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Chiral vortical effect generated by chiral anomaly in vortex-skyrmions

G. E. Volovik[1)]

Low Temperature Laboratory, Aalto University, School of Science and Technology, P.O. Box 15100, FI-00076 AALTO, Finland

Landau Institute for Theoretical Physics RAS, 119334 Moscow, Russia

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The problems with Weyl materials – Weyl semimetals and Weyl superconductors (see latest reviews in Ref. [1] and Ref. [2] correspondingly) – is that at first glance they should possess any type of a bulk response that exists in conventional non-Weyl materials with the same symmetry [3, 4]. The task is to resolve the contribution of the Weyl physics from that based on symmetry consideration. Here we consider the situation when the effect is fully determined by chiral anomaly. This is the chiral vortical effect (CVE) – the type of the parity violating effects [5–7] when there is the current along the vortex axis concentrated in the vortex core.

We consider the vortex-skyrmion in Weyl superfluids – the continuous (non-singular) texture in Fig. 1. It

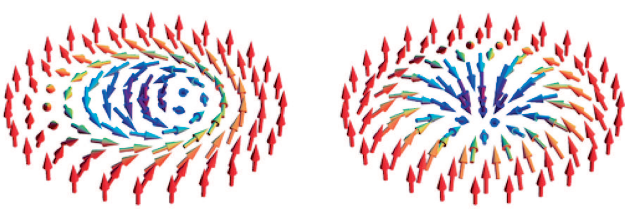

Fig. 1. (Color online) *Right*: Neel-type vortex skyrmion, which is symmetric under the combined symmetry $PTO_{x,\pi}$, which forbids the current along the vortex axis, since $J_z = PTO_{x,\pi}J_z = -J_z$. *Left*: Bloch-type skyrmion, which is symmetric under the combined symmetry $TO_{x,\pi}$. The broken P-symmetry allows the current along the vortex axis – the chiral vortical effect. The main difference from the Neel and Bloch skyrmions discussed in the non-superconducting magnetic materials in Ref. [16] is that in the chiral superfluids there is the circulation of superfluid velocity around the skyrmion. In the p-wave superfluids the skyrmion represents the vortex with two quanta of circulation [8]

has the skyrmionic structure of the orbital magnetization and the superflow with two quanta of circulation around the skyrmion, see review [8]. In such inhomogeneous case the traditional reponse theory, which results depend on the order of limits $q \to 0$ and $\omega \to 0$ [3], is not applicable. Instead we use the trick, which has been used for the calculation of the angular momentum of the texture in chiral Weyl superfluids [9] (see also Refs. [10, 11]), and for calculation of different manifestations of the chiral magnetic effect (CME) [12]. We calculate the current generated by deformation of the order parameter within the skyrmion, when it is deformed from the state obeying the space inversion P to the state with the violated space inversion symmetry. In the initial state there is no current along the axis of the skyrmion, since it is forbidden by the P-symmetry.

If the texture has no dependence on the coordinate z along the vortex-skyrmion, then in the process of deformation of the texture there is no force applied in z direction. In this situation the change of the total linear momentum, which in Galilean invariant systems is equivalent to the mass current J_z, may come only from the spectral flow from the occupied negative energy states. Such spectral flow may take place either through the nodes in the bulk spectrum [9], or through the boundaries [10, 11]. The skyrmion is the localized object, which is not connected to the side walls of the cylindrical container, and thus the boundary effects can be ignored. Then, if the bulk state is fully gapped, there will be no current along the skyrmion axis, even if the space inversion (or the combined space inversion) is broken. In superfluids with Weyl points in bulk, the spectral flow through the Weyl nodes regulated by the Adler–Bell–Jackiw equation [13, 14] for the chiral anomaly gives rise to the CVE.

