IMPURITY-ASSISTED TUNNELING IN A QUANTUM BALLISTIC MICROCONSTRICTION

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Submitted 5 September 1991

We calculate the conductance of a pinch-off saddle-point potential ballistic microconstriction with a short-range impurity in the conducting channel. Resonant tunneling via impurity bound states causes narrow peaks in conductance vs Fermi energy

Recent experiments demonstrate that even a single impurity can affect the conductance of a quantum ballistic microconstriction [1]. The effect of the impurity is especialy strong near the thresholds, corresponding to steps between adjacent quantization plateaus. This result is confirmed by calculations [2,3]. It follows from these calculations that in the case of a short-range attractive impurity a bound state appears below each threshold. Resonant scattering by the bound state causes a narrow dip in conductance G vs Fermi energy E.In [2,3] the microconstriction was modeled as an infinit waveguide with a constant cross-section. We adopt the saddle-point potential [4,5], which is a more realistic model [6,7]. In this model bound states exist for repulsive impurities too, and resonant scattering display not only dips in G vs E, but also peaks. The peaks are due to electron resonant tunneling via bound states.

The saddle-point potential is taken to be

$$V(x,y) = \frac{\hbar^2}{2md^2}\left[-\frac{x^2}{L^2}+\frac{y^2}{d^2}\right]. \tag{1}$$

Here d is the width of the channel, L – its length, d is of the order of the Fermi wavelength λ_F, and $L \gg d$.The waveguide modes for the potential (1) are

$$\Psi^{\pm}_{E,n}(x,y) = \Phi_n(y)E(-\varepsilon_n, \pm\xi). \tag{2}$$

Here E is the electron energy, $\Phi_n (n = 0,1,...)$ are the harmonic oscillator wave functions corresponding to energies $E_n = \hbar\Omega(n+\frac{1}{2})$ with $\hbar\Omega = \hbar^2/md^2$, $E(-\varepsilon,\xi)$ is the Weber function [8] with $\varepsilon_n = (E-E_n)/\hbar\omega$ and $\hbar\omega = \hbar^2/mdL$, $\xi = x(2/Ld)^{1/2}$ Functions $\Psi^{\pm}_{E,n}$ correspond to modes n, coming from $x = \mp\infty$. The threshold energy for mode n is E_n.

When there is no impurity the conductance of microjunction (in units of $2e^2/h$) is [5]

$$G_0 = \sum_{n=0}^{\infty} t^2(\varepsilon_n). \tag{3}$$

with the transmission coefficient

$$t^2(\varepsilon) = 1 - r^2(\varepsilon) = (1 + \exp(-2\pi\varepsilon))^{-1}. \tag{4}$$

The impurity change the conductance due to mixing and additional reflection of modes. Following [2,3] we assume the impurity to be a short-range one. The scattered field for such type of impurity is

$$\psi'(r) = -2\pi\psi^\circ(r_0)\frac{G_E(r, r_0)}{D_E(r_0)}. \tag{5}$$

Here r_0 is the impurity position, $\psi^0(r)$ is the field in absence of the impurity,$G_E(r, r')$ is the Green function of the Schrodinger equation with confining potential $V(x, y)$.The denominator D_E is expressed in terms of the near asymptotic behaviour of the Green function

$$G_E(r, r')|_{r,r' \to r_0} = \frac{1}{2\pi}\left[\ln\frac{d}{|r - r'|} + K_E(r_0)\right], \tag{6}$$

namely

$$D_E(r_0) = \Lambda + K_E(r_0). \tag{7}$$

Here $\Lambda = \ln(d/a)$, where a is the 2D scattering length of the impurity potential.

