Pis'ma v ZhETF, vol. 109, iss. 8, pp. 509 – 510

Two roads to antispacetime in distorted B-phase of ^{3}He

G. E. Volovik[1)]

Low Temperature Laboratory, Aalto University, School of Science and Technology, FI-00076 AALTO, Finland

Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia

Submitted 19 March 2019
Resubmitted 19 March 2019
Accepted 20 March 2019

DOI: 10.1134/S0370274X19080022

The topological materials with emergent analogs of gravity demonstrate the possibility of realization of different exotic spacetimes, including the transition to antispacetime, see, e.g., [1] and references therein. There are several routes to the effective gravity. One of them is the tetrad gravity emerging in the vicinity the Weyl or Dirac points [2–5] – the exceptional crossing points in the fermionic spectrum [6, 7]. Also the degenerate $2+1$ gravity emerges near the Dirac nodal line in the spectrum [1]. Another important source of gravity is the formation of the tetrads as bilinear combinations of the fermionic fields [8–10].

Emergent gravity provides different types of the antispacetime obtained by the space reversal P and time reversal T operations, including those where the determinant of the tetrads e changes sign [9–12]. In cosmology, the antispacetime Universe was in particular suggested as analytic continuation of our Universe across the Big Bang singularity [13]. There were speculations, that antispacetime may support nonequilibrium states with negative temperature as a result of analytic continuation across the singularity [14, 15]. Here we consider the antispacetime realized in experiments [16] on the analog of cosmological walls bounded by strings [17] – Kibble walls (KWs).

In notations [18] used in [19], the Green's function of the relativistic massive Dirac particle has the form:

$$S = \frac{Z(p^2)}{-i\gamma^a e_a^\mu p_\mu + M(p^2)}. \tag{1}$$

Here e_a^μ are tetrads with $\mu, a = 0,1,2,3$; the residue $Z(p^2)$ and the mass $M(p^2)$ are the functions of $p^2 = g^{\mu\nu}p_\mu p_\nu$, where $g^{\mu\nu} = e_a^\mu e_b^\nu \eta^{ab}$. It is convenient to express γ-matrices in terms of two sets of Pauli matrices: σ^1, σ^2 and σ^3 for conventional spin, and τ_1, τ_2, τ_3 for the isospin in the left-right space:

$$\gamma^0 = -i\tau_1, \quad \gamma^a = \tau_2\sigma^a, \quad a = (1,2,3). \tag{2}$$

$$\gamma_5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \tau_3. \tag{3}$$

In some phases of superfluid ^{3}He, the Green's function for fermionic Bogoliubov quasiparticles is similar to that in Eq. (1). Now instead of the mass function $M(p^2)$, the energy of quasiparticles in the normal Fermi liquid enters, $\epsilon(\mathbf{p}) = v_{\rm F}(|\mathbf{p}| - p_{\rm F})$. The spin matrices σ^a act in the spin space of ^{3}He atoms; the matrices τ_b act in the isotopic Bogoliubov–Nambu space. The function Z can be ignored. The tetrads come from the spin-triplet p-wave order parameter in ^{3}He superfluids – the 3×3 matrix A_a^i with spin index $a = (1,2,3)$ and orbital index $i = (1,2,3)$: $\sum_{\mathbf{k}} k^i \langle a_{\mathbf{k}\alpha} a_{-\mathbf{k}\beta}\rangle \sim A_a^i \left(\sigma^a\sigma^2\right)_{\alpha\beta}$. For the time reversal symmetric phases [20]:

$$A_a^i = p_{\rm F} e^{i\Phi} e_a^i \ , \ a, i = (1,2,3). \tag{4}$$

The tetrads e_a^i emerge due to the spontaneously broken symmetries $SO(3)_S \times SO(3)_L$ under spin and orbital rotations. This is analogous to the formation of the tetrads in relativistic theories as bilinear combinations of the fermionic fields [9, 10]. In addition to tetrads, the order parameter (4) contains the phase Φ coming from spontaneous breaking of $U(1)$-symmetry, and the Green's function depends both on e_a^μ and on Φ:

$$\tilde{S}(e_a^\mu, \Phi) = e^{-\gamma_0\Phi/2} S(e_a^\mu) e^{\gamma_0\Phi/2}. \tag{5}$$

For $\Phi = \pi$ the symmetry transformation $e^{-\gamma_0\Phi/2}$ is equivalent to the conventional space reversal transformation – the parity $P = e^{-\gamma_0\pi/2} = \gamma_0$, with $P^2 = -1$. This suggests that in relativistic theories the discrete symmetry, such as the space inversion P, could be the residual Z_2 symmetry after breaking of the more fundamental symmetry group.

