Pis'ma v ZhETF, vol. 108, iss. 11, pp. 779 – 780

Two-qubit operation on Majorana qubits in ordinary-qubit chains

Yu. Makhlin[+*1)], *S. Backens*[×], *A. Shnirman*[×°]

[+]*Condensed-matter physics Laboratory, National Research University Higher School of Economics, 101000 Moscow, Russia*

[*]*Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia*

[×]*Institut für Theorie der Kondensierten Materie, Karlsruhe Institute of Technology, D-76131 Karlsruhe, Germany*

[°]*Institute of Nanotechnology, Karlsruhe Institute of Technology, D-76344 Eggenstein-Leopoldshafen, Germany*

Submitted 30 October 2018

DOI: 10.1134/S0370274X1823008X

Majorana zero modes (MZMs) in topological materials are extensively studied theoretically and experimentally [1]. They could demonstrate nontrivial exchange statistics, and allow for implementation of topological quantum computation [2–4]. They arise, e.g., in the Kitaev chain [5], equivalent to an Ising spin chain in a transverse field. Thus, it was suggested that the MZMs may be observed in systems based on spins or qubits [6] and one could physically simulate the MZM braiding [7] in spin or qubit chains [8]. Prospective experimental implementations based on superconducting qubits were discussed [8]. Due to tunability of qubit systems and the growing interest in quantum simulations, such settings attract attention of researchers as they allow for physical simulation of fermionic degrees of freedom.

While the braiding operation for fermions is well established, analysis in the spin language is relevant for the interpretation of the prospective experiments and more importantly for the analysis of the effects specific to the spin system, such as the noise, which is highly non-local in the fermionic picture. Although these systems lack complete topological protection, they are interesting experimentally for simulations of the Majorana physics. Furthermore, such qubit systems may mimic fermionic quantum computers, relevant for quantum simulations of many-body systems and quantum chemistry.

In a fermionic system, braiding two MZMs from different qubits realizes a two-qubit operation. However, identification of the two-qubit gate in the qubit (or spin) language is non-trivial [9]. One can straightforwardly find the effect of the operation on the basis (classical) states of the qubits and the ancilla coupler, however, the relative phases remain unidentified. Here we calculate these phases and demonstrate that their values are robust towards modifications in the details of the operation. This allows us to identify the resulting two-qubit gate. This result is relevant for physical simulation of the Majorana qubits in Josephson-qubit chains and other spin or qubit structures.

We consider a T-junction of spin chains, the simplest geometry which allows for braiding (Fig. 1a). The system, including the "coupler" spin [8, 10] (see [8, 11, 12] about relevance and implementation of 3-spin coupling with superconducting and trapped-ion qubits), is described by the Hamiltonian:

$$H = \sum_{\alpha=1}^{3} H_{0,\alpha} - \frac{1}{2}\sum_{\alpha\beta\gamma} |\epsilon^{\alpha\beta\gamma}|\, J_{\alpha\beta}\, S^{\gamma}\, \sigma^z_\alpha(1)\, \sigma^z_\beta(1), \quad (1)$$

$$H_{0,\alpha} = -\sum\nolimits_j h_\alpha(j)\, \sigma^x_\alpha(j) - J\sum\nolimits_j \sigma^z_\alpha(j)\, \sigma^z_\alpha(j+1). \quad (2)$$

In this system of Ising spin chains with local transverse fields $h_\alpha(j)$, any interval with $h \ll J$ is ferromagnetic, whereas $h \gg J$ results in a trivial (paramagnetic) phase.

This system may be mapped to free fermions on the same lattice [13, 8], which allows one to consider analogs of phenomena in the fermionic system. In particular, a ferromagnetic interval is an analog of a topologically protected Majorana qubit. Braiding of two MZMs at its ends can be simulated in a spin system [8]. Here we consider the spin analog of braiding Majorana modes from two different topological intervals, which can be viewed as a two-qubit logical gate.

The braiding requires shifting the "topological" intervals along the chains and through the junction as shown in Fig. 1b. This is achieved via locally controlled fields h (their physical realization depends on the type of qubits). When $h \to \infty$, the respective spin is frozen along the $+x$ direction; when $h \to 0$, it is released.

Initially, the two topological intervals are prepared in the first and third chain at some distance from the coupler spin S. During stages 1 and 4 in Fig. 1b, only one component of the coupler spin is involved, which simplifies the analysis. However, during stages 2 and 3, at least two 3-spin couplings in Eq. (1) are relevant at each step. We follow the adiabatic evolution in Fig. 1 and show, that the details of these manipulations are irrelevant. First, we find the result for the basis initial states of the three chains and the coupler, those with