We consider the p-wave superfluid with two Weyl points of opposite chiralities at $\mathbf{K}_\pm = \pm k_F \hat{\mathbf{l}}$, where $\hat{\mathbf{l}}$ is the unit vector in the direction of the angular momentum of Cooper pairs (extension to the superfluids with several Weyl points [15] is straightforward). In the axisymmetric vortex-skyrmion, the positions of Weyl nodes in momentum space are the functions of the coordinates:

$$\mathbf{K}_\pm(\mathbf{r}) = \pm k_F \left(\hat{\mathbf{z}} \cos\eta(r) + \sin\eta(r)(\hat{\mathbf{r}}\cos\alpha + \hat{\boldsymbol{\phi}}\sin\alpha) \right). \qquad (1)$$

Here (z, r, ϕ) are cylindrical coordinates; $\eta(r)$ changes from π to 0 ($\eta(0) = \pi$ and $\eta(\infty) = 0$); the param-

[1)]e-mail: volovik@boojum.hut.fi

eter α is constant. For $\alpha = 0$, the texture is shown in Fig. 1 (*right*). It is the Neel-type skyrmion, which is symmetric under the combined symmetry $PTO_{x,\pi}$, where P is space inversion, T is time inversion, and $O_{x,\pi}$ is π rotation about horizontal axis. This symmetry forbids the current along the vortex axis, since $J_z = PTO_{x,\pi}J_z = -J_z$. For $\alpha \neq 0$ this symmetry is violated and the current along the vortex axis may exist. For $\alpha = \pi/2$ it is the Bloch-type skyrmion in Fig. 1, which is symmetric under the combined symmetry $TO_{x,\pi}$. Neel and Bloch skyrmions in nonsuperconducting magnetic materials see, e.g., in Ref. [16].

We consider deformation, at which $\alpha(t)$ changes from zero to the finite value. The positions of the Weyl points play the role of the effective vector potentials $\mathbf{A}_\pm(\mathbf{r}, t) = \mathbf{K}_\pm(\mathbf{r}, t)$ acting on fermions living at these two Weyl points (in superconductors such gauge fields are induced by strain [17]). This results in the effective electric and magnetic fields $\mathbf{E}_\pm = -\partial_t \mathbf{K}_\pm$ and $\mathbf{B}_\pm = \nabla \times \mathbf{K}_\pm$ with

$$\mathbf{E}_+ \cdot \mathbf{B}_+ = \mathbf{E}_- \cdot \mathbf{B}_- = -k_F^2 \frac{d\cos\eta}{dr} \sin\eta \frac{d\sin\alpha}{dt}. \qquad (2)$$

The productions of the chiral charge at two Weyl points due to chiral anomaly compensate each other, $\dot{n} = \frac{1}{4\pi^2}(\mathbf{E}_+ \cdot \mathbf{B}_+ - \mathbf{E}_- \cdot \mathbf{B}_-) = 0$. But the created chiral charge carries the linear momentum $\mathbf{K}_\pm$, and the momentum production per unit time per unit volume is:

$$\dot{j}_z = \frac{1}{4\pi^2}\left(K_{z+}(\mathbf{E}_+ \cdot \mathbf{B}_+) - K_{z-}(\mathbf{E}_- \cdot \mathbf{B}_-)\right) = -\frac{k_F^3}{2\pi^2} \cos\eta \frac{d\cos\eta}{dr} \sin\eta \frac{d\sin\alpha}{dt}. \qquad (3)$$

The momentum accumulated by the vortex in the process of deformation, and thus the mass current is:

$$J_z(\text{vortex}) = \int d^2r \int dt \dot{j}_z = -\frac{k_F^3}{6\pi} \sin\alpha \int_0^\infty dr \sin^3\eta. \qquad (4)$$

For $\alpha = \pi/2$ this coincides with the current obtained by traditional methods for the so-called w-vortex [8]. The CVE current (4) is determined by positions of nodes and does not depend on microscopic parameters of the system, such as the mass of the atom. It is generated by the chiral anomaly experienced by Weyl fermions and vanishes when Weyl nodes merge at $\mathbf{K}_\pm = 0$.

In the equilibrium state the total current should vanish, see, e.g., Ref. [18]. The current (4) within the skyrmion is compensated by the bulk current $J_z(\text{bulk}) = \int d^2r \, \rho v_{sz}$ [8]. In the normal (non-superfluid) state of the liquid the equilibrium CVE is not possible: there is no non-dissipative supercurrent in bulk which could compensate the CVE current. The same refers to the chiral magnetic effect (CME): there is no current in response to a static magnetic field [19, 20]. In superconductors with broken parity the non-uniform current along the magnetic field is possible. But again the total current is zero in equilibrium [21, 22], and it is difficult to resolve between the conventional CME and the CME originating from Weyl nodes, though in some cases the Weyl contribution can be dominating [23].

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