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Dark matter from dark energy in q-theory (short version)

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A condensed-matter-type approach to the cosmological constant problem [1] is given by q-theory [2–5]. It was already noted in Ref. [3] that a rapidly-oscillating q-field could give a significant contribution to the inferred dark-matter component of our present universe. Here, we expand on this dark-matter aspect of q-theory.

Consider the particular realization of q-theory based on a 3-form gauge field A with a corresponding 4-form field strength $F \propto q$ (see Refs. [2, 3] and further references therein). The action is now taken to include a kinetic term for the q-field [6]:

$$S = -\int_{\mathbb{R}^4} d^4x \sqrt{-g}\,\Bigg(\frac{R}{16\pi G_N} + \epsilon(q) + \frac{1}{8}\,K(q)\,g^{\alpha\beta}\,\nabla_\alpha(q^2)\nabla_\beta(q^2) + \mathcal{L}^{\rm SM}\Bigg), \tag{1a}$$

$$F_{\alpha\beta\gamma\delta} = q\sqrt{-g}\,\epsilon_{\alpha\beta\gamma\delta}, \quad F^{\alpha\beta\gamma\delta} = q\,\epsilon^{\alpha\beta\gamma\delta}/\sqrt{-g}, \tag{1b}$$

where the functions $\epsilon(q)$ and $K(q)$ in (1a) involve only even powers of q. In the 4-form realization, the mass dimension of q is 2. The Lagrange density $\mathcal{L}^{\rm SM}$ in the action (1a) involves the fields of the standard model (SM) of elementary particle physics. Here, and elsewhere, we use natural units with $c = \hbar = 1$.

With the definition $C(q) \equiv K(q)\,q^2$, the equations of motion for the 3-form gauge field can be written as a generalized Maxwell equation, which has the following solution:

$$\frac{d\epsilon(q)}{dq} - \frac{1}{2}\,\frac{dC(q)}{dq}\,\nabla_\alpha\, q\,\nabla^\alpha q - C(q)\,\Box\, q = \mu \tag{2}$$

for an integration constant μ.

The Einstein equation from (1a) reads

$$R_{\alpha\beta} - \frac{1}{2}\,g_{\alpha\beta}\,R = -8\pi G_N\left(T^{(q)}_{\alpha\beta} + T^{\rm (SM)}_{\alpha\beta}\right). \tag{3}$$

The contribution of the 3-form gauge field to the energy-momentum tensor is given by

$$T^{(q)}_{\alpha\beta} = -\,g_{\alpha\beta}\left(\epsilon(q) - \mu\, q + \frac{1}{2}C(q)\,\nabla_\alpha\, q\,\nabla^\alpha q\right) + C(q)\nabla_\alpha\, q\,\nabla_\beta\, q, \tag{4}$$

where the solution (2) with integration constant μ has been used to simplify the expression. Observe that, for nonconstant q-fields, terms with $(dC/dq)\,(\nabla\, q)^2$ and $C\,\Box\, q$ have been absorbed completely by the constant μ in (4) and that the two remaining terms with $C\,(\nabla\, q)^2$ have the same structure as if q were a fundamental (pseudo-)scalar.

In equilibrium, the cosmological constant $\Lambda_{\rm bare}$ (from quantum-field zero-point-energies, cosmological phase transitions, or other origins) is cancelled by a spacetime-independent q-field of appropriate magnitude q_0.

Specifically, we have for the equilibrium state

$$q(x) = \text{constant} = q_0, \tag{5a}$$

$$\mu = \mu_0 = d\epsilon/dq\,\big|_{q=q_0}, \tag{5b}$$

$$\epsilon(q_0) - \mu_0\, q_0 = 0. \tag{5c}$$

A further stability condition is given by the positivity of the inverse isothermal vacuum compressibility [2]:

$$\big(\chi_0\big)^{-1} \equiv \left[q^2\,\frac{d^2\epsilon(q)}{dq^2}\right]_{q=q_0} > 0. \tag{6}$$

Without additional matter, the Einstein equation (3) for the equilibrium q-field (5) gives Minkowski spacetime with the metric

$$g_{\alpha\beta}(x)\,\big|_{\rm equil.} = \eta_{\alpha\beta} = \big[\text{diag}(-1,\,1,\,1,\,1)\big]_{\alpha\beta} \tag{7}$$

for standard Cartesian coordinates $(x^0,\, x^1,\, x^2,\, x^3) = (t,\, x,\, y,\, z)$.

