\partial z},\ v_x = \beta_2\frac{\partial E_x}{\partial z},\ v_y = \beta_2\frac{\partial E_y}{\partial z}.$$

It is seen, a magnetic field does exist in a plane perpendicular to $\vec{k}_0$.

The crystal lattice being inhomogeneously distorted, an effective magnetic field appears in the second order of the perturbation theory by simultaneous taking account of $\vec{k}\vec{p}$-term and deformation potential. By symmetry, the connection between $\vec{v}$ and strain tensor ϵ is similar to (1) with changing $\partial E_i/\partial x_j \Leftrightarrow \epsilon_{ij}$: $v_z = \alpha'\mathrm{Sp}\epsilon + \beta_1'\epsilon_{zz}$, $v_x = \beta_2'\epsilon_{xz}$, $v_y = \beta_2'\epsilon_{yz}$. Note z-component of a magnetic field does exist under deformation but only under that is attended with rotation providing $\partial u_x/\partial y - \partial u_y/\partial x \neq 0$, $\vec{u}$ being the displacement vector[2)]. However nontrivial it is, constants α', β_1', β_2' should be determined spectroscopically by the valleys minima shift with an external stress. In silicon, the proximity of $\vec{k}_0$ to the Brillouin zone boundary furthers an indirect measurement of this shift. In the presence of nondiagonal strain component ϵ_{xz}, the electron spectrum contains the term $\hbar^2 k_x(k_z + k_0 - 2\pi/a)/2m_{xz}$ with an effective mass $m_{xz} \propto \epsilon_{xz}^{-1}$. If k_0 is close to $2\pi/a$, it is the same term that gives rise to the shift of the minimum along x at $k_z = 0$, so that $\beta_2' = \hbar(k_0 - 2\pi/a)/2m_{xz}\epsilon_{xz}$. The value of m_{xz} may be extracted from the variation of the cyclotron mass at the band edge vs. uniaxial stress orientation. (Such a dependence has been obtained in Ref.[2], but the measurements accuracy was not enough for orientation being of interest to us.) The difference $k_0 - 2\pi/a$ is expressed in terms of the longitudinal mass $m_{\parallel}$ and the velocity v_0 which defines the linear in k_z splitting $\hbar v_0(k_z - 2\pi/a)$ of the spectrum near the Brillouin zone boundary, $k_0 = 2\pi/a - m_{\parallel}v_0/\hbar$. Thus, knowledge $m_{\parallel}$ and v_0 as functions of $\mathrm{Sp}\epsilon$ and ϵ_{zz} will enable us to find constants α' and β_1' (see [2] for rough estimation).

In weak localization range, the interference phenomena are caused by a spatial coherency of waves passing the loop of the diffusion trajectory in opposite directions [3]. As temperature decreases, even weak interactions which do not preserve this coherency manifest themselves. An electron being scattered by a

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[1)] Given the crystal having no inversion symmetry, an interaction similar to spin-orbit one with $\vec{k}_0$ substituting for the spin vector should exist.

[2)] It is interesting that the Burgers dislocation oriented along $\vec{k}_0$ induces the magnetic field just the same as the solenoid does. Being zero everywhere except for the dislocation axis such a field is responsible for the Aharonov - Bohm effect.

random magnetic field, the amplitude $f(\vec{k}_1, \vec{k}_2)$ of $\vec{k}_1 \to \vec{k}_2$ transition is not equal to $f(-\vec{k}_2, -\vec{k}_1)$, which breaks coherency. Within the Born approximation

$$f(\vec{k}_1, \vec{k}_2) = -\frac{m}{2\pi\hbar}(\vec{k}_1 + \vec{k}_2)\vec{v}_{\vec{k}_1 - \vec{k}_2},$$

$\vec{v}_{\vec{k}_1 - \vec{k}_2}$ being the Fourier transform of the velocity $\vec{v}(\vec{r})$, m the density of states mass. Then, amplitudes $f(\vec{k}_1, \vec{k}_2)$ and $f(-\vec{k}_2, -\vec{k}_1)$ differ in sign only. On this account, the phase relaxation time τ_M is simply half the reciprocal probability of scattering by a random magnetic field, τ_M being supposed much larger the free path time. Let us evaluate τ_M in heavily doped semiconductors deep in the metallic range where the Fourier component of the electric field correlator has a form $< E_i E_j >_{\vec{k}} = 16\pi^2 n e^2 k_i k_j / (k^2 + r_s^{-2})^2 \kappa^2$, here n is the charged impurity concentration, r_s the screening radius, κ dielectric constant. Note should be taken, a dispersion law being anisotropic, the momentum relaxation time depends on a momentum orientation, however, τ_M is got by averaging over wave vectors both of final states and of initial ones. In this way

