Pis'ma v ZhETF, vol. 105, iss. 11, pp. 691 – 692

Fermi points and the Nambu sum rule in the polar phase of ^{3}He[1)]

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Submitted 10 May 2017

DOI: 10.7868/S0370274X17110078

Polar phase of superfluid ^{3}He has been recently observed in anisotropic aerogel [1–5]. The possibility that this phase may appear in aerogel has been proposed earlier in [6]. The superfluid phases of ^{3}He were widely discussed (see [7, 8] and references therein). The BCS theory of ^{3}He based on the functional integral representation has been developed in a number of publications (see, for example, [9–12]) and was summarized recently, for example, in [13].

One of the most interesting properties of the superfluid ^{3}He is that it is able to simulate the phenomena typical for the high energy physics theory. In particular, the emergent relativistic Weyl fermions appear naturally in the A phase [7, 14–16]. This possibility is related to the appearance of the Fermi points. At the same time, it is generally believed, that in the polar phase the Fermi line appears [7]. (For the discussion of Fermi lines in superconductors and semimentals see [17–25].) If we neglect the spin-orbit interactions, then the order parameter matrix $A_{\alpha i}$ in the polar phase vacuum has the form: $A^{(0)}_{\alpha i} = (\beta V)^{1/2}\frac{\Delta}{2}\delta_{p0}e^{i\Phi}\hat{d}_\alpha \hat{m}_i$, where $\hat{\mathbf{d}}$ and $\hat{\mathbf{m}}$ are unit vectors while phase Φ may be set to zero; β is the inverse temperature, V is the system volume. This gives dispersion that does not depend on spin as well as on $\hat{d}$ and Φ: $E(k) = \pm\sqrt{v_{\mathrm{F}}^2(|k|-k_{\mathrm{F}})^2 + \Delta^2(\hat{m}\hat{k})^2}$. Here v_{F} is Fermi velocity. In the present paper we demonstrate that the Fermi line predicted by this equation disappears if we take into account the spin-orbit interaction. Instead of it the two Fermi points appear, which allows, in principle, to simulate relativistic physics in the ^{3}He inside aerogel.

Collective modes in superfluids and superconductors [26–29] possess an analogy to the Higgs modes of particle physics (for the review see [30]). Quasi–Nambu–Goldstone modes in nonrelativistic systems were discussed in [31] and observed in ^{3}He-B [32]. In ^{3}He-B [33–36] and later in ^{3}He-A, and in the 2D phases of ^{3}He the Nambu sum rule was established that relates the energy gaps of the bosonic modes and the fermion "gap" [37, 38, 34]. The Nambu sum rule also takes place in a number of relativistic models of the Nambu–Jona–Lasinio type [34]. In the present paper we analyse the spectrum of bosonic collective modes in the polar phase and demonstrate, that here a kind of the Nambu sum rule exists as well.

The extra interaction that stabilizes the direction of vector $\hat{m}$ along the anisotropy axis of aerogel is added to the BCS model. Without this interaction the polar phase is unstable, and it becomes stable only when this extra interaction is sufficiently strong. Without the spin-orbit interaction there should be 3 Goldstone bosons. Direct calculation of energy gaps confirms this expectation. Our analysis of the bosonic excitations demonstrates that in the channels with vanishing orbital momentum there are gapped modes. We calculated both real and imaginary parts of those gaps. The imaginary parts appear because the fermions are gapless, and those bosonic states may decay into them. The real parts are given by $E_{v,S=0,L=0} = E_{u,S=1,L=0} = 0$, $E_{u,S=0,L=0} = E_{v,S=1,L=0} \approx \sqrt{12/5}\Delta$. Correspondingly, there is the Nambu sum rule satisfied by these gaps: $E_u^2 + E_v^2 \approx 4\langle\Delta_\theta^2\rangle = \frac{12}{5}\Delta^2$. Here Δ_θ is the angle depending fermionic energy "gap". It appears that when the interaction stabilizing the direction of $\hat{m}$ is sufficiently strong, and vacuum is stable, our one loop results do not predict the appearance of the Higgs modes with $L = 1$. The mentioned values of the energy gaps were estimated neglecting the spin-orbit interaction that keeps the direction of $\hat{d}$ in the plane orthogonal to $\hat{m}$. We do not consider the effect of spin-orbit interaction on the spectrum of bosonic collective modes in the present paper.

However, we discuss in details the effect of the spin-orbit interaction on the fermionic quasiparticles. Without the spin-orbit interaction the considered BCS model predicts the appearance of the Fermi lines, along which the quasiparticle energy vanishes. However, the smooth deformation of the system is able to eliminate those Fermi lines. Indeed, this occurs due to the spin-orbit interaction. We show, that it changes the

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"gap" equations in such a way, that the conventional expression for the vacuum value of the order parameter is already not the solution of the "gap" equation. Instead, the corrected form of the condensate appears: $A^{(0)}_{\alpha i} = (\beta V)^{1/2}\frac{\Delta}{2}\,\delta_{p0}\Big(\hat{d}^{\alpha}\hat{m}^{i} + a\,\hat{d}^{\alpha}\hat{m}^{i} + b\,\hat{d}^{i}\hat{m}^{\alpha}\Big)$, where $a, b \ll 1$. This form of the condensate gives rise to the dispersion: $E(k) = \pm\sqrt{v_{\rm F}^2(|k|-k_{\rm F})^2 + \Delta^2\Big((1+a)^2(\hat{m}\hat{k})^2 + b^2(\hat{d}\hat{k})^2\Big)}$ and to the two Fermi points $\pm k_{\rm F}\,\hat{m}\times\hat{d}$ instead of the Fermi line. Close to those two Fermi points the emergent Dirac fermions apppear. In turn, those Fermi points are not protected by topology, and the next corrections are able to eliminate them, so that small effective mass of the Dirac fermions might appear.

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364017110029

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