Calculating the scattered field for $\psi^0 = \Psi^+_{En}$ one can find the transmission coefficients $T_{n \to n'}$ and the conductance

$$G = \sum_{nn'} |T_{n \to n'}|^2. \tag{8}$$

In what follows we consider the case of a pinch-off microconstriction , $E < E_0$, and assume the impurity to be in the narrow part of the constriction , $x_0 \ll L$. In this simple case

$$K_E = p + \beta H(\varepsilon, \xi_0), \tag{9}$$

where

$$H(\varepsilon, \xi) = \frac{1}{\sqrt{2}}t(\varepsilon)E(-\varepsilon, \xi)E(-\varepsilon, -\xi) \equiv P(\varepsilon, \xi) + iQ(\varepsilon, \xi) \tag{10}$$

with $\varepsilon \equiv \varepsilon_0$ and $\beta = \pi(L/d)^{1/2}\Phi_0^2(y_0)$. Here βH is the singular contribution due to the threshold mode $n = 0$. The contribution due to below barrier modes $n > 0$ is given by the constant p , which is real and of the order of unity. The conductance of a pinch-off constriction is reduced to

$$G = G_0\frac{|\tilde{\Lambda}|^2}{|D_E|^2}, \tag{11}$$

where $G_0 = t^2(\varepsilon)$ is the conductance in the absence of the impurity , $\tilde{\Lambda} = \Lambda + p$ and $D_E = \tilde{\Lambda} + \beta H$.

Sharp features in G vs E occur because of Breit-Wigner resonances [9], which correspond to resonance scattering by the impurity bound states. The bound states are defined as scattering amplitude poles $\bar{E} - i\Gamma$, located near the real E-axis. These poles are roots of equation $D_E = 0$. The width of the bound state Γ is small in the energy domain, where $Q \ll P$. That is why $\bar{E}$ can be sought from equation

$$\tilde{\Lambda} + \beta P = 0, \tag{12}$$

and also

$$\Gamma = \left(\frac{Q}{dP/dE}\right)_{E=\bar{E}}. \tag{13}$$

If one developes D_E near the pole, (11) is reduced to

$$G(E) = G_0(E)\left(\frac{P}{Q}\right)^2_{E=\bar{E}} \cdot \frac{\Gamma^2}{(E-\bar{E})^2+\Gamma^2}. \tag{14}$$

It follows from this expression that each impurity bound state exhibits as a peak in G vs E, the peak value of the conductance with impurity being larger by a factor of $(P/Q)^2$ than the conductance without impurity.

The investigation of the bound states and the conductance peaks is greatly simplified, since $Q \ll P$ only if $\varepsilon < 0$ and $|\varepsilon| \gg 1$. In this energy domain the conductance (14) can be represented in the following form

$$G(E) = \frac{4\Gamma'\Gamma''}{(E-\bar{E})^2+\Gamma^2}. \tag{15}$$

Here $\Gamma = \Gamma' + \Gamma''$, where Γ' and Γ'' are partial widths due to the escape of the electron from the bound state to $x = +\infty$ and $x = -\infty$, respectively. Now it is obvious that the peaks in G vs E are due to resonant tunneling via the impurity bound state . To find the bound states explicitly we express the Weber functions in terms of Airy functions [8].The situation is relatively simple in the case when the impurity is in the central cross section, $\xi_0 = 0$. Only one bound state exist, and only in the case of a strong enough attractive impurity, i.e. when $\tilde{\Lambda} < 0$ and $|\tilde{\Lambda}| \ll \beta$. The binding energy and the width are

$$\Delta = E_0 - \bar{E} = \hbar\omega\frac{\beta^2}{2|\tilde{\Lambda}|^2}, \qquad \Gamma' = \Gamma'' = \Delta\exp(-\pi\Delta/\hbar\omega). \tag{16}$$

Consider now the more complicated situation, when the impurity is displaced from the central cross section, $\xi_0 \gg 1$. In this case bound state exist if $|\tilde{\Lambda}| \ll \beta\xi_0^{-1/3}$, and two types of such states appear. The first type of bound state is of the same nature as that for the central impurity and exist only for attractive impurities. The corresponding energy level is below the level $E_0 + V(x_0, 0)$. The binding energy if reckoned from this level coinsides with Δ given by (16)-see fig.1. Second type bound states exist for both signs of the impurity potential. The energy levels are above the level $E_0 + V(x_0, 0)$, the wave functions being "miror confined" waves reflected from the saddle-point potential on the left and from the impurity potential on the right — see fig.2. The number of miror confined states is of the order of ξ_0^2.

