Pis'ma v ZhETF, vol. 109, iss. 1, pp. 10 – 11

Negative temperature for negative lapse function

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Submitted 25 October 2018
Resubmitted 6 November 2018
Accepted 8 November 2018

DOI: 10.1134/S0370274X19010028

Fermion dynamics distinguishes spacetimes having the same metric $g_{\mu\nu}$, but different tetrads $e_{\mu a}$, and in particular, it distinguishes a lapse with negative sign, $N < 0$ [1]. Here we show that the quasiequilibrium thermodynamic state may exist, in which the region with $N < 0$ has negative local temperature $T(x) < 0$, while the global Tolman temperature T_0 remains positive.

Tolman's law [2] (see also [3]) states that in a static gravitational field with the shift $N^i = 0$, the locally measured temperature $T(x)$ obeys:

$$T(x) = \frac{T_0}{\sqrt{g_{00}(x)}} = \frac{T_0}{|N(x)|}, \qquad (1)$$

where T_0 is spatially constant in thermal equilibrium, and N is the lapse function with $g_{00}(x) = N^2(x)$. In the ADM parametrization with $N^i = 0$, one has $g_{\mu\nu}dx^\mu dx^\nu = N^2 dt^2 - g_{ik}dx^i dx^k$.

In the effective gravity emerging for quasirelativistic quasiparticles in superfluids [4], T_0 is the conventional temperature of the liquid as measured by external observer. It is constant in space in thermal equilibrium. The local $T(x)$ is measured by the local "internal observer", who uses quasiparticles for measurements.

Fermions interact with gravity via the tetrads instead of the metric, $g_{\mu\nu} = \eta^{ab} e_{a\mu} e_{b\nu}$. In terms of tetrads, one has $N^2 = g_{00} = (e_{00})^2 = (e^0_0)^{-2}$. The general Lorentz transformations acting on fermions include two discrete operations: the reversal of time, and parity transformation. Under time reversal we have $\mathbf{T}\, e_{00} = -e_{00}$ and $\mathbf{T}\det(e) = -\det(e)$, and under parity transformation – $\mathbf{P}\, e_{00} = e_{00}$ and $\mathbf{P}\det(e) = -\det(e)$. Correspondingly, the fermionic vacuum has the four-fold degeneracy.

In condensed matter the analog of parity transformation takes place in a topological Lifshitz transition, when the chiral vacuum with Weyl nodes in the polar distorted superfluid ^{3}He-A[5, 6] crosses the vacuum state of the polar phase with a degenerate fermionic tetrad, $\det(e) = 0$ [7]. In this transition from "spacetime" to "antispacetime", the chirality of Weyl fermions changes: the left-handed fermions living in the spacetime transform to the right-handed fermions in "antispacetime". This transition experiences the nonanalytic behavior of the action at the crossing point. Here we discuss the similar transition from "spacetime to antispacetime" by the time reversal and show that this transition may have analytical properties suggested in [8–10].

Let us assume that the lapse function $N(x)$ is the analytical function of the tetrad field. Then instead of the conventional Tolman law, $T(x) = T_0/|N(x)|$ in Eq. (1), one would have the modified Tolman law

$$T(x) = \frac{T_0}{N(x)}, \quad N(x) = e_{00}(x) = \frac{1}{e^0_0(x)}. \qquad (2)$$

For negative $e_{00}(x)$ (but still positive $g_{00}(x)$), the local temperature $T(x)$ of fermions becomes negative, see Fig. 1a.

Formation of negative temperature in the island with negative $e_{00}(x)$ can be explained in the following way. For the fermions, the crossing $e_{00} = 0$ corresponds to the change of the Hamiltonian $\mathcal{H} \to -\mathcal{H}$. When the island is formed, then immediately after formation one has the state with inverse filling of the particle energy levels, which corresponds to the negative local temperature, $T(x) < 0$.

Though in general the negative temperature state with inverse population is not in full equilibrium, in principle, it can be made locally stable, see, e.g., [11]. Anyway, finally the state in the island relaxes to the fully equilibrium state in Fig. 1b with positive temperature, $T(x) = T_0/|e_{00}(x)| = T_0/|N(x)| > 0$.

So, while the dynamics in the negative lapse region may correspond to the inverse arrow of time for fermions [1], the thermodynamics in this region may correspond to the negative temperature.

