Pis'ma v ZhETF, vol. 105, iss. 8, pp. 504 – 505

Lifshitz transitions via the type-II Dirac and type-II Weyl points

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Submitted 14 March 2017

DOI: 10.7868/S0370274X17080094

Massless Weyl fermions [1] are the building blocks of Standard Model. In the chiral gauge theory of weak interactions, the fundamental elementary particles are Weyl fermions with a pronounced asymmetry between the $SU(2)$ doublet of left-handed Weyl fermions and the $SU(2)$ singlet of right-handed Weyl ferrmions. The masslessness of the Weyl fermions is topologically protected [2]. The corresponding topological invariant – the Chern number – has values $N_3 = -1$ and $N_3 = +1$ for the left and right particles respectively [3]. The gapless Weyl fermions are at the origin of the anomalies in quantum field theories, such as chiral anomaly, and the coresponding symmetry protected Chern numbers characterize the anomalous action [3]. The Dirac particles, which emerge below the symmetry breaking electroweak transition, are the composite objects obtained by the doublet-singlet mixing of Weyl fermions with opposite chirality. The topological invariants $N_3 = \pm 1$ of left and right Weyl fermions cancel each other, and without the topological and symmetry protection the Dirac particles become massive.

The areal of Weyl fermions is not restricted by the Standard Model or by elementary particle physics in general. Investigations in condensed matter reveal abundant and novel physics originating from the Weyl fermionic excitations, that live in the vicinity of the topologically protected touching point of two bands [4–11]. Such diabolical (conical) point represents the monopole in the Berry phase flux,[12, 13]. Recently the attention is attracted to the so-called type-II Weyl points [14]. A remarkable property of this type of Weyl point is that it is the node of co-dimension 3 in the 3D momentum space, which is accompanied by the nodes of the co-dimension less than three: the nodes of co-dimension 1 (Fermi surfaces) or nodes of co-dimension 2 (Dirac lines). Many new types of the topological Lifshitz transitions become possible, where the topologically protected nodes of other co-dimensions are involved [15]. There is a variety of topological numbers, which characterize the momentum space manifolds of zeroes. Together with the geometry of the shapes of the manifolds, this makes the Lifshitz transitions widespread in fermionic system.

The type II Weyl point may also emerge as the intermediate state of the topological Lifshitz transition, at which the Fermi surfaces exchange their global topological charge N_3 [16, 17]. Several examples of such transfer of global topological invariants between Fermi surfaces is considered, including the peculiar one in Fig. 1, where Fermi surfaces loose the Berry monopole after Lifshitz transition. This class of emergent type-II Weyl point, in which the Berry phase monopole is transported across the Fermi surface can be represented by the following Hamiltonian:

$$H = c\boldsymbol{\sigma} \cdot (\mathbf{p} - \mathbf{p}^{(0)}) + \frac{p^2 - p_{\mathrm{F}}^2}{2m}. \qquad (1)$$

Fig. 1 plots the Lifshithz transitions and the evolution of configuration of Berry monopole in momentum space driven by the change of the position $\mathbf{p}^{(0)}$ of the Weyl point. The regime with $p_{\mathrm{F}} > mc$ is considered. For $|\mathbf{p}^{(0)}| < p_{\mathrm{F}}$ we have two Fermi surfaces, one inside the other, but both embracing the Weyl point with $N_3 = 1$, the Berry phase monopole. At the Lifshitz transition, which occurs at $|\mathbf{p}^{(0)}| = p_{\mathrm{F}}$, the inner and outer Fermi surfaces touch each other at the type-II Weyl point. As distinct from the conventional type-II Weyl point, which connects two Fermi pockets, this Weyl point connects the inner and outer Fermi surfaces. After the Lifshitz transition, at $|\mathbf{p}^{(0)}| > p_{\mathrm{F}}$, the Weyl point leaves both Fermi surfaces. The Fermi surfaces are again one inside the other, but both without the Berry flux. Finally at the second Lifshitz transition, at $|\mathbf{p}^{(0)}| = (m^2c^2 + p_{\mathrm{F}}^2)/(2mc)$, the inner Fermi surface collapses to the point and disappears, since the point is no more supported by the topological invariant N_3.

