Pis'ma v ZhETF, vol. 77, iss. 7, pp. 407 – 411 © 2003 April 10

Evolution of cosmological constant in effective gravity

G. E. Volovik[1)]

Low Temperature Laboratory, Helsinki University of Technology, FIN-02015 HUT, Finland

Landau Institute for Theoretical Physics RAS, 117334 Moscow, Russia

Submitted 20 February 2003
Resubmitted 5 March 2003

In contrast to the phenomenon of nullification of the cosmological constant in the equilibrium vacuum, which is the general property of any quantum vacuum, there are many options in modifying the Einstein equation to allow the cosmological constant to evolve in a non-equilibrium vacuum. An attempt is made to extend the Einstein equation in the direction suggested by the condensed-matter analogy of the quantum vacuum. Different scenarios are found depending on the behavior of and the relation between the relaxation parameters involved, some of these scenarios having been discussed in the literature. One of them reproduces the scenario in which the effective cosmological constant emerges as a constant of integration. The second one describes the situation, when after the cosmological phase transition the cosmological constant drops from zero to the negative value; this scenario describes the relaxation from this big negative value back to zero and then to a small positive value. In the third example the relaxation time is not a constant but depends on matter; this scenario demonstrates that the vacuum energy (or its fraction) can play the role of the cold dark matter.

PACS: 04.90.+e, 98.80.–k

1. Introduction. It is clear that the pressure of the vacuum in our Universe is very close to zero as compared to the Planck energy scale, and thus the experimental cosmological constant is close to zero. However, if at this almost zero pressure one starts calculating the vacuum energy by summing all the positive and negative energy states, one obtains a huge vacuum energy which by about 120 orders of magnitude exceeds the experimental limit. This is the main cosmological constant problem [1 – 4].

Exactly the same 'paradox' occurs in any quantum liquid (or in any other condensed matter) at zero pressure. The experimental energy of the ground state of, say, the quantum liquid at zero pressure is zero. On the other hand, if one starts calculating the vacuum energy summing the energies of all the positive and negative energy modes up to the corresponding Planck (Debye) scale, one obtains the huge energy. However, there is no real paradox in quantum liquids, since if one adds all the trans-Planckian (microscopic, atomic) modes one immediately obtains the zero value, irrespective of the details of the microscopic physics [5]. The fully microscopic consideration restores the Gibbs-Duhem relation $\epsilon = -p$ between the energy (the relevant thermodynamic potential) and the pressure of the quantum liquid at $T = 0$, which ensures that the energy of the vacuum state $\epsilon = 0$ if the external pressure is zero.

This is the first message from condensed matter to the physics of the quantum vacuum: One should not worry about the huge vacuum energy, the trans-Planckian physics with its degrees of freedom will do all the job of the cancellation of the vacuum energy without any fine tuning and irrespective of the details of the trans-Planckian physics. There are other messages which are also rather general and do not depend much on details of the trans-Planckian physics. For example, if the cosmological constant is zero above the cosmological phase transition, it will become zero below the transition after some transient period.

Thus from the quantum-liquid analog of the quantum vacuum it follows that the cosmological constant is not a constant but is an evolving physical parameter, and our goal is to find the laws of its evolution. In contrast to the phenomenon of the cancellation of the cosmological constant in the equilibrium vacuum, which is the general property of any quantum vacuum, there are many options in modifying the Einstein equation to allow the cosmological constant to evolve. However, the condensed matter physics teaches us that we must avoid the discussion of the microscopic models of the quantum vacuum [6] and use instead the general phenomenological approach. That is why we do not follow the traditional way of description in terms of, say, the scalar field which mediates the decay of the dark energy [7], and present an attempt of the pure phenomenological description by introducing the dissipative terms directly into the Einstein equation.

