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Two-sphere partition functions and Kähler potentials on CY moduli spaces

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As it is known [1] it is important to study superstring compactifications on Calabi–Yau (CY) manifolds. In particular, the dynamics of a part of massless modes is governed by Kähler potential of the complex moduli space of CY. It means that in order to study the low energy dynamics one has to compute this quantity. In general it is a complicated task to perform. In these notes we suggest to use recently discovered relation [2] of Kähler potentials and the exact partition functions of gauged linear sigma-models on S^2 [3, 4].

It was argued by Witten [5] that $N = (2,2)$ supersymmetric sigma-models with CY target spaces admit ultraviolet description as two-dimensional $N = (2,2)$ gauged linear sigma-models (GLSM). It is well known that any GLSM can be successfully placed on a sphere while preserving supersymmetry. The enough amount of supersymmetry allows one to compute the finite volume partition function of this theory exactly, using the localization technic [3, 4]. For a class of models with an abelian gauge group $U(1)^k$ the exact partition function is given by the Mellin–Barnes type integral

$$Z = \sum_{m_l\in\mathbb{Z}} \prod_{l=1}^{k} e^{-i\theta_l m_l} \int \dots \int \prod_{l=1}^{k} \frac{d\tau_l}{(2\pi i)} e^{4\pi r_l \tau_l} \times \\ \times \prod_{i=1}^{N+1} \frac{\Gamma\Big(\sum_{l=1}^{k} Q_{il}(\tau_l - \frac{m_l}{2})\Big)}{\Gamma\Big(1 - \sum_{l=1}^{k} Q_{il}(\tau_l + \frac{m_l}{2})\Big)}, \tag{1}$$

where Q_{il} is a matrix of $U(1)$ charges.

We argue that the conjecture of Jockers et al. [2] can be casted in the form

$$Z = -i \int_X \Omega \wedge \bar{\Omega}, \tag{2}$$

where X is some CY manifold uniquely defined [6] by the charge matrix Q_{il} and Ω is a distinguished holomorphic form on X. In these notes we show, by explicit computation that (2) holds for a class of Fermat hypersurfaces in weighted projective space [7].

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