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Noise in the helical edge channel anisotropically coupled to a local spin

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Experiments revealed that the actual conductance of the edge states of two-dimensional topological insulators (2D TI) is much smaller than the theoretical value e^2/h [1], which implies a presence of spin-flip processes. Different mechanisms of such processes were proposed [2–7], but none of them has obtained a definite experimental confirmation.

An efficient tool for determining the mechanism of conduction are measurements of nonequilibrium noise. So far, most theoretical papers dealing with noise in 2D TI addressed the electron tunneling between the helical states at opposite edges of the insulator [8–11]. The noise in the edge states themselves due to the hyperfine interaction of the electrons with nuclear spins and nonuniform spin-orbit coupling was calculated in [12]. The shot noise that results from the exchange of electrons between the edge states and conducting puddles in the bulk of the insulator was calculated in [13]. Very recently, the current noise generated by a local magnetic moment coupled to the edge states was considered in [14]. These authors calculated the noise spectrum for the case of isotropic coupling by extrapolating the Nyquist relation to finite voltages. They also presented an expression for the noise at vanishingly small anisotropic coupling, zero frequency, and high bias.

In this paper, we microscopically calculate the nonequilibrium electrical noise for an arbitrary anisotropy of exchange coupling of the edge states to a local spin 1/2 and arbitrary classical frequencies. Our results coincide with [14] in the limiting cases.

Consider a pair of helical edge states with linear dispersion $\varepsilon(k) = \pm v_0 k$. These states connect two electron reservoirs kept at constant voltages $\pm V/2$ and are coupled to a magnetic impurity via a Hamiltonian [15]

$$\begin{aligned} H_{\rm int} = J_z S_z s_z + J_0\,(S_+ s_- + S_- s_+) + \\ + J_a\,(S_+ + S_-)\,s_z + \\ + J_1 S_z\,(s_+ + s_-) + J_2\,(S_+ s_+ + S_- s_-), \end{aligned} \tag{1}$$

where S_z, $S_\pm = S_x \pm iS_y$ and $s_z, s_\pm$ are the operators of the impurity spin and of the spin density of electrons at its location. In the weak-coupling limit[2)], the master equation for the occupation numbers of the spin-up and spin-down states $N_\uparrow$ and $N_\downarrow = 1 - N_\uparrow$ is of the form

$$\frac{dN_\uparrow}{dt} = (\Gamma_0^+ + \Gamma_a + \Gamma_2^+)\,N_\downarrow - (\Gamma_0^- + \Gamma_a + \Gamma_2^-)\,N_\uparrow. \tag{2}$$

The transition rates $\Gamma_0^\pm$, Γ_a, and $\Gamma_2^\pm$ are proportional to $\alpha_0 = |J_0|^2/4v_0^2$, $\alpha_a = |J_a|^2/4v_0^2$, and $\alpha_2 = |J_2|^2/4v_0^2$, respectively. The electrical current equals

$$\begin{aligned} I = I_\uparrow^{\rm in} + I_\downarrow^{\rm in} - e\,(\Gamma_0^+\,N_\downarrow - \Gamma_0^-\,N_\uparrow) + \\ + e\,(\Gamma_1^+ - \Gamma_1^-) + e\,(\Gamma_2^+\,N_\downarrow - \Gamma_2^-\,N_\uparrow), \end{aligned} \tag{3}$$

where the currents injected into the edge states from the left and right reservoirs are given by equations $I_{\uparrow,\downarrow}^{\rm in} = \pm(e/2\pi\hbar)\int d\varepsilon\, f_{\uparrow,\downarrow}(\varepsilon)$ and the rest of terms describe spin-flip backscattering of electrons from the impurity. The rates of scattering events that do not change the impurity spin $\Gamma_1^\pm$ are proportional to $\alpha_1 = |J_1|^2/4v_0^2$.

The two sources of noise in the edge states are the partition noise and the occupation-number noise [16]. The former may be taken into account by introducing into Eqs. (2) and (3), the external sources associated with each scattering process whose spectral density is twice the sum of scattering fluxes in both directions [17]. The latter comes into play through the fluctuations of the distribution functions of injected electrons, whose correlation function is well known [18].