1)e-mail: makhlin@itp.ac.ru

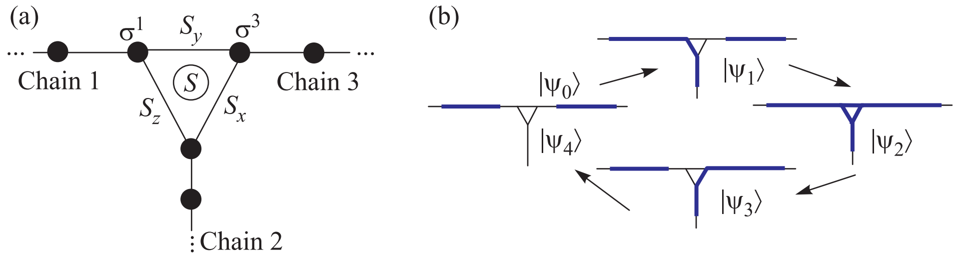

Fig. 1. (Color online) (a) – T-junction of three Ising spin chains. (b) – Stages of the braiding operation for two qubits. The individual spins in the chains and the coupler spin are not shown. The thick lines indicate the topological (ferromagnetic) regions: outside of these regions, the spins are frozen by the transverse field

spin $a = \pm 1$ in the first qubit, spin $b = \pm 1$ in the second qubit, and $S_z = S = \pm 1$ for the coupler. We find that

$$(a, b, S\hat{z}) \mapsto (a, b, Sab\hat{x}), \tag{3}$$

i.e., the coupler is disentangled and in the eigenstate of S_x with the eigenvalue Sab. However, the phase factors of these final states cannot be inferred from the adiabatic analysis and remain ambiguous.

To identify these relative phases, we note that if the initial state before the operation is (a, b, S), then after stage 1 we have $\sigma_z^1 = a$, $\sigma_x^3 = +1$, $S_z = S$ and the spins states in the ferromagnetic regions of the three chains near the junction are a, Sa, and b, respectively. No relative phase factors are acquired during stage 1. During stages 2 and 3 the edge spin σ^3 in chain 3 unfreezes, and the edge spin σ^1 in chain 1 is frozen, which is governed by the Hamiltonian

$$H_c = -J_1 a\sigma_z^1 - J_3 b\sigma_z^3 - h_1\sigma_x^1 - h_3\sigma_x^3 - \\ - J_{13} S_y \sigma_z^1 \sigma_z^3 - J_{12} S_z \sigma_z^1 (Sa) - J_{23} S_x \sigma_z^3 (Sa). \tag{4}$$

Along with the h-fields, other parameters may also be varied in time.

The following observation allows us to find the two-qubit gate: the operator of the coupler spin π-rotation about the y-axis $R_S \equiv R_y^S(\pi)$ transforms the dynamics for the initial condition (a, b, S) to that for $(a, b, -S)$: it changes the Hamiltonian H_c (4) and the initial conditions before stage 2 accordingly:

$$R_S^\dagger H_c(S = +1) R_S = H_c(S = -1). \tag{5}$$

Similarly, the operators $R_a = R_x^{\sigma^1}(\pi) R_z^S(\pi)$ and $R_b = R_x^{\sigma^3}(\pi) R_z^S(\pi)$ flip signs of a and b. Thus, if we know the final state for one initial condition, e.g., $a = +1, b = +1, S = +1$, even up to a phase, we can immediately find the final states for the remaining seven initial basis states by acting on it with a proper combination of the R operators.

This allows one to restore the whole two-qubit operation, $\exp[-i(\pi/4) S^y \tau_1^z \tau_2^z]$, in terms of the Pauli matrices τ for the two qubits. This is fully equivalent to the braiding operation in the fermionic representation [9]. It entangles the coupler with the qubits, which can be avoided by placing both qubits in the same chain before the operation [8, 9].

In conclusion, we found a two-qubit logical gate realized by physically simulating braiding of Majorana zero modes in spin or qubit chains. This provides the basis for the further analysis of imperfections in the braiding operation in a qubit system. Demonstration of such operations with present-day superconducting quantum bits is within reach [8].

This research was supported by the DFG-RSF under grant # 16-42-01035, SH 81/4-1, MI 658/9-1.

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364018230029

1. R. M. Lutchyn, E. P. A. M. Bakkers, L. P. Kouwenhoven, P. Krogstrup, C. M. Marcus, and Y. Oreg, Nat. Rev. Mater. **3**, 52 (2018).
2. C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, Rev. Mod. Phys. **80**, 1083 (2008).
3. S. Das Sarma, M. Freedman, and C. Nayak, NJP Quantum Information **1**, 15001 (2015).
4. A. Yu. Kitaev, Ann. Phys. **303**, 2 (2003).
5. A. Yu. Kitaev, Phys. Usp. **44**, 131 (2001).
6. M. Pino, A. M. Tsvelik, and L. B. Ioffe, Phys. Rev. Lett. **115**, 197001 (2015).
7. J. Alicea, Y. Oreg, G. Refael, F. von Oppen, and M. P. A. Fisher, Nat. Phys. **7**, 412 (2011).
8. S. Backens, A. Shnirman, Yu. Makhlin, Y. Gefen, J. E. Mooij, and G. Schön, Phys. Rev. B **96**, 195402 (2017).
9. S. Backens, A. Shnirman, and Yu. Makhlin, e-print arXiv:1810.02590.
10. A. M. Tsvelik, Phys. Rev. Lett. **110**, 147202 (2013).
11. C. Hutter, A. Shnirman, Yu. Makhlin, and G. Schön, Europhys. Lett. **74**, 1088 (2006).
12. P. Hauke, D. Marcos, M. Dalmonte, and P. Zoller, Phys. Rev. X **3**, 041018 (2013).
13. N. Crampé and A. Trombettoni, Nucl. Phys. B **871**, 526 (2013).