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On chiral magnetic effect in Weyl superfluid ^{3}He-A

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In relativistic quantum field theories (RQFT) many results crucially depend on the regularization scheme in the high-energy corner (ultraviolet). This especially concerns the effect of chiral anomaly, see, e.g., review [1] and discussion of consistent anomaly vs conventional anomaly in Ref. [2]. The results may differ for example by the factor 1/3. We discuss here the effects of chiral anomaly which exist and have been experimentally observed in chiral Weyl superfluid ^{3}He-A.

Superfluid ^{3}He-A has two well separated Weyl points in the fermionic spectrum. Close to the Weyl points, quasiparticles obey the Weyl equation and behave as Weyl fermions, which interact with emergent gauge and gravity fields, see book [3] and references therein, and also recent review on topological superfluids [4]. The dynamics of ^{3}He-A is accompanied by the hydrodynamic anomalies which reflect the chiral anomaly in the Weyl superfluids. The anomaly is described by the Adler–Bell–Jackiw equation [5, 6], which has been experimentally verified in experiments with skyrmions in ^{3}He-A [7]. The anomaly also gives rise to the analog of chiral magnetic effect (CME), which has been also experimentally identified in ^{3}He-A [8–11]. The CME is the appearance of the non-dissipative current along the magnetic field due to the chirality imbalance [12, 13]. It is now under investigation in relativistic heavy ion collisions, where strong magnetic fields are created by the colliding ions, see review [14]. The theory of CME also experiences the problems, such as with the choice of the proper ultraviolet cut-off, see e.g. Ref. [15], and the choice of the proper order of limits in the infrared [16].

In ^{3}He-A the hydrodynamic anomalies can be calculated either using the equations of the microscopic theory (analog of the trans-Planckian physics), or using the spectral flow described by the effective RQFT which emerges in the vicinity of two Weyl points at $\mathbf{K}_\pm = \pm k_F \hat{\mathbf{l}}$, where $\hat{\mathbf{l}}$ is the unit vector along the angular nomentum of Cooper pairs. The chiral fermions experience the effect of chiral anomaly in the presence of the synthetic electric and magnetic fields

$$\mathbf{A} = k_F \hat{\mathbf{l}}, \quad \mathbf{E} = -\partial_t (k_F \hat{\mathbf{l}}), \quad \mathbf{B} = \nabla \times (k_F \hat{\mathbf{l}}). \qquad (1)$$

The fermions created from the superfluid vacuum carry the momentum $\mathbf{K}_\pm = \pm k_F \hat{\mathbf{l}}$. This gives the momentum creation from the vacuum state of ^{3}He-A:

$$\dot{\mathbf{P}} = \frac{1}{2\pi^2} k_F \hat{\mathbf{l}} \, (\mathbf{B} \cdot \mathbf{E}). \qquad (2)$$

Let us choose the spacetime dependence of effective gauge field in such a way that k_F depends only on time, $k_F(t)$, while the orbital unit vector depends only on space, $\hat{\mathbf{l}}(\mathbf{r})$. We consider the process in which $k_F(t)$ changes from zero, where the anomaly is absent, to the final value. Then $\mathbf{E} = -\hat{\mathbf{l}} \, \dot{k}_F$, and the final momentum density is

$$\mathbf{P} = -\frac{1}{2\pi^2} \int dt \, \dot{k}_F k_F^2 \hat{\mathbf{l}} (\hat{\mathbf{l}} \cdot \nabla \times \hat{\mathbf{l}}) = -\frac{k_F^3}{6\pi^2} \hat{\mathbf{l}} (\hat{\mathbf{l}} \cdot \nabla \times \hat{\mathbf{l}}). \qquad (3)$$

This is the same anomalous term in the current density, which has been obtained by Cross [17] using the gradient expansion of the free energy and supercurrents starting with the microscopic Hamiltonian in a weak coupling approximation. In the presence of superflow with velocity $\mathbf{v}_\mathrm{s}$, the current in Eq. (3) gives the following contribution to the free energy density, which is also consistent with the results by Cross [17]:

$$f_{CS}^{(1)} = \mathbf{v}_\mathrm{s} \cdot \mathbf{P} = -\frac{k_F^3}{6\pi^2} \, (\mathbf{v}_\mathrm{s} \cdot \hat{\mathbf{l}}) \, (\hat{\mathbf{l}} \cdot \nabla \times \hat{\mathbf{l}}). \qquad (4)$$

