Pis'ma v ZhETF, vol. 106, iss. 12, pp. 780 – 781 

# Probing the critical point of the Jaynes–Cummings second-order dissipative quantum phase transition

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Submitted 28 August 2017
Resubmitted 18 October 2017

DOI: 10.7868/S0370274X17240110

We highlight the importance of quantum fluctuations in organizing a dissipative quantum phase transition for the driven Jaynes–Cummings interaction with variable qubit-cavity detuning. Quantum fluctuations are responsible for the substantial difference from the predictions of the semiclassical theory, the extent of which is revealed in the properties of quantum bistability, and visualized with the help of *quasi*-distribution functions for the cavity field. The nonlinear system dynamics is subject to an appropriate scale parameter, based on which a "thermodynamic limit" can be defined.

The Master Equation (ME) in the interaction picture and the rotating wave approximation (RWA) for a two-level atom with frequency $\omega_q$ and raising (lowering) operators $\sigma_+(\sigma_-)$ interacting with a single cavity mode with frequency $\omega_c$ and raising (lowering) operators $a^\dagger(a)$, driven by a coherent field with strength $\varepsilon_d$ and frequency $\omega_d$ in the presence of dissipation at zero temperature, reads:

$$\begin{aligned}\dot{\rho} = \mathrm{i}\Delta\omega_c[a^\dagger a, \rho] + \mathrm{i}\Delta\omega_q[\sigma_+\sigma_-, \rho] - \mathrm{i}\varepsilon_d[a + a^\dagger, \rho] - \\ - \mathrm{i}g[a\sigma_+ + a^\dagger\sigma_-, \rho] + \kappa\left(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a\right) + \\ + (\gamma/2)\left(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-\right),\end{aligned} \tag{1}$$

where $\Delta\omega_{(c,q)} = \omega_d - \omega_{(c,q)}$ is the detuning between the laser driving field and the (cavity field, qubit) respectively. The coupling strength between the cavity mode and the atom, detuned by $\delta \equiv \omega_q - \omega_c$ [with $|\delta| \ll \omega_{(c,q)}$], is denoted by $g$, an interaction which is assumed to be much stronger than the cavity decay rate $2\kappa$ and the spontaneous emission rate $\gamma$ in the *strong coupling regime*.

The $Q$ function in the steady-state of the cavity field:

$$Q(x + \mathrm{i}y) = (1/\pi)\langle x + \mathrm{i}y|\rho_{cv,\mathrm{ss}}|x + \mathrm{i}y\rangle \tag{2}$$

is used to provide a "classical" visualization of the intracavity radiation field in the quantum-classical correspondence provided by *quasi*-probability distributions. In Eq. (2), $\rho_{cv,\mathrm{ss}}$ is the reduced cavity density matrix $\rho_{cv,\mathrm{ss}} = \lim_{t\to\infty}[\langle+|\rho(t)|+\rangle + \langle-|\rho(t)|-\rangle]$, where $|+\rangle$ and $|-\rangle$ are the upper and lower states of the two-level atom, respectively. The *quasi*-probability distribution in the $Q$ representation for a cavity field in the coherent state $|\alpha_c\rangle = |x_c + \mathrm{i}y_c\rangle$ with average photon occupation $\langle n\rangle \equiv \langle a^\dagger a\rangle = |\alpha_c|^2$ assumes a Gaussian form: $Q_c(x + \mathrm{i}y) = (1/\pi)\exp\{-[(x - x_c)^2 + (y - y_c)^2]\}$.

As we can observe in Fig. 1, there is a sharp drop in the cavity photon number as we move from $\delta < 0$ to $\delta > 0$ since the probabilities of occupying the two neoclassical states are reversed, with the low-photon state (closer to the center of coordinates) becoming dominant [see Frame (d)]. At the same time, the states of complex-amplitude quantum bistability remain centered at the same positions in the phase portrait for the same $|\delta|$. Moreover, at $\delta = 0$ [Frame (c)] there appears a third state very close to the center of co-ordinates along the excitation path of the JC ladder. The two states in Frames (a, b, d) satisfy the mean-field state equation of the Kerr nonlinearity:

$$\alpha = -\mathrm{i}\varepsilon_d\left\{\kappa - \mathrm{i}\left[\Delta\omega_c + \frac{g^2}{\delta}\left(1 + \frac{4g^2}{\delta^2}|\alpha|^2\right)^{-1/2}\right]\right\}^{-1}, \tag{3}$$

one for $\delta < 0$ (high-photon) and one for $\delta > 0$ (low-photon state). We note, remarkably, that both states are present in the phase portrait *quasi*-distribution, even if the value of $\delta$ has a definite sign, while the variation of qubit-cavity detuning results only in the change of their relative weights. At the same time, the Maxwell–Bloch equations do not predict any bistability for the corresponding drive parameters and vanishing spontaneous emission rate. On the other hand, the very low amplitude state of Frame (c) is a prediction of the *neoclassical* theory of radiation, satisfying the state equation:

$$\begin{aligned}\alpha = -\mathrm{i}\varepsilon_d\left[\kappa - \mathrm{i}\left(\Delta\omega_c - \frac{g^2}{\sqrt{\Delta\omega_c^2 + 4g^2|\alpha|^2}}\right)\right]^{-1} \approx \\ \approx -\mathrm{i}\varepsilon_d\left[\kappa - \mathrm{i}\left(\Delta\omega_c - \frac{g^2}{\Delta\omega_c}\right)\right]^{-1}, \ \mathrm{Re}(\alpha) \approx -\frac{\varepsilon_d\Delta\omega_c}{g^2}.\end{aligned} \tag{4}$$