$$\frac{1}{\tau_M} = \frac{64\pi^3}{3\hbar}\rho_F m_{\|}\epsilon_F \frac{ne^2}{\kappa^2}(\alpha + \beta_1 - \frac{m_\perp}{m_{\|}}\beta_2)^2,$$

where is the density of states with the Fermi energy ϵ_F (allowing for one spin projection). It has been taken into account that an effective mass $m_{\|}$ along [100] in silicon is much larger than that $m_\perp$ of transverse motion (ratio of α, β_1 and β_2 was kept arbitrary). Notice an absence of Coulomb divergence when evaluating τ_M, for which reason $r_s = \infty$ has been put. The rate of phase relaxation increases rapidly with doping level ($\tau_M^{-1} \propto n^2$). The intervalley transition time and τ_M may be of any ratio, depending as they do on a potential core strength and interband energy distances. Similarly, for the scattering in a magnetic field caused by short-range distortions (whose correlation radius is less than the Fermi wavelength),

$$\frac{1}{\tau_M} = \frac{4\pi}{3\hbar}\rho_F m_{\|}\epsilon_F d, \; d = (d_1 + 2d_2)(3\alpha'^2 + 2\alpha'\beta_1') + d_1\beta_1'^2 + 2\frac{m_\perp}{m_{\|}}d_3\beta_2'^2$$

with $d_{1,2,3}$: identifying the correlators of strain fields:

$$< \epsilon_{xx}\epsilon_{xx} >_{\vec{k}} = d_1, \; < \epsilon_{xx}\epsilon_{yy} >_{\vec{k}} = d_2, \; < \epsilon_{xy}\epsilon_{xy} >_{\vec{k}} = d_3.$$

In n-type Si-MOS structure with (100) surface, two dimensional electron gas occupies two lowest lying equivalent valleys oriented normal to the boundary plane. In this geometry, an effective magnetic field arises from the lattice deformation only. Therefore, the phase relaxation time in high-quality Si(100) structures may be much larger than with another surface orientation.

Thus, the interference phenomena are determined by three characteristic times: phase relaxation time τ_E associated with inelastic processes, intervalley transition time τ_v and τ_M. If $\tau_v \gg \tau_M$, an experimentally measured phase breaking time $(\tau_E^{-1} + \tau_M^{-1})^{-1})$, rising with temperature decreasing, should tend to saturation at the value of τ_M . Of course, an effective magnetic field, destroying the Cooperon attributed to one valley, does not affect the Cooperon constructed with wave functions of different valleys in a symmetrical way. Hence, at lowest temperatures, τ_E coming larger τ_v , it is the symmetrized Cooperon that governs interference phenomena. In the opposite case $\tau_v \ll \tau_M$, though an electron changes many times valleys differing in $\vec{v}(\vec{r})$ sign in time τ_M, the expression for τ_M does remain the same (if the space fluctuations of a magnetic field be smooth enough, it will not be the case). Note that the decay of the Cooperon due to the intervalley transitions takes place only because of the nonequivalency of differently oriented valleys. If the symmetry of the valleys were the same (and τ_M^{-1} were neglected, see below) the intervalley scattering would not manifest itself at all. Correspondingly, when considering the total Cooperon time evolution in Ref.[4], only the number of nonequivalent valleys should be of importance. Notice the statement of Ref.[5] about the destroying action of intervalley transitions in Si(100)-MOS structure is at variance with that above. The point is the processes of multiple intervalley scattering which are finished in the same valley as the electron waves started from have not been taken into account in [5]. To take an example, the expression for the Cooperon which describes the interference in two equivalent valleys (in Si-MOSFET or uniaxially strained bulk Si, e.g.) is proportional to the sum

$$\frac{1}{x+\tau_M^{-1}}+\frac{1}{x+\tau_M^{-1}+2\tau_v^{-1}}+\frac{1}{x}-\frac{1}{x+2\tau_v^{-1}}.$$

where $x = \vec{Q}D\vec{Q} + \tau_E^{-1}$, $\vec{Q}$ being the Cooperon momentum, D the diffusion coefficient. Provided $\tau_v \gg \tau_M$ (and $x \gg (\tau_M\tau_v)^{-1/2}$), this sum is reduced to $2/(x+\tau_M^{-1})$, the temperature dependence being cut off at $\tau_E \sim \tau_M$. If $\tau_v \ll \tau_M$, then it turns into $(x+\tau_M^{-1})^{-1} + x^{-1}$ and half of the Cooperon retains the pole form.

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