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Non-exponential decoherence of radiofrequency resonance rotation of spin in storage rings

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Radiofrequency (RF) spin rotator driven resonance up-down oscillations of polarization of charged particles in storage rings are of interest in a broad class of spin experiments incuding the search for electric dipole moments of charged particles [1–3]. Inherent to particles in storage rings is a modulation of the spin-tune by synchrotron oscillations (SO) which drives the spin precession and RF frequencies apart [4] and decoheres evolution of the polarization of particles in an ensemle. It becomes still more important at large spin rotation times imperative in searches for extremely small EDM's [5–7]. Here we report a fully analytic theory of SO driven spin decoherence. These analytic results are new and do substantially extend the early considerations by Benati et al. [4].

Synchrotron oscillations driven decoherence depends on the spread of SO amplitudes a_z in an ensemble, which we relate by Abel transform [8] to longitudinal density $N(z)$ in the bunch. Spin phase after n revolutions will be $\theta_s(n) = 2\pi\nu_s n + \psi_s\xi[\cos(2\pi\nu_z n + \lambda) - \cos\lambda]$, where λ is a random SO phase and $\xi = a_z/B$ is a normalized SO amplitude. Typically the spin phase slip parameter

$$\psi_s = \frac{2G\beta^2\gamma}{\sqrt{3}\nu_z} \cdot \left\langle \frac{\Delta p^2}{p^2} \right\rangle^{1/2} \ll 1, \qquad (1)$$

especially for deuterons with small magnetic anomaly $G = (g-2)/2$. Simultaneously, SO's change the revolution time and generate a slip of the RF phase $\Delta\theta_{\rm rf}(n) = (C_{\rm rf}+1)\Delta\theta_s(n)$, for a connection of $C_{\rm rf}$ to the RF phase slip factor see Ref. [9, 10].

In the solution of the spin evolution equations, we follow a formalism of Appendix A in Ref. [11] and resort to the Bogolyubov–Krylov–Mitropolsky averaging method [12]. Remarkably, evolution of the vertical polarization does not depend on the SO phase λ. Upon averaging of the up-down oscillations of one-particle polarizations over an ensemble, we obtain our main result for evolution of the average vertical polarization:

$$\begin{aligned} A(n) &= \Re\langle\exp\{-in\epsilon(\xi)\}\rangle_\xi \simeq \\ &\simeq \Re\left\langle\exp\left\{-in\epsilon_0 J_0(2\sqrt{\rho}\xi)\right\}\right\rangle_\xi = \\ &= (1-i\rho n)^{-1/2}\exp\left\{-i\epsilon_0 n + \frac{i\rho n}{1-i\rho n}\right\} = \\ &= \frac{\exp\left\{-\sin^2\varphi(n)\right\}}{(1+\rho^2 n^2)^{1/4}}\cos\left\{\epsilon_0 n - \kappa(n)\right\}, \end{aligned} \qquad (2)$$

where ϵ_0 is a spin resonance strength for a central particle on a reference orbit, $\rho = \epsilon_0 C_{rf}^2\psi_s^2/4$, $\varphi(n) = \arctan(\rho n)$ and $\kappa(n) = [\varphi(n) + \sin 2\varphi(n)]/2$.

Typical decoherence pattern is shown by solid line in Fig. 1. Here we evaluated $A(n)$ for deuterons of momentum $p = 970$ MeV/c, $\eta = -0.61$ [4], the momentum spread $\langle\Delta p^2/p^2\rangle^{1/2} = 3\cdot 10^{-4}$ and the RF driven up-down spin oscillations with the period $\tau_s = 2.4$ s. This corresponds to RF solenoid with $\int Bdl = 0.0264$ T mm. Then in this example we have $\rho = 1.37\cdot 10^{-7}$.

The overall attenuation of the envelope of the up-down spin oscillations comes from two non-exponential factors. The first one, $\exp\left\{-\sin^2\varphi(n)\right\}$ tends to a constant $1/e$ as soon as $\varphi(n) > 1$. A significance of this attenuation factor can be judged from a comparison of the solid line with dashed line – *2* in Fig. 1 – in the latter case we took out the factor $\exp\left\{-\sin^2\varphi(n)\right\}$.

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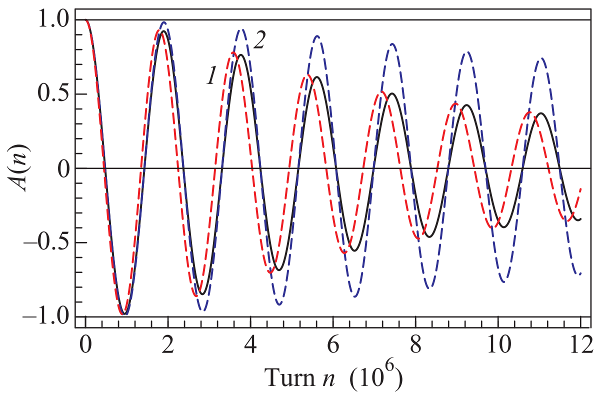

Fig. 1. (Color online) Non-exponential attenuation of vertical polarization oscillations: the solid curve is given by Eq. (2), the dashed (red, *1*) curve is Eq. (2) where we put spin phase walk $\kappa(n) = 0$ and the dashed line (blue, *2*) curve is for suppressed attenuation factor $\exp\{-\sin^2\varphi(n)\}$

As such, dashed line – *2* in Fig. 1 shows the effect of the second attenuation factor, $(1+\rho^2 n^2)^{-1/4}$, which decreases continuously $\propto 1/\sqrt{n}$. The customary attempts to describe this decoherence by conventional exponential attenuation will be entirely misleading. Besides the non-exponential damping of spin oscillations, we predict a nontrivial walk of the spin phase $\kappa(n)$.

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