1) e-mail: volovik@boojum.hut.fi

2. Einstein equation. *Standard formulation.* Let us start with the non-dissipative equation for gravity – the Einstein equation. It is obtained from the action:

$$S = S_{\rm E} + S_\Lambda + S_{\rm M}, \tag{1}$$

where $S_{\rm M}$ is the matter action;

$$S_{\rm E} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\mathcal{R} \tag{2}$$

is the Einstein curvature action; and

$$S_\Lambda = -\frac{\Lambda}{8\pi G}\int d^4x\sqrt{-g}, \tag{3}$$

where Λ is the cosmological constant [8]. Variation over the metric $g^{\mu\nu}$ gives

$$\frac{1}{8\pi G}\left(G_{\mu\nu} - \Lambda g_{\mu\nu}\right) = T^{\rm M}_{\mu\nu}, \tag{4}$$

where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}\mathcal{R}g_{\mu\nu} \tag{5}$$

is the Einstein tensor, and $T^{\rm M}_{\mu\nu}$ is the energy-momentum tensor for matter. This form of the Einstein equation implies that the matter fields on the right-hand side of Eq.(4) serve as the source of the gravitational field, while the Λ-term belongs to the gravity.

Moving the Λ-term to the rhs of the Einstein equation changes the meaning of the cosmological constant. The Λ-term becomes the energy-momentum tensor of the vacuum, which in addition to the matter is the source of the gravitational field [9]:

$$\frac{1}{8\pi G}G_{\mu\nu} = T^{\rm M}_{\mu\nu} + T^{\Lambda}_{\mu\nu},\ T^{\Lambda}_{\mu\nu} = \rho_\Lambda g_{\mu\nu} = \frac{\Lambda}{8\pi G}g_{\mu\nu}. \tag{6}$$

Here ρ_Λ is the vacuum energy density and $p_\Lambda = -\rho_\Lambda$ is the vacuum pressure.

Einstein equation in induced gravity. In the induced gravity introduced by Sakharov [10], the gravity is the elasticity of the vacuum, say, fermionic vacuum, and the action for the gravitational field is induced by the vacuum fluctuations of the fermionic matter fields. Such kind of the effective gravity emerges in quantum liquids [5]. In the induced gravity the Einstein tensor must be also moved to the matter side, i.e. to the rhs:

$$0 = T^{\rm M}_{\mu\nu} + T^{\Lambda}_{\mu\nu} + T^{\rm curv}_{\mu\nu}, \tag{7}$$

where

$$T^{\rm curv}_{\mu\nu} = -\frac{1}{8\pi G}G_{\mu\nu} \tag{8}$$

has the meaning of the energy-momentum tensor produced by deformations of the fermionic vacuum. It describes such elastic deformations of the vacuum, which distort the effective metric field $g_{\mu\nu}$ and thus play the role of the gravitational field. As distinct from the $T^{\Lambda}_{\mu\nu}$ term which is of the 0-th order in gradients of the metric, the $T^{\rm curv}_{\mu\nu}$ term is of the 2-nd order in gradients of $g_{\mu\nu}$. The higher-order gradient terms also naturally appear in induced gravity.

In the induced gravity the free gravitational field is absent, since there is no gravity in the absence of the quantum vacuum. Thus the total energy-momentum tensor comes from the original (bare) fermionic degrees of freedom. That is why all the contributions to the energy-momentum tensor are obtained by the variation of the total fermionic action over $g^{\mu\nu}$:

$$T^{\rm total}_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = -\frac{1}{8\pi G}G_{\mu\nu} + \rho_\Lambda g_{\mu\nu} + T^{\rm M}_{\mu\nu}. \tag{9}$$

According to the variational principle, $\delta S/\delta g^{\mu\nu} = 0$, the total energy-momentum tensor is zero, which gives rise to the Einstein equation in the form of Eq.(7). Since $T^{\rm total}_{\mu\nu} = 0$, it satisfies the conventional and covariant conservation laws, $T^{{\rm total}\ \nu}_{\mu,\nu} = 0$ and $T^{{\rm total}\ \nu}_{\mu;\nu} = 0$, and thus serves as the covariant and localized energy-momentum tensor of matter and gravity.