As is demonstrated in [7] on example of the Weyl superfluid, the action in terms of tetrads is non-analytic. For example, the action for the effective gauge field is shown to be proportional to $\sqrt{-g} = |\det(e)|$. This is contrary to the action proportional to $\det(e)$, which

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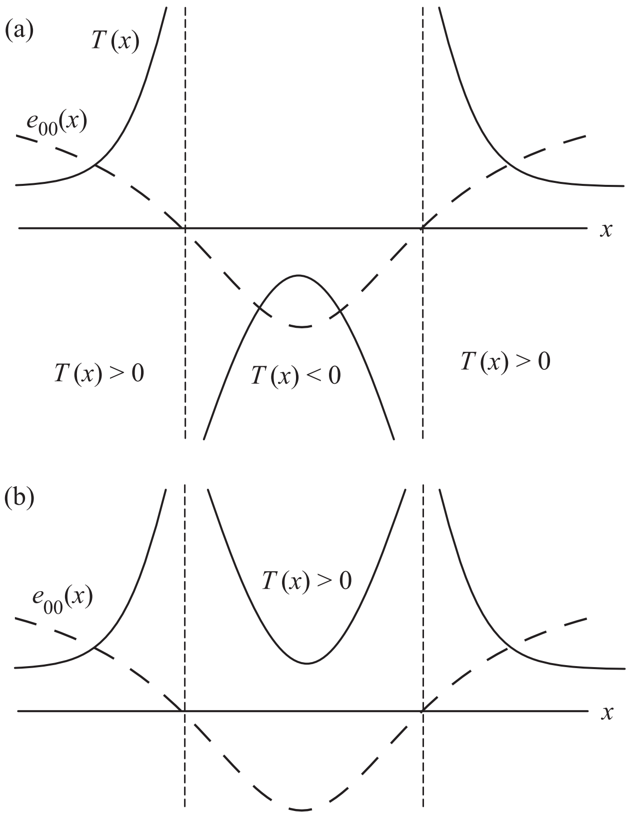

Fig. 1. Island of negative lapse function, $N(x) = e_{00}(x) < 0$. (a) – After formation of the island, the metastable state with negative local temperature is formed according to the modified Tolman law, $T(x) = T_0/e_{00}(x) < 0$. The global Tolman temperature T_0 is constant in space, $T_0 = \mathrm{const} > 0$. It is the temperature at infinity, where $e_{00}(\pm\infty) = 1$. In this scenario, $e_{00}(x)$ crosses zero, while and temperature $T(x)$ crosses infinity. The negative temperature state in the island is nonequilibrium, and finally it relaxes to the equilibrium state in Fig. b with positive temperature obeying the conventional Tolman law, $T(x) = T_0/|e_{00}(x)| > 0$

has been suggested in [8–10]. The nonanalytic behavior of the action takes place when the boundary is crossed between two equilibrium degenerate states with different signs of $\det(e)$ or $N(x)$. In both equilibrium states in Fig. 1b the temperature is positive, and the Tolman law is given by the nonanalytic equation (1). The analytic action suggested in [8–10] can be realized in the quasiequilibrium state in Fig. 1a, when the boundary is crossed between the equilibrium state with $N(x) > 0$ and the nonequilibrium state in the island with $N(x) < 0$ and negative $T(x) < 0$. In this case the Tolman law is the analytic function of $N(x)$ in Eq. (2), as well as the action.

Note that the realization of the hypersurfaces, at which $\det(e)$ crosses zero or infinity, may require consideration beyond the Einstein general relativity. The similar problem arises for the hypersurfaces, at which the Newton constant changes sign [12].

In [1] three alternatives to the problem of antispacetime were suggested: (i) Antispacetime does not exist, and $\det(e) > 0$ should constrain the gravity path integral; (ii) Antispacetime exists, but the action depends on $|\det(e)|$. (iii) Antispacetime exists and contributes nontrivially to quantum gravity.

Our consideration suggests that the antispacetime may exist with two possible realizations. The option (ii) takes place in case of full equilibrium both in spacetime and in antispacetime in Fig. 1b, and in this case the action is non-analytic [7] together with conventional Tolman law in Eq. (1). The option (iii) with the analytic behavior of the action takes place in the quasiequilibrium state in Fig. 1a with the negative temperature in the island. This state is formed immediately after formation of the island, and obeys the analytic Tolman law in Eq. (2). After relaxation to the full equilibrium the nonanalytic behavior of the action and of the Tolman law in Eq. (1) is restored.

This work has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement # 694248).

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S002136401901003X

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