Pis'ma v ZhETF, vol. 106, iss. 4, pp. 220 – 221

Effective Minkowski to Euclidean signature change of the magnon BEC pseudo-Goldstone mode in polar ^{3}He

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Submitted 5 July 2017

DOI: 10.7868/S0370274X17160068

Introduction. There are different classes of broken symmetry spin superfluid states [1]. The first of them contains magnetic systems with spontaneously broken spin rotation symmetry leading to the associated Nambu–Goldstone (NG) modes (spin waves or magnons), to spin supercurrents and to topological defects, such as spin vortices with circulating spin supercurrent [2–4] and half-quantum vortices [5].

The second class of spin superfluid states encompasses states which are periodic in time. A state with spontaneously formed phase-coherent precession of magnetization was first observed in ^{3}He-B [6, 7]. The lifetime of the coherent precession is extremely large compared to thermalization, and if dissipation is neglected, this spontaneously time-periodic state is an example of a time crystal [8–10]. From a different point of view, it is an out-of-equilibrium Bose–Einstein condensate (BEC) of optical magnons [11, 12].

The spontaneous breaking of time translation symmetry leads to the new NG mode – the propagating oscillations of the phase of precession [13–20].

Experimentally the magnon BEC in superfluid ^{3}He arises when the system is either continuosly driven with an external RF field or after a short RF pulse is applied. After the RF pulse is turned off, the spin precession experiences dephasing, but the phase coherence is rapidly restored, and spins enter a long-lived state – the magnon BEC, where the total spin $\mathbf{S}$ is freely precessing at an angle β with respect to $\mathbf{H}$, with the off-diagonal order parameter $\langle \hat{S}^+\rangle = \langle \hat{S}^x + i\hat{S}^y\rangle = S\sin\beta e^{i\omega t+i\alpha}$, where $n = S(1-\cos\beta)/\hbar$ is the condensate magnon number density, α the phase, and the precession frequency ω plays the role of a chemical potential μ.

Magnon BEC in polar ^{3}He. The order parameter of the polar phase is $A_{\alpha i} = \Delta_P \hat{d}_\alpha \hat{n}_i e^{i\Phi}$ [4], where Δ_P is the gap amplitude with phase Φ, $\hat{\mathbf{n}}$ the fixed orbital anisotropy, $\hat{\mathbf{d}}$ the spin-anisotropy axis. Spin dynamics is governed by the Leggett equations for $\mathbf{S}$ and $\hat{\mathbf{d}}$, with the Hamiltonian $F = F_{\rm spin} + F_{\rm grad} + F_{\rm so}$ [4]:

$$f_{\rm spin} = \frac{1}{2}\gamma_m^2 \mathbf{S}\boldsymbol{\chi}^{-1}\mathbf{S} - \gamma_m \mathbf{H}\cdot\mathbf{S}, \tag{1}$$

$$f_{\rm grad} = \frac{1}{2}K_{ij}\nabla_i \hat{d}_\alpha \nabla_j \hat{d}_\alpha,\ f_{\rm so} = g_D(\hat{\mathbf{d}}\cdot\hat{\mathbf{n}})^2. \tag{2}$$

The spin susceptibility is $\chi_{\alpha\beta} = \chi_\parallel \hat{d}_\alpha\hat{d}_\beta + \chi(\delta_{\alpha\beta} - \hat{d}_\alpha\hat{d}_\beta)$, $K_{ij} = K_\parallel \hat{n}_i\hat{n}_j + K_\perp(\delta_{ij} - \hat{n}_i\hat{n}_j)$ and $g_D = \frac{\chi\Omega_P^2}{2\gamma_m^2}$, where Ω_P is the Leggett frequency and γ_m the ^{3}He gyromagnetic ratio. In equilibrium $\mathbf{S} = \chi\mathbf{H}/\gamma_m$. The Hamiltonian for slow modes is obtained by averaging spin-orbit and gradient terms in Eq. (2) over the fast precession:

$$H(\alpha, n) = \\ = \int d^3\mathbf{r} \frac{1}{4S^2} n(n_{\max}+n) K_{ij}\nabla_i\alpha\nabla_j\alpha + n\mathbf{w}\cdot\nabla\alpha + \\ + \frac{K_{ij}}{4n(n_{\max}-n)}\nabla_i n\nabla_j n + (\omega_L - \mu)n + \epsilon(n) + \frac{M^2}{2}\alpha^2. \tag{3}$$