In this respect there is no much difference between different contributions to the energy-momentum tensor: all come from the original fermions. However, in the low-energy corner, where the gradient expansion for the effective action works, one can distinguish between different contributions: (i) some part of the energy-momentum tensor ($T^{\rm M}_{\mu\nu}$) comes from the excited fermions – quasiparticles – which in the effective theory form the matter. The other parts come from the fermions forming the vacuum – the Dirac sea. The contribution from the vacuum fermions contains: (ii) The zeroth-order term in the gradients of $g_{\mu\nu}$; this is the energy-momentum tensor of the homogeneous vacuum – the Λ-term. Of course, the whole Dirac sea cannot be sensitive to the change of the effective infrared fields $g_{\mu\nu}$: only small infrared perturbations of the vacuum, which we are interested in, are described by these effective fields. (iii) The stress tensor of the inhomogeneous distortion of the vacuum state, which plays the role of gravity; the second-order term $T^{\rm curv}_{\mu\nu}$ in the stress tensor represents the curvature term in the Einstein equation.

The same occurs in induced QED [11], where the electromagnetic field is induced by the vacuum fluctuations of the same fermionic field. The total electric current

$$j^\mu = \frac{\delta S}{\delta A_\mu} = \frac{\delta S^{\rm int}}{\delta A_\mu} + \frac{\delta S^{\rm Maxwell}}{\delta A_\mu} = j^\mu_{\rm charged\ particles} + j^\mu_{\rm field}, \tag{10}$$

is produced by excited fermions (the 1-st term) and by fermions in the quantum vacuum (the 2-nd term). The electric current in the second term is produced by such elastic deformations of the fermionic vacuum which play the role of the electromagnetic field, and the lowest-order term in the effective action describing such distortion of the vacuum state is the induced Maxwell action

$$S^{\rm Maxwell} = \int dt d^3x \frac{\sqrt{-g}}{16\pi\alpha} F^{\mu\nu} F_{\mu\nu}, \tag{11}$$

which is of the second order in gradients of A_μ. According to the variational principle, $\delta S/\delta A_\mu = 0$, the total electric current produced by the vacuum and excited fermions is zero. This means that the system is always locally electro-neutral. This ensures that the homogeneous vacuum state without excitations has zero electric charge, i.e. our quantum vacuum is electrically neutral. The same occurs with the hypercharge, weak charge, and color charge of the vacuum: they are zero in the absence of matter and fields.

In the traditional approach the cosmological constant is fixed, and it serves as the source for the metric field: in other words the input in the Einstein equation is the cosmological constant, the output is de Sitter expansion, if matter is absent. In effective gravity, where the gravitational field, the matter fields, and the cosmological 'constant' emerge simultaneously in the low-energy corner, one cannot say that one of these fields is primary and serves as a source for the other fields thus governing their behavor. The cosmological constant, as one of the players, adjusts to the evolving matter and gravity in a self-regulating way. In particular, in the absence of matter ($T^{\rm M}_{\mu\nu} = 0$) the non-distorted vacuum ($T^{\rm curv}_{\mu\nu} = 0$) acquires zero cosmological constant, since, according to the 'gravi-neutrality' condition Eq.(7), it follows from equations $T^{\rm M}_{\mu\nu} = 0$ and $T^{\rm curv}_{\mu\nu} = 0$ that $T^{\Lambda}_{\mu\nu} = 0$. In this approach, the input is the vacuum configuration (in a given example there is no matter, and the vacuum is homogeneous), the output is the vacuum energy. In contrast to the traditional approach, here the gravitational field and matter serve as a source of the induced cosmological constant.

This conclusion is supported by the effective gravity and effective QED which emerge in quantum liquids or any other condensed matter system of the special universality class [5]. The nullification of the vacuum energy in the equilibrium homogeneous vacuum state of the system also follows from the variational principle, or more generally from the Gibbs-Duhem relation applied to the equilibrium vacuum state of the fermionic system if it is isolated from the environment. In the absence of the environment one has $p_\Lambda = 0$, while from the Gibbs-Duhem relation $\rho_\Lambda = -p_\Lambda$ at $T = 0$ it follows that $\rho_\Lambda = 0$. This corresponds to $T^{\Lambda}_{\mu\nu} = 0$ for quiescent flat Universe at $T = 0$, i.e. quiescent flat Universe without matter is not gravitating.

