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Linear NMR in the polar phase of ^{3}He in aerogel

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^{3}He is an example of the system with non-trivial Cooper paring. A few superfluid phases are known in this system. Recently the new one, the polar phase, have been observed in ^{3}He confined in nematically ordered aerogel [1]. A number of various topological defects including half-quantum vortices can exist in the polar phase. Half-quantum vortices have been originally predicted for A-phase in [2] but have not been observed in experiments. This is because of energetically unfavorable solitons which should always connect half-quantum vortex pairs in the A-phase. In the polar phase of ^{3}He in aerogel there are no such solitons if the magnetic field is parallel to aerogel strands. Also vortices in the polar phase are strongly pinned and cannot move when the field is tilted and solitons appear. In our experimental work [3] half-quantum vortices were created by rotating the ^{3}He sample. Then magnetic field was tilted and spin waves localized in solitons were observed by nuclear magnetic resonance (NMR). In this paper we develop a theory for textures, topological defects and spin dynamics in the polar phase of ^{3}He. We also present results of numerical simulations of spin waves in the presence of half-quantum vortices.

We are studying the polar phase of ^{3}He in nematically ordered aerogel. The order parameter in this system ([4]) is

$$A_{aj} = \frac{1}{\sqrt{3}}\Delta\ e^{i\varphi} d_a l_j, \qquad (1)$$

where φ is the phase, and $\mathbf{d}$ and $\mathbf{l}$ are unit vectors in spin and orbital spaces, respectively. The orbital unit vector $\mathbf{l}$ is directed along the aerogel strands and can not move. There are three components of the Hamiltonian which are important for spin dynamics: magnetic energy, energy of spin-orbit interaction and gradient energy:

$$F_M = -(\mathbf{S}\cdot\gamma\mathbf{H}) + \frac{\gamma^2}{2}\chi^{-1}_{ab} S_a S_b,$$
$$F_{SO} = 3g_D\Big[A^*_{jj}A_{kk} + A^*_{jk}A_{kj} - \frac{2}{3}A^*_{jk}A_{jk}\Big],$$
$$F_\nabla = \frac{3}{2}\Big[K_1(\nabla_j A^*_{ak})(\nabla_j A_{ak}) +$$
$$+ K_2(\nabla_j A^*_{ak})(\nabla_k A_{aj}) + K_3(\nabla_j A^*_{aj})(\nabla_k A_{ak})\Big], \qquad (2)$$

where $\mathbf{S}$ is spin and $\mathbf{H}$ is the magnetic field. Susceptibility χ_{ab} is anisotropic, the axis of anisotropy is $\mathbf{d}$ and minimum of the magnetic energy corresponds to $\mathbf{S}\perp\mathbf{d}$.

We use a coordinate system where $\mathbf{H}\parallel\hat{\mathbf{z}}$ and $\mathbf{l}$ is in $\hat{\mathbf{z}}-\hat{\mathbf{y}}$ plane (see Fig. 1a):

$$\mathbf{H} = \hat{\mathbf{z}}H, \qquad \mathbf{l} = \hat{\mathbf{y}}\sin\mu + \hat{\mathbf{z}}\cos\mu,$$
$$\mathbf{d} = (\hat{\mathbf{x}}\cos\alpha + \hat{\mathbf{y}}\sin\alpha)\sin\beta + \hat{\mathbf{z}}\cos\beta. \qquad (3)$$

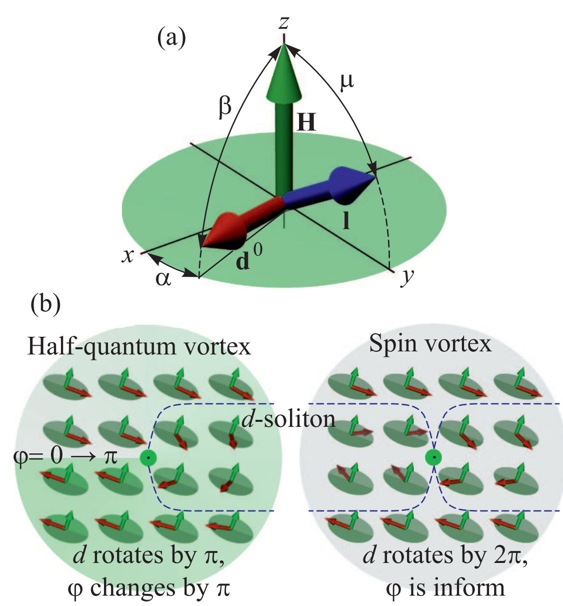

Fig. 1. (Color online) (a) – Angles, used in the texture calculations. (b) – The half-quantum vortex and the spin vortex in the polar phase of ^{3}He. Vector $\mathbf{l}$ is perpendicular to the picture plane. Angle $\alpha = 0$ is changing by π between upper and lower parts of the picture. This can be done via either a d-soliton or a π jump in the phase (which is shown by color gradient)

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Here μ is angle between $\mathbf{l}$ and magnetic field, this angle is set by the experimental setup because direction of $\mathbf{l}$ is determined by aerogel; β is angle between $\mathbf{d}$ and the field; α is azimuthal angle of $\mathbf{d}$ in the plane, perpendicular to the magnetic field.