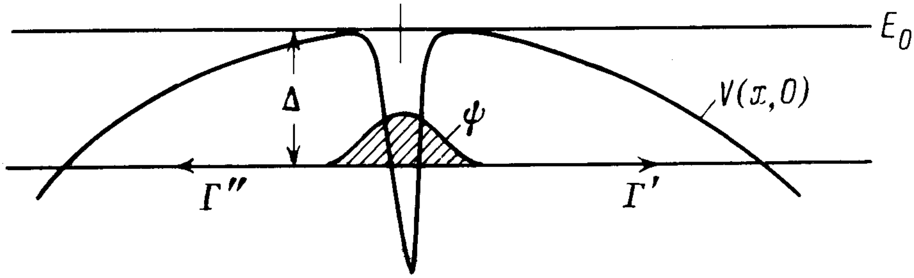

Fig.1

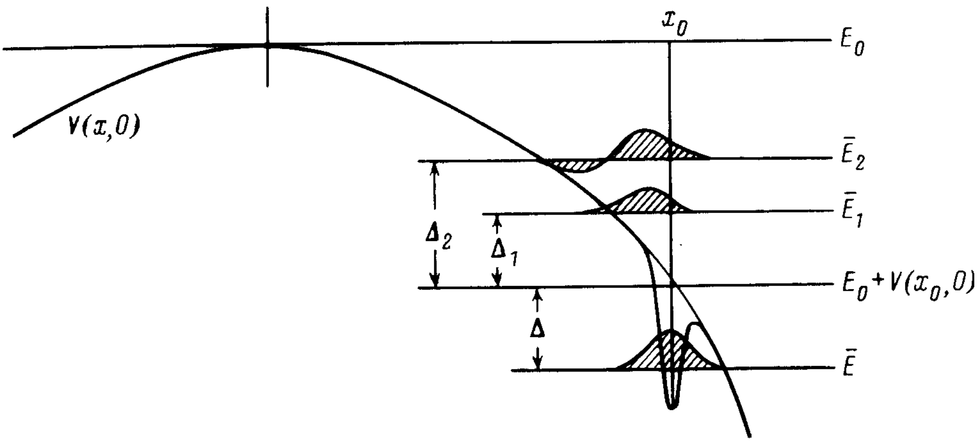

Fig.2

To avoid complicated formulal we assume in what follows that $|\tilde{\Lambda}| \gg \beta\xi_0^{-1}$. Then the partial widths of the first type bound state are

$$\Gamma' = \Delta \exp(-\sigma^3/3),$$

$$\Gamma''/\Gamma' = G_0(\bar{E})(\Delta/\Gamma')^2 \ll 1. \tag{17}$$

where $\sigma = \sqrt{2}\beta/|\tilde{\Lambda}|\xi_0^{1/3}$. Both Γ' and Γ'' are due to tunneling through wide barriers and hence are exponentially small.

The position of the "miror confind" energy levels is given by

$$\Delta_s = \hbar\omega(\xi_0/2)^{2/3}t_s, \tag{18}$$

where t_s is a zero of the Airy function : $Ai(-t_s) = 0$.The partial widths are

$$\Gamma'_s = c_1\Delta_s/\sigma^2,$$

$$\Gamma''_s/\Gamma'_s = c_2 G_0(\bar{E})(\Delta_s/\Gamma'_s) \ll 1, \tag{19}$$

where

$$c_1^{-1} = 2^{2/3}\pi t_s Bi^2(-t_s), \qquad c_1 c_2 = t_s^{-2}. \tag{20}$$

The right barrier created by the impurity is narrow and hence Γ_s' is not exponentially small.

Introducing (17) or (19) into (15) one can calculate the enhancement of the conductance due to impurity-assisted resonant tunneling.

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