3. Modification of Einstein equation and relaxation of the vacuum energy. *Dissipation in Einstein equation.* The Einstein equation does not allow us to obtain the time dependence of the cosmological constant, because of the Bianchi identities $G^{\nu}_{\mu;\nu} = 0$ and covariant conservation law for matter fields (quasiparticles) $T^{\nu{\rm M}}_{\mu;\nu} = 0$ which together lead to $\partial_\mu \Lambda = 0$. But they allow us to obtain the value of the cosmological constant in different static Universes, such as the Einstein closed Universe [8], where the cosmological constant is obtained as a function of the curvature and matter density. To describe the evolution of the cosmological constant the relaxation term must be added.

The dissipative term in the Einstein equation can be introduced in the same way as in two-fluid hydrodynamics [12] which serves as the non-relativistic analog of the self-consistent treatment of the dynamics of the vacuum (the superfluid component of the liquid) and matter (the normal component of the liquid) [5]

$$T^{\rm M}_{\mu\nu} + T^{\Lambda}_{\mu\nu} + T^{\rm curv}_{\mu\nu} + T^{\rm diss}_{\mu\nu} = 0, \tag{12}$$

where $T^{\rm diss}_{\mu\nu}$ is the dissipative part of the total energy-momentum tensor. In contrast to the conventional dissipation of the matter, such as viscosity and thermal conductivity of the cosmic fluid, this term is not the part of $T^{\rm M}_{\mu\nu}$. It describes the dissipative back reaction of the vacuum, which does not influence the matter conservation law $T^{\nu{\rm M}}_{\mu;\nu} = 0$. The condensed-matter example of such relaxation of the variables describing the fermionic vacuum is provided by the dynamic equation for the order parameter in superconductors – the time-dependent Ginzburg-Landau equation which contains the relaxation term (see e.g. the book [13]).

Let us consider how the relaxation occurs on the example of the spatially flat Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t) d\mathbf{r}^2. \tag{13}$$

The Ricci tensor and scalar and the Einstein tensor are

$$R^0_0 = -3\frac{\partial_t^2 a}{a}; \quad R^i_j = -\delta^i_j \left(\frac{\partial_t^2 a}{a} + 2\frac{(\partial_t a)^2}{a^2} \right), \tag{14}$$

$$\mathcal{R} = -6\left(\frac{\partial_t^2 a}{a} + \frac{(\partial_t a)^2}{a^2}\right), \qquad (15)$$

$$G_0^0 = R_0^0 - \frac{1}{2}\mathcal{R} = 3\frac{(\partial_t a)^2}{a^2} \equiv 3H^2, \qquad (16)$$

$$G_i^j = R_i^j - \frac{1}{2}\mathcal{R}\delta_i^j = \delta_i^j\left(H^2 + 2\frac{\partial_t^2 a}{a}\right) = \delta_i^j\left(3H^2 + 2\dot{H}\right). \qquad (17)$$

In the lowest order of the gradient expansion, the dissipative part $T^{\rm diss}_{\mu\nu}$ of the stress tensor describing the relaxation of Λ must be proportional to the first time derivative.

Cosmological constant as integration constant. Let us start with the following guess

$$T^{\rm diss}_{\mu\nu} = \tau_\Lambda \frac{\partial \rho_\Lambda}{\partial t} g_{\mu\nu}. \qquad (18)$$

This non-covariant term implies the existence of the preferred reference frame, which is the natural ingredient of the trans-Planckian physics where the general covariance is violated. The modified Einstein equation in the absence of matter are correspondingly

$$3H^2 = \Lambda + \tau_\Lambda \dot{\Lambda}, \qquad (19)$$

$$3H^2 + 2\dot{H} = \Lambda + \tau_\Lambda \dot{\Lambda}. \qquad (20)$$

This gives the constant Hubble parameter $H =$ constant, i.e. the exponential de Sitter expansion or contraction. The cosmological 'constant' Λ relaxes to the value determined by the expansion rate:

$$\Lambda(t) = 3H^2 + \left(\Lambda(0) - 3H^2\right)\exp\left(-\frac{t}{\tau_\Lambda}\right). \qquad (21)$$

This is consistent with the Bianchi identity, which requires that $\partial_t(\Lambda + \tau_\Lambda\dot{\Lambda}) = 0$. Actually this situation corresponds to the well known case when the cosmological constant arises as an integration constant (see reviews [1, 4]). Here it is the integration constant $\Lambda_0 = \Lambda + \tau_\Lambda\partial_t\Lambda$. Such scenario emerges because the dissipative term in Eq.(18) is proportional to $g_{\mu\nu}$.