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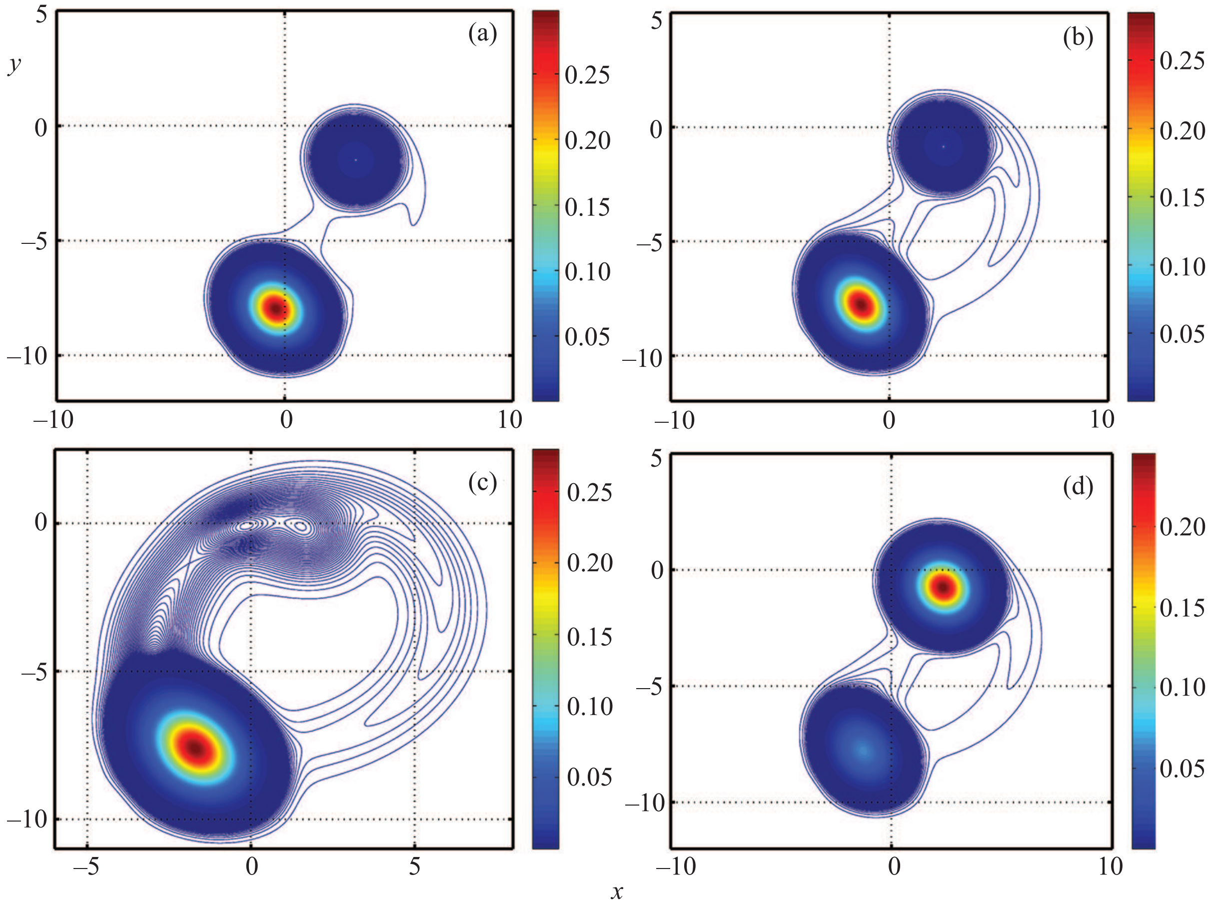


Fig. 1. (Color online) Towards qubit-cavity resonance. *Quasi*-probability function $Q(x+\mathrm{i}y)$ of the intracavity field for varying cavity-qubit detuning $\delta/g$: $-10, -5, 0, +5$ in (a)–(d) respectively. Parameters: $\Delta\omega_c/\kappa = 0.8$, $g/\kappa = 16$, $\gamma/(2\kappa) = 0$, and $\varepsilon_d = g/2$

The approximation in the second line would also give the Rabi resonances at $\Delta\omega_c = \pm g$ in the linear regime, for a much weaker drive and a larger drive-cavity detuning.

For $\Delta\omega_c = 0$, setting $\delta = \pm|\delta|$ in Eq. (3) yields two complex-conjugate neoclassical field amplitudes $\mathrm{i}\alpha$. Taking now the limit $|\delta| \to 0$ recovers the two states of phase bistability, $\alpha = -\mathrm{i}\varepsilon_d[\kappa \pm \mathrm{i}g/(2|\alpha|)]^{-1}$, which is reflected by two symmetrically located peaks of equal height in the $Q$ function. In the opposite limit, when $g/|\delta| \ll 1$ and $|\alpha|^2 \ll n_{\mathrm{nc,\ Kerr}}$, where $n_{\mathrm{nc,\ Kerr}} = [\delta/(2g)]^2$, the resonances of the linear strongly dispersive regime are located at $\Delta\omega_c = \pm g^2/|\delta|$.

Consequently, varying the detuning between the JC oscillator constituents and the drive allows us to extract information on the departure from the semiclassical theory, bringing together the dispersive ($|\delta| \gg g$) and the resonant ($\delta = 0$) behaviour around the critical point of a second-order phase transition. States obeying the neoclassical equations of radiation, together with their corresponding scaling, coexist in the quantum picture and differ from the predictions of the Maxwell-Bloch theory, following the occurrence of spontaneous dressed-state polarization and phase bistability at resonance.

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S002136401724002X