The question is still what the orders of magnitude are of q_0 and $1/\chi_0$. If we assume that the theory (1a) without the SM term only contains a single energy scale, then that scale must be of order of the Planck energy

$$E_{\rm P} \equiv (G_N)^{-1/2} \approx 1.22\times 10^{19}\,\text{GeV}. \tag{8}$$

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In that case, we have

$$q_0 \overset{?}{\sim} (E_\mathrm{P})^2, \;\; 1/\chi_0 \overset{?}{\sim} (E_\mathrm{P})^4\,, \tag{9}$$

where the question marks are to remind us that these estimates are based on an assumption.

Now consider a small spacetime-dependent change of the equilibrium q-field,

$$q(x) = q_0 + q_0\, \xi(x), \tag{10}$$

in terms of a dimensionless real scalar field $\xi(x)$ with $|\xi(x)| \ll 1$. Also take

$$K(q) = \text{constant} = (q_0)^{-3}\,, \;\; q_0 > 0, \tag{11}$$

so that $C(q_0)$ is positive.

The reduced Maxwell equation (2) for $\mu = \mu_0$ gives the following Klein–Gordon equation [6]:

$$\Box\,\xi - \frac{1}{q_0}\left[q^2\,\frac{d^2\epsilon(q)}{dq^2}\right]_{q=q_0}\xi = 0, \tag{12}$$

where higher-order ξ terms have been omitted. Neglecting, at first, the spatial derivatives of ξ, the solution of (12) is a rapidly-oscillating homogeneous function,

$$\xi(t) = a_\xi\,\sin\big(\omega\, t + \varphi_\xi\big), \tag{13a}$$

$$\omega^2 = (q_0)^{-1}(\chi_0)^{-1} \overset{?}{\sim} (E_\mathrm{P})^2, \tag{13b}$$

where the small amplitude a_ξ and the phase φ_ξ in (13a) are determined by the boundary conditions and where the last estimate in (13b) follows from (9).

For a time-dependent homogeneous perturbation $\xi(t)$ in (10) given by the rapidly-oscillating solution (13), the time-averaged energy-momentum tensor (4) corresponds to a perfect fluid with the following values for the energy density and pressure:

$$\rho^{\,(q-\text{perturbation})} = \tfrac{1}{2}\,(\chi_0)^{-1}\,(a_\xi)^2, \tag{14a}$$

$$P^{\,(q-\text{perturbation})} = 0, \tag{14b}$$

where a_ξ with $|a_\xi| \ll 1$ is determined by the initial boundary conditions [in a cosmological context, taken at the moment when the homogeneous vacuum energy density $\rho_V(t)$ has reached its final near-zero value].

Next, consider additional space-dependence of the ξ field with a typical length-scale

$$L \gg c/\omega \overset{?}{\sim} \hbar\, c/E_\mathrm{P} \sim 10^{-35}\,\mathrm{m}, \tag{15}$$

where $\hbar$ and c have been temporarily reinstated and the estimate (13b) for ω has been used. (For applications in a cosmological context, the length-scale L must be less than the cosmological length-scale $c/H_0 \sim 10^{26}\,\mathrm{m}$.) The corrections to the perfect-fluid energy-momentum tensor will then be small, namely of order $q_0\,\xi^2/L^2$, which is a factor $(L\,\omega)^{-2} \ll 1$ times the leading term (14a). For such large-scale perturbations, there will be, to high precision, a pressureless perfect fluid and this fluid will cluster gravitationally, just as dark matter with standard Newtonian gravitation and dynamics.

In the present article, we have shown that a small perturbation of the equilibrium q-field behaves gravitationally as a pressureless perfect fluid. As such, the fluctuating part of the q-field is a candidate for the inferred dark-matter component of the present universe (see, e.g., Sec. 26 of Ref. [7] for a review). If correct, this implies that direct detection of dark-matter particles will fail, at least in the foreseeable future with the currently available energies.

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