In the analogy with RQFT the superflow velocity plays the role of the chiral chemical potential [3]. Experimentally the effective imbalance between the chiral chemical potentials of the left-handed and right-handed Weyl fermions is provided by the superflow due to the Doppler shift:

$$2 k_F (\mathbf{v}_\mathrm{s} \cdot \hat{\mathbf{l}}) = \mu_R - \mu_L. \qquad (5)$$

Then Eq. (4) can be fully expressed in terms of the effective relativistic fields, and it becomes the Chern–Simons term in the energy functional:

$$F_{CS}^{(1)} = \frac{1}{12\pi^2} \, (\mu_R - \mu_L) \int d^3 r \; (\mathbf{A} \cdot \mathbf{B}). \qquad (6)$$

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In the RQFT the variation of energy (6) over the vector potential $\mathbf{A}$ results in the equilibrium current along magnetic field $\mathbf{B}$, i.e. in the chiral magnetic effect.

The experimental signature of the CME in ^{3}He-A is the helical instability of the superflow generated by the Chern–Simons term, which contains the helicity $\mathbf{A}\cdot\mathbf{B}$ of the effective gauge field. To study the helical instability, we need the quadratic form of energy in terms of perturbations $\delta\hat{\mathbf{l}}$ of the homogeneous flow state. For that one should consider another route of deformation leading to spectral flow. Let us take $k_F = \text{const}$; the constant superfluid velocity $\mathbf{v}_\text{s} \parallel \hat{\mathbf{z}}$; $\hat{\mathbf{l}}_0 = \hat{\mathbf{z}}$; and $\hat{\mathbf{l}}(\mathbf{r},t) = \hat{\mathbf{z}} + \delta\hat{\mathbf{l}}(z,t)$. Since $\hat{\mathbf{l}}(z,t)$ is unit vector, $\delta\hat{\mathbf{l}}(z,t) \perp \hat{\mathbf{z}}$, and we take $\delta\hat{\mathbf{l}}(z,t) = \mathbf{m}(z)t$, where $\mathbf{m}(z) \perp \hat{\mathbf{z}}$. Then the final momentum is

$$\mathbf{P} = -\frac{1}{2\pi^2}\int dt\, t\, k_F^3\, \hat{\mathbf{z}}\,(\mathbf{m}\cdot\nabla\times\mathbf{m}) = -\frac{k_F^3}{4\pi^2}\,\hat{\mathbf{l}}_0\,(\delta\hat{\mathbf{l}}\cdot\nabla\times\delta\hat{\mathbf{l}}). \tag{7}$$

The corresponding energy density in the presence of the superflow is

$$f_{CS}^{(2)} = \mathbf{v}_\text{s}\cdot\mathbf{P} = -\frac{k_F^3}{4\pi^2}\,(\mathbf{v}_\text{s}\cdot\hat{\mathbf{l}}_0)\,(\delta\hat{\mathbf{l}}\cdot\nabla\times\delta\hat{\mathbf{l}}). \tag{8}$$

In terms of the effective fields this gives the following Chern–Simons term:

$$F_{CS}^{(2)} = \frac{1}{8\pi^2}\,(\mu_R - \mu_L)\int d^3r\,(\mathbf{A}\cdot\mathbf{B})\,. \tag{9}$$

This term has been also independently derived from the equations describing the “trans-Planckian” dynamics of the quantum liquid, see Ref. [11].

We obtained two expressions, Eq. (6) and Eq. (9), for the Chern-Simons term emerging in superfluid ^{3}He-A. For derivation we used the RQFT emerging in the vicinity of the Weyl points. These two expressions differ by the factor 2/3. However, both terms are correct, since they coincide with the results of the microscopic non-relativistic theory, which is complete and thus does not require any regularization schemes. These two terms describe two different physical situations in the same underlying non-relativistic system. Eq. (6) represents the anomalous term in the hydrodynamic free energy, while the Eq. (9) determines the threshold of the helical instability of the superfluid flow. This demonstrates that the results obtained within the RQFT may reflect the specific properties of the underlying quantum vacuum, and thus can be different depending on what physical phenomena are considered in the same vacuum.

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