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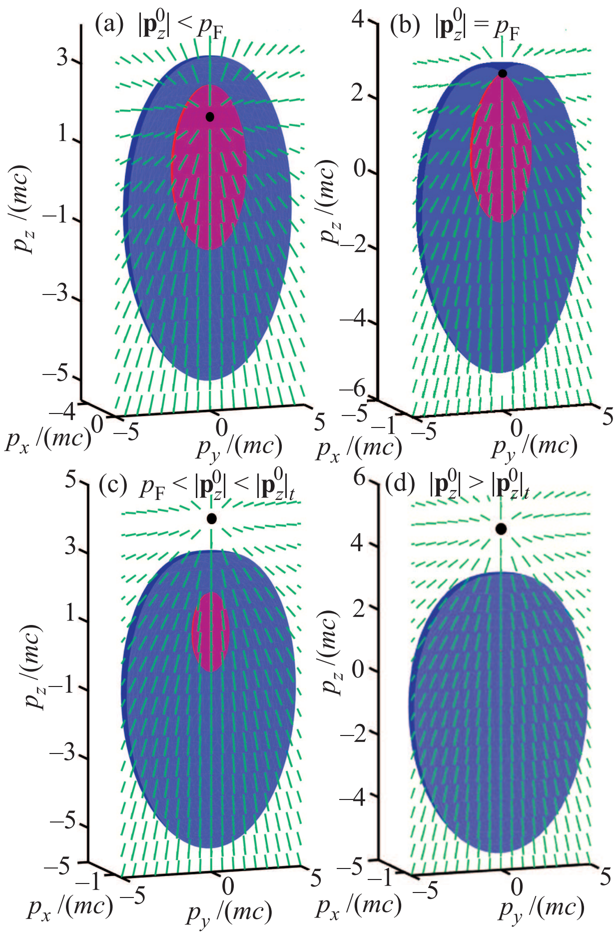

Fig. 1. (Collor online) Illustration of the process in which Fermi surfaces loose their global topological charge. (a) – Both blue and red Fermi surfaces enclose the Berry monopole with topological charge $N_3 = +1$, when $|\mathbf{p}^{(0)}| < p_{\rm F}$. (b) – The intermediate state of Lifshitz transition at $|\mathbf{p}^{(0)}| = p_{\rm F}$. The inner and outer Fermi surfaces touch each other, at the peculiar type-II Weyl point with topological charge $N_3 = +1$. (c) – On the other side of the Lifshitz transition, at $|\mathbf{p}^{(0)}| > p_{\rm F}$, the Weyl point is outside of the Fermi surfaces, i.e. after the transition the Fermi surfaces lost the Berry phase flux. (d) – After the second Lifshitz transition, which takes place at $|\mathbf{p}^{(0)}| = (m^2c^2 + p_{\rm F}^2)/(2mc)$, the inner Fermi surface disappears, since it is not protected by the topological charge N_3

Conclusion. The interplay of different topological invariants enhances the variety of the topological Lifshitz transitons. Here we discussed the examples of the transitions, which involve the Fermi surfaces with topological charge N_1 and Weyl points with the topological charge N_3. In general also Dirac lines with topological charge N_2 are involved. Depending on the type of the transition, the intermediate state has the type-II Dirac point, the type-II Weyl point or the Dirac line.

Many other Lifshitz transitions are expected, with topological invariants describing the shape of the Fermi surface; the shape of the Dirac nodal lines; their interconnections; etc. For the interacting fermions more types of Lifshitz transitions are possible, in partucular the transitions which involve the Weyl points of type-III and type-IV [18].

The work by GEV has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement # 694248). The work by KZ has been supported in part by the National Natural Science Foundation of China (NSFC) under Grants # 11674200, # 11422433, # 11604392.

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364017080094

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