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Tetrads and q-theory

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As the microscopic structure of the deep relativistic quantum vacuum is unknown, a phenomenological approach (q-theory) has been proposed to describe the vacuum degrees of freedom and the dynamics of the vacuum energy after the Big Bang. The original q-theory was based on a four-form field strength from a three-form gauge potential. However, this realization of q-theory, just as others suggested so far, is rather artificial and does not take into account the fermionic nature of the vacuum. We now propose a more physical realization of the q-variable. In this approach, we assume that the vacuum has the properties of a plastic (malleable) fermionic crystalline medium. The new approach unites general relativity and fermionic microscopic (trans-Planckian) degrees of freedom, as the approach involves both the tetrad of standard gravity and the elasticity tetrad of the hypothetical vacuum crystal. This approach also allows for the description of possible topological phases of the quantum vacuum.

The q-theory framework [1, 2] provides a general phenomenological approach to the dynamics of vacuum energy, which may be useful for the resolution of problems related to the cosmological constant in the Einstein equation (a brief review of q-theory appears in Appendix A of [3]). The advantage of q-theory is that, at the classical level, the field equations of the theory essentially do not depend on the detailed microscopic (trans-Planckian) origin of the q-field. In the classical limit, the field equations of q-theory (i.e., the equation for the microscopic variable q and the modified Einstein equation for the metric) are universal.

The q-theory approach to the cosmological constant problem aims to describe the decay of the vacuum energy density from an initial Planck-scale value to the present value of the cosmological constant. However, the correct description of this decay requires the quantum version of q-theory, which may be different for different classes of realizations of the q-variable (see Sec. 1 in [4] for a general discussion of quantum-dissipative effects and [5] for a sample calculation).

Up till now, our discussions of q-theory have primarily used the nonlinear theory of a four-form field strength from a three-form gauge potential (the linear theory of the vacuum energy in terms of the four-form field strength has been considered by Hawking [6], in particular). However, the four-form field strength, though useful for the construction of the general phenomenological equations for the quantum vacuum, is rather abstract. The physical origin of such a field is not clear. In the new realization, the corresponding vacuum variable q is expressed in terms of both the gravitational tetrad and the elasticity tetrad of the underlying crystal. This realization in terms of tetrad fields is more appropriate for the quantum theory of the fermionic vacuum of our Universe than the realization with a bosonic four-form field strength.

Throughout, we use natural units with $c = \hbar = 1$ and take the metric signature $(-+++)$.

The tetrad formalism of torsion-less gravity is given by the following equations:

$$g_{\mu\nu} = \eta_{ab}\, e^a_\mu\, e^b_\nu,\ \nabla_\mu\, g^{\mu\nu} = 0, \tag{1a}$$

$$D_\mu\, e^a_\nu \equiv \nabla_\mu\, e^a_\nu + \omega^a_{\mu b}\, e^b_\nu = 0, \tag{1b}$$

where ∇_μ is the conventional covariant derivative of general relativity and $\omega^a_{\mu b}$ the spin connection,

$$\omega^a_{\mu b} = e^a_\nu\, \nabla_\mu\, e^\nu_b. \tag{2}$$

We interpret the vacuum as a plastic (malleable) fermionic crystalline medium. At each point of spacetime, we have a local system of four deformed crystallographic manifolds of constant phase $X^a(x) = 2\pi n^a$, for $n^a \in \mathbb{Z}$ with $a = 0, 1, 2, 3$. In addition to the conventional tetrad e^a_μ of gravity, we then introduce the following elasticity tetrad E^a_μ (cf. [7–9]):

$$E^a_\mu(x) = D_\mu\, X^a(x), \tag{3}$$

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where both indices a and μ take values from the set $\{0, 1, 2, 3\}$. Invariance under the local $SO(1, 3)$ group of rotations is implemented by defining

$$D_\mu X^a \equiv \nabla_\mu X^a + \omega^a_{\mu b} X^b = \partial_\mu X^a + \omega^a_{\mu b} X^b. \tag{4}$$

Let us assume that the vacuum energy density $\epsilon(q)$ in the action depends on the following type of q-field:

$$q(x) = \frac{1}{4}\, e^\mu_a(x)\, E^a_\mu(x). \tag{5}$$

The action in its simplest form is then given by

$$S = \int_{\mathbb{R}^4} d^4x\, e \left(\frac{R}{16\pi G_N} + \epsilon(q)\right), \tag{6}$$

where R is the Ricci curvature scalar and e the tetrad determinant. Variation of Eq. (6) over e^μ_a gives the Einstein equation [10],

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = 8\pi G_N\, \rho_V(q)\, g_{\mu\nu}, \tag{7}$$

where $\rho_V(q)$ will be discussed shortly, and variation over X^a gives the following differential equation for q (which is both a coordinate scalar and a Lorentz scalar):

$$\partial_\mu \left(\frac{d\epsilon(q)}{dq}\right) = 0, \tag{8}$$

where (1b) has been used. The vacuum energy density $\rho_V(q)$, which enters the Einstein equation (7) through a cosmological-constant-type term, is given by

$$\rho_V(q) \equiv \epsilon(q) - q\,\frac{d\epsilon(q)}{dq}, \tag{9}$$

with an extra term $-q\, d\epsilon/dq$.

Equation (8) for q has the following general solution:

$$\frac{d\epsilon(q)}{dq} = \mu = \text{constant}, \tag{10}$$

with the arbitrary constant μ interpreted as a "chemical potential" in [1], and the gravitating vacuum energy density (9) becomes $\rho_V(q) = \epsilon(q) - \mu\, q$.

The quantum vacuum in perfect equilibrium has a constant nonzero value of the q-field,

$$q(x) = q_0 = \text{constant}, \tag{11}$$

which gives a particular value μ_0 for μ in (10),

$$\mu_0 = \left[\frac{d\epsilon(q)}{dq}\right]_{q=q_0}. \tag{12}$$

In addition, there are the equilibrium conditions:

$$\rho_V(q_0) = 0, \tag{13a}$$

$$\left[\frac{d\,\rho_V(q)}{dq}\right]_{q=q_0} = 0, \tag{13b}$$

$$\left[\frac{d^2\,\rho_V(q)}{dq^2}\right]_{q=q_0} > 0. \tag{13c}$$

Equations (13a) and (13b) result from the self-adjustment of the conserved vacuum variable q, as follows from the Gibbs–Duhem relation for an isolated self-sustained system without external pressure; Eq. (13c) corresponds to positive isothermal compressibility of the vacuum.

To summarize, we have obtained with (5) one further realization of the q-variable, in addition to the four-form realization [1] and the brane realization [2]. The advantage of this new realization is that it has a more direct physical origin. The quantum version of q-theory is sensitive to the particular realization of the q-field. Assuming q-theory to be relevant, the comparison with experiment may then provide information on the detailed structure of the fermionic quantum vacuum and, in particular, on the types of quantum anomalies. The fermionic crystalline model of the vacuum is one of the possible structures of the deep fermionic vacuum, distinct from a structure described the abstract four- form field strength. This new fermionic structure gives, for example, rise to new types of quantum anomalies, where elasticity tetrads E with dimensions of inverse length or inverse time are mixed with gauge and gravity fields [9].

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