Here $n_{\max} = 2S$; $\mathbf{w}$ – the counterflow velocity; $\epsilon(n) = \langle f_{\rm so}(t)\rangle$ corresponds to a magnon-magnon interaction:

$$\epsilon(n) = \tfrac{g_D}{4}\Big(1 + \cos^2\lambda + (1 - 3\cos^2\lambda)\cos^2\beta - \\ - \frac{1}{2}(1+\cos\beta)^2\sin^2\lambda\Big), \tag{4}$$

and λ is the angle between $\hat{\mathbf{n}}$ and $\mathbf{H}$. The small mass $M^2 = \frac{\Omega_P^2}{8}\frac{H_{\rm rf}}{H}(1 - 5\cos^2\lambda)\sin\beta$ of the NG mode arises only for the continuous RF field $H_{\rm rf} \ll H$ [18, 19]. Setting $\mathbf{w} = 0$ one obtains the quadratic phonon spectrum: $\omega^2(\mathbf{k}) = c_\parallel^2 k_\parallel^2 + c_\perp^2 k_\perp^2 + M^2$. The anisotropic metric is $g^{ij} = (c_\parallel^2 - c_\perp^2)\hat{n}^i\hat{n}^j + c_\perp^2\delta^{ij}$, where the "speeds of light" for phonons are expressed in terms of the magnon's velocities [20] $c_{\parallel,\perp S}^2 = \gamma_m^2 K_{\parallel,\perp}/\chi$:

$$c_{\parallel,\perp}^2 = \frac{1}{16S^2}\frac{\Omega_P^2}{\omega_L^2}(1 - 5\cos^2\lambda)n(n_{\max}+n)c_{\parallel,\perp S}^2. \tag{5}$$

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The dispersion changes sign at $\epsilon'' \equiv d^2\epsilon/dn^2 = 0$ when $\tan\lambda = 2$, which implies the transition from a Minkowski to Euclidean signature metric for the dynamical phonon modes [21] and an instability.

The signature change of the full acoustic metric is obtained by considering the quadratic action $S = \int dt \left(\int d^3\mathbf{r}\, n\dot{\alpha} - H(\alpha, n)\right)$ of perturbations $\alpha \ll 1$:

$$S = \frac{1}{2}\int dt d^3\mathbf{r} \left(\frac{1}{\epsilon''}(\dot{\alpha} - \mathbf{w}\cdot\nabla\alpha)^2 - \gamma^{ij}\nabla_i\alpha\nabla_j\alpha\right) \equiv \equiv \frac{\operatorname{sgn}\epsilon''}{2}\int dt d^3\mathbf{r}\,\sqrt{|g|}g^{\mu\nu}\nabla_\mu\alpha\nabla_\nu\alpha, \quad (6)$$

where $\gamma_{ij} = \frac{n}{2S^2}(n_{\max} + n)K_{ij}$. For $\tan\lambda > 2$ the spin-orbit interaction is repulsive and the magnon BEC is stable. The phonon acoustic metric ($\gamma = \det\gamma^{ij}$):

$$g^{00} = \frac{1}{\sqrt{\gamma\epsilon''}}\ ,\ g^{0i} = -\frac{w^i}{\sqrt{\gamma\epsilon''}}\ ,\ g^{ij} = \frac{w^i w^j}{\sqrt{\gamma\epsilon''}} - \gamma^{ij}\sqrt{\frac{\epsilon''}{\gamma}}. \quad (7)$$

For $\tan\lambda < 2$, the spin-orbit interaction is attractive, the magnon BEC is unstable to phonon NG modes, $\omega(\mathbf{k})^2 < 0$, and the metric has Euclidean signature,

$$g_{\mathrm{E}}^{00} = \frac{1}{\sqrt{\gamma|\epsilon''|}},\ \ g_{\mathrm{E}}^{0i} = -\frac{w^i}{\sqrt{\gamma|\epsilon''|}}, \quad g_{\mathrm{E}}^{ij} = \frac{w^i w^j}{\sqrt{\gamma|\epsilon''|}} + \gamma^{ij}\sqrt{\frac{|\epsilon''|}{\gamma}}. \quad (8)$$

When the instability develops, it is cut-off at higher energies by the gradient term of n in Eq. (3) that have Minkowski signature in the dispersion [21, 22]. The magnon BEC in the unstable region serves as an non-relativistic version of a "ghost condensate" [22].

Outlook. In magnon superfluids there are two effective metrics: one for magnons and another for phonons in the magnon BEC in Eqs. (7), (8). The latter experiences a transition between repulsive and attractive spin-orbit interaction at $\tan\lambda_c = 2$, which corresponds to the transition between Minkowski and Euclidean signature of the metric. This does not correspond to the imaginary time *thermal* partition function, but to the *dynamic* phonon modes of the condensate. Such transition has been considered for ^{3}He-A [23, 24] and observed in disordered ^{3}He in aerogel [25]. Signature change occurs in many models of the early universe in cosmology, for quantum gravity and for cosmic strings [26–28]. It triggers a ghost instability of the quantum vacuum [22, 29], where the BEC decays as a false vacuum.

This work has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement # 694248).

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364017160032

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