In the time reversal symmetric states realized in experiments [16, 21] the tetrads are:

$$e_a^i = c_1\hat{\mathbf{f}}_a\hat{\mathbf{x}}^i + c_2\hat{\mathbf{g}}_a\hat{\mathbf{y}}^i + c_3\hat{\mathbf{d}}_a\hat{\mathbf{z}}^i, \quad (a,i) = (1,2,3), \tag{6}$$

where $\hat{\mathbf{d}}$, $\hat{\mathbf{f}}$ and $\hat{\mathbf{g}}$ are orthogonal unit vectors in spin space; $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are orthogonal unit vectors in orbital space; c_1, c_2 and c_3 are "speeds of light". In the

1) e-mail: volovik@boojum.hut.fi

pure B-phase $|c_1| = |c_2| = |c_3|$; in the polar phase [21] $c_1 = c_2 = 0$; in the polar disorted B phase (PdB) $|c_2| = |c_1| < |c_3|$. The particular states of these phases:

$$e_a^\mu = \mathrm{diag}(-1, c_1, c_2, c_3). \tag{7}$$

In the PdB phase, the states with $c_2 = +c_1$ and $c_2 = -c_1$ in Eq. (7) can be separated by the nontopological domain wall – the analog of the KW bounded by strings [17]. The KW typically appears in the two phase transitions: at first transition the linear defect becomes topologically stable; at the second transition the linear defect looses its topological stability and becomes the termination line of the KW. In superfluid ^{3}He, the HQVs (half-quantum vortex – HQV) appear at first transition from the mormal liquid to the polar phase [22], and at further transition to the PdB phase they become the end lines of the KWs [16]. Across KW, $e_2^2 = c_2$ changes sign, and the spacetime analytically transforms to the antispacetime. The intermediate state within the KW has the degenerate tetrad $e_a^\mu = \mathrm{diag}(-1, c_1, 0, c_3)$ – the distorted planar phase (for planar phase $|c_1| = |c_3|$ and $c_2 = 0$ [20]).

Figure 1 demonstrates the loop of HQV, which terminates the KW. In cosmology, the HQV corresponds

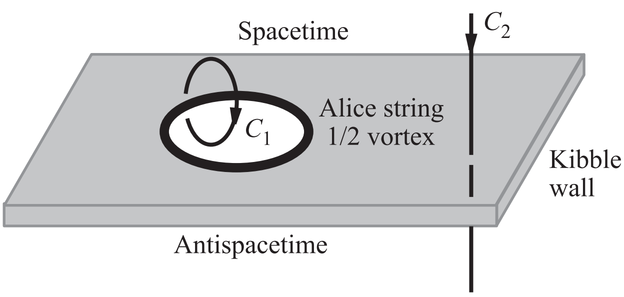

Fig. 1. Roads to antispacetime: the safe route around the Alice string (along C_1) or dangerous route along C_2 across the Kibble wall (through the Alice looking glass)

to the Alice string [23]: by circling around the HQV the phase Φ changes by π, the vectors $\hat{\mathbf{d}}$ and $\hat{\mathbf{f}}$ rotate by π, and one continuously arrives at opposite e_2^2:

$$\mathrm{diag}(-1, c_1, c_2, c_3) \to \mathrm{diag}(-1, c_1, -c_2, c_3), \tag{8}$$

i.e., to the same antispacetime as across the KW.

In conclusion, in the polar distorted B-phase of superfluid ^{3}He, the half-quantum vortex (Alice string) and the Kibble wall bounded by strings demonstrate the two ways to enter the mirror world in Fig. 1: either to go around the HQV or to cross the Kibble wall. The polar distorted B-phase also suggests the scenario of the formation of the discrete symmetry – the parity P in particle physics – from the continuous symmetry existing on the more fundamental level.

This work has been supported by the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (Grant Agreement # 694248).

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364019080034

1. J. Nissinen and G. E. Volovik, Phys. Rev. D **97**, 025018 (2018).
2. H. B. Nielsen and M. Ninomiya, Nucl. Phys. B **185**, 20 (1981).
3. C. D. Froggatt and H. B. Nielsen, *Origin of Symmetry*, World Scientific, Singapore (1991).
4. G. E. Volovik, *The Universe in a Helium Droplet*, Clarendon Press, Oxford (2003).
5. P. Hořava, Phys. Rev. Lett. **95**, 016405 (2005).
6. C. Herring, Phys. Rev. **52**, 365 (1937).
7. A. A. Abrikosov and S. D. Beneslavskii, JETP **32**, 699 (1971).
8. G. E. Volovik, Physica B **162**, 222 (1990).
9. D. Diakonov, arXiv:1109.0091.
10. A. A. Vladimirov and D. Diakonov, Phys. Rev. D **86**, 104019 (2012).
11. M. Christodoulou, A. Riello, and C. Rovelli