If the electron-impurity coupling is rotationally symmetric with respect to the z axis, the total spin of the electrons and the impurity is conserved by the scattering, and therefore the dc current and the spectral density of noise at zero frequency are not affected by it. At nonzero frequencies, we reproduce the results of [14]. The fluctuation-dissipation relation is valid in this particular case because for an isotropic coupling, the nonequilibrium system may be mapped onto an equilibrium one in external magnetic field [19].

The coupling may be anisotropic even for a point-like impurity [15]. In this case, the coupling constant J_a is nonzero as well as J_0, so both Γ_a and $\Gamma_0^\pm$ have to

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2)This implies that we are well above Kondo temperature.

be taken into account. The scattering process described by Γ_a leads to a relaxation of the impurity spin, and therefore the scattering correction to the dc current is nonzero. However Γ_a is proportional to $T/\hbar$, while Γ_0^+ is proportional to $eV/\hbar$ at $eV \gg T$. This is why the backscattering current $I_{bs} = e^2V/2\pi\hbar - I$ initially grows with voltage but eventually saturates at $I_{bs} = \alpha_a eT/\pi\hbar$. The nonequilibrium noise shows a similar behavior and tends to $S_I = (2e^2/\pi\hbar)(1+\alpha_a)T$. The Fano factor of the excess noise with respect to the backscattering current $F_{bs} \equiv (S_I - 2e^2T/\pi\hbar)/2eI_{bs}$ is unity. This suggests that the backscattering of different electrons from the impurity is totally uncorrelated.

If the impurity has a finite size, all the coupling parameters in the Hamiltonian (1) may be nonzero. As the three scattering rates $\Gamma_0^\pm$, $\Gamma_1^\pm$, and $\Gamma_2^\pm$ are proportional to $eV/\hbar$ at $eV \gg T$, the backscattering current and the current noise also increase proportionally to the voltage. As the voltage increases, the Fano factor with respect to backscattering current becomes

$$F_{bs} = \frac{\alpha_1(\alpha_0+\alpha_2)^3 + 4\alpha_0\alpha_2(\alpha_0^2+\alpha_2^2)}{[\alpha_1(\alpha_0+\alpha_2)+2\alpha_0\alpha_2](\alpha_0+\alpha_2)^2}. \quad (4)$$

Depending on the ratios α_1/α_0 and α_2/α_0, it varies between 1 and 2 and reaches maximum in the limit $\alpha_1 \ll \alpha_2 \ll \alpha_0$ (see Fig. 1). The increase of F_{bs} above 1

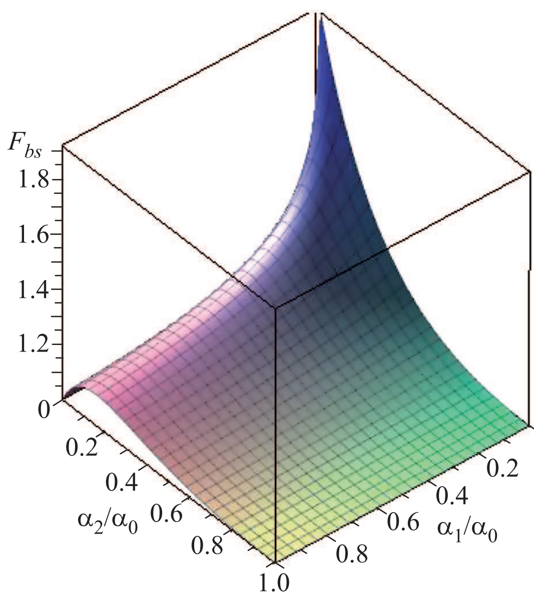

Fig. 1. (Color online) Fano factor vs. α_1/α_0 and α_2/α_0

suggests that the events of electron backscattering from the impurity are correlated. A similar increase of the Fano factor above unity was observed in resonant tunneling via interacting localized states [20].

At $\alpha_1 \ll \alpha_2 \ll \alpha_0$, the frequency dependence of spectral density is consistent with a picture of a random sequence of current pulses of the form

$$I_p(t) = e\left[\sqrt{2}\,\delta(t) + (2-\sqrt{2})\,\Gamma_0\,\Theta(t)\exp(-\Gamma_0 t)\right], \quad (5)$$

which carry a charge of $2e$ each.

The only experimental paper on electrical noise in the edge states of 2D TI we are aware of is [21], which reported the conductance much smaller than $e^2/\hbar$ and the Fano factor smaller than one. To test the current theory, one could controllably implant magnetic impurities like Mn near the edges of a 2D TI.

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