Model with two relaxation parameters. Since $T^{\rm diss}_{\mu\nu}$ is tensor, the general description of the vacuum relaxation requires introduction of several relaxation times. This also violates the Lorentz invariance, but we already assumed that the dissipation of the vacuum variables due to trans-Planckian physics implies the existence of the preferred reference frame. In the isotropic space we have only two relaxation times: in the energy and pressure sectors. In the presence of matter one has

$$3H^2 = \Lambda + \tau_1\dot{\Lambda} + 8\pi G\rho^{\rm M}, \qquad (22)$$

$$3H^2 + 2\dot{H} = \Lambda + \tau_2\dot{\Lambda} - 8\pi G p^{\rm M}. \qquad (23)$$

Since the covariant conservation law for matter does not follow now from the Bianchi identities, these two equations must be supplemented by the covariant conservation law to prevent the creation of matter:

$$a\frac{\partial}{\partial a}(\rho^{\rm M}a^3) = p^{\rm M}a^3. \qquad (24)$$

Let us consider the simplest case when the relaxation occurs only in the pressure sector, i.e. $\tau_1 = 0$. We assume also that the ordinary matter is cold, i.e. its pressure $p^{\rm M} = 0$, which gives $\rho^{\rm M} \propto a^{-3}$. Then one finds two classes of solutions: (i) $\Lambda =$ constant; and (ii) $H = 1/3\tau_2$. The first one corresponds to the conventional expansion with constant Λ-term and cold matter, so let us discuss the second solution, $H = 1/3\tau_2$.

Relaxation after cosmological phase transition. In the simplest case when $\tau_2 \equiv \tau =$ const, the Λ-term and the energy density of matter $\rho^{\rm M}$ exponentially relax to $1/3\tau^2$ and to 0 respectively:

$$H = \frac{1}{3\tau}, \quad \Lambda(t) = \frac{1}{3\tau^2} - 8\pi G\rho^{\rm M}(t), \qquad \frac{\rho^{\rm M}(t)}{\rho^{\rm M}(0)} = \exp\left(-\frac{t}{\tau}\right). \qquad (25)$$

Such solution describes the behavior after the cosmological phase transition. According to the condensed-matter example of the phase transition, the cosmological 'constant' is (almost) zero before the transition; while after the transition it drops to the negative value, and then relaxes back to zero [5]. Eq.(25) corresponds to the latter stage, but it demostrates that in its relaxation after the phase transition Λ crosses zero and finally becomes a small positive constant determined by the relaxation parameter τ which governs the exponential de Sitter expansion.

Dark energy as dark matter. Let us now allow τ to vary. Usually the relaxation and dissipation are determined by matter (quasiparticles). The term which violates the Lorentz symmetry or the general covariance must contain the Planck scale $E_{\rm Planck}$ in the denominator, since it must disappear at infinite Planck energy. The lowest-order term, which contains the $E_{\rm Planck}$ in the denominator, is $\hbar/\tau \sim T^2/E_{\rm Planck}$, where T is the characteristic temperature or energy of matter. In case of radiation it can be written in terms of the radiation density:

$$\frac{1}{3\tau^2} = 8\pi\alpha G\rho^{\rm M}, \quad (26)$$

where α is the dimensionless parameter. If Eq.(26) can be applied to the cold baryonic matter too, then the solution of the class (ii) becomes again $H = 1/3\tau$, but now τ depends on the matter field. This solution gives the standard power law for the expansion of the cold flat Universe and the relation between Λ and the baryonic matter $\rho^{\rm M}$:

$$a \propto t^{2/3}, \quad 8\pi G\rho^{\rm M} = \frac{4}{3\alpha t^2}, \qquad H = \frac{2}{3t}, \quad \Lambda = (\alpha - 1)8\pi\; G\rho^{\rm M}. \quad (27)$$

In terms of the densities normalized to $\rho_c = 3H^2/8\pi G$ (the critical density corresponding to the flat Universe in the absence of the vacuum energy) $\Omega_\Lambda = \rho_\Lambda/\rho_c$ and $\Omega^{\rm M} = \rho^{\rm M}/\rho_c$ one has

$$\Omega^{\rm M} = \frac{1}{\alpha}, \quad \Omega_\Lambda = \frac{\alpha - 1}{\alpha}. \quad (28)$$

Since the effective vacuum pressure in Eq.(23) is $p_\Lambda \propto \propto -(\Lambda + \tau\dot{\Lambda}) = 0$, in this solution the dark energy behaves as the cold dark matter. Thus the vacuum energy can serve as the origin of the non-baryonic dark matter.

4. Conclusion. In the effective gravity the equilibrium time-independent vacuum state without matter is non-gravitating, i.e. its relevant vacuum energy which is responsible for gravity is zero. In a non-equilibrium situation the cosmological constant is non-zero, but it is an evolving parameter rather than the constant. The process of relaxation of the cosmological constant, when the vacuum is disturbed and out of the equilibrium, requires some modification of the Einstein equation violating the Bianchi identities to allow the cosmological constant to vary. In contrast to the phenomenon of nullification of the cosmological constant in the equilibrium vacuum, which is the general property of any quantum vacuum and does not depend on its structure and on details of the trans-Planckian physics, the deviations from the general relativity can occur in many different ways, since there are many routes from the low-energy effective theory to the high-energy 'microscopic' theory of the quantum vacuum. However, it seems reasonable that such modification can be written in the general phenomenological way, as for example the dissipative terms are introduced in the hydrodynamic theory. Here we suggested to describe the evolution of the Λ-term by two phenomenological parameters (or functions) – the relaxation times. The corresponding dissipative terms in the stress tensor of the quantum vacuum are determined by trans-Planckian physics and do not obey the general covariance.

We discussed here simplest examples of the relaxation of the vacuum to equilibrium described by a single relaxation parameter. The first example ($\tau_1 = \tau_2$) reproduces the well known scenario in which the effective cosmological constant emerges as a constant of integration. The second example ($\tau_1 = 0$ and $\tau_2 = {\rm const}$) describes the situation which occurs if, after the cosmological phase transition, Λ acquires a big negative value: Λ relaxes back to zero and then to a small positive value. The third example, when τ_2 is determined by the baryoinic matter density, demonstrates that the vacuum energy (or its fraction) can play the role of the cold dark matter.

These examples are too simple to describe the real evolution of the present Universe and are actually excluded by observations [2]. The general consideration with two relaxation functions is needed. In this general case, it corresponds to the varying in time parameter $w_Q = p_Q/\rho_Q$ describing the equation of state of the quintessence with $w_Q(t) = -(\Lambda + \tau_2\dot{\Lambda})/(\Lambda + \tau_1\dot{\Lambda})$. The recent observational bounds on w_Q can be found, for example, in [14].

I thank A. Achúcarro, T. W. B. Kibble, I. I. Kogan, L. B. Okun, A. K. Rajantie and A. A. Starobinsky for fruitful discussions. This work was supported by ESF COSLAB Programme and by the Russian Foundations for Fundamental Research.

1. S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
2. V. Sahni and A. Starobinsky, Int. J. Mod. Phys. **D9**, 373 (2000).
3. P. J. E. Peebles and B. Ratra, astro-ph/0207347.
4. T. Padmanabhan, hep-th/0212290.
5. G.E. Volovik, *The Universe in a Helium Droplet*, Clarendon Press, Oxford, 2003.
6. R. B. Laughlin, gr-qc/0302028.
7. U. Alam, V. Sahni, and A. A. Starobinsky, astro-ph/0302302.
8. A. Einstein, Sitzungsber. Konigl. Preuss. Akad. Wiss. **1**, 142 (1917).
9. M. Bronstein, Physikalische Zeitschrift der Sowietunion **3**, 73 (1933).
10. A. D. Sakharov, Sov. Phys. Dokl. **12**, 1040 (1968).
11. Ya. B. Zel'dovich, JETP Lett. **6**, 345 (1967).
12. I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity*, Benjamin, New York, 1965.
13. N. B. Kopnin, *Theory of nonequilibrium superconductivity*, Clarendon Press, Oxford, 2001.
14. D. Pogosyan, J. R. Bond, and C. R. Contaldi, astro-ph/0301310.