There are two characteristic scales introduced by energies (2). Ratio of magnetic and gradient energies gives the magnetic length ξ_H and ratio of spin-orbit and gradient energies gives the dipolar length ξ_D. Since the gradient energy is anisotropic, we have different values in directions perpendicular and parallel to the $\mathbf{l}$ vector:

$$\xi^2_{Hjk} = \frac{K_{jk}\Delta^2}{H^2(\chi_\perp - \chi_\parallel)}, \quad \xi^2_{Djk} = \frac{K_{jk}}{4g_D}, \tag{4}$$

where $K_{jk} = K_1\delta_{jk} + (K_2 + K_3)l_j l_k$.

In the high-field limit $\xi_D \gg \xi_H$ magnetic energy is in the minimum everywhere excluding small regions of the ξ_H size (for example cores of spin vortices). The small volume of this regions makes them invisible in NMR experiments. In the rest of the volume $\beta = \pi/2$ and only variations of α are important.

By minimizing the energy (2) with the order parameter (1) we find equation for the distribution of α:

$$\bar{\xi}^2_{jk}\ \nabla_j\nabla_k\alpha = \frac{1}{2}\sin 2\alpha, \quad \text{where} \quad \bar{\xi}_{jk} = \frac{\xi_{Djk}}{\sin\mu}. \tag{5}$$

In the case of $\mathbf{H} \parallel \mathbf{l}$ (or $\mu = 0$) there is no length scale in this problem. $\mathbf{d}$ can freely move in the plane perpendicular to the field and only the gradient term is important. Tilting the magnetic field from the $\mathbf{l}$ direction makes the length $\bar{\xi}$ finite. In the tilted field the equation for the texture has a form of static sine-Gordon equation. It has two uniform solutions with $\alpha = 0$ and $\alpha = \pi$. Transition between these solutions is a *d-soliton*. In one-dimensional case it has the form:

$$\alpha(x) = 2\arctan\left(\exp(x/\bar{\xi})\right), \tag{6}$$

where $\bar{\xi}$ depends on the soliton orientation: if x coordinate goes perpendicular or parallel to $\mathbf{l}$, it should be $\bar{\xi}_\perp$ or $\bar{\xi}_\parallel$, respectively.

Looking at the order parameter formula (1) one can see that there can be also a *half-quantum vortex*, in which both vector $\mathbf{d}$ and phase ϕ rotate by π around the vortex line. This is possible because $A_{\alpha j}(\mathbf{d},\phi) = A_{\alpha j}(-\mathbf{d},\phi+\pi)$. In the tilted magnetic field one d-soliton should end at the half-quantum vortex. The texture can also form a *spin vortex* in which vector $\mathbf{d}$ rotates by 2π around the vortex line. In this case two d-solitons should end at this vortex. On Figure 1b both types of vortices are shown.

Linearized spin dynamics is described by equation:

$$(\omega^2 - \omega_L^2)s_+ = \Omega_P^2\left\{\cos^2\mu - \sin^2\alpha\sin^2\mu\right\} s_+ - \\ - c^2_{jk}\left\{-\left(\frac{\nabla}{i} + \nabla\alpha\right)^2_{jk} + (\nabla\alpha)^2_{jk}\right\} s_+, \tag{7}$$

where $s_+ = (s_x + is_y)/\sqrt{2}$ is a complex deviation of spin from the equilibrium value. This is similar to the equation of motion of a charged particle in a magnetic field with a vector potential $\mathbf{A} = \nabla\alpha$. The "magnetic field" $\nabla\times\mathbf{A}$ is zero everywhere except half-quantum vortex cores but it affects the motion of the spin wave because of Aharonov-Bohm effect [5]. This effect for half-quantum vortices in ^{3}He-A is discussed in [6].

Equation (7) can be used to find an NMR frequency in a uniform texture as well as a frequency of a spin-wave localized in the d-soliton (6):

$$\omega_u = \sqrt{\omega_L^2 + \Omega_P^2\ \cos^2\mu}, \tag{8}$$

$$\omega_s = \sqrt{\omega_L^2 + \Omega_P^2\cos 2\mu}. \tag{9}$$

These two frequencies have been observed in NMR experiment in [3].

To study d-solitons with finite length and interaction between d-solitons we do a 2D numerical simulation of texture and spin waves. Five structures have been calculated: a single soliton between two half-quantum vortices with a length D; A periodic structures of infinite solitons with the period D and same or alternating soliton orientations; the combination of both effects, periodic structures of finite solitons with equal length and period D (this corresponds to a square lattice of vortices). For large values of structure dimension D calculated frequency coincides with (9). Noticeable deviations appear only when D is comparable with $\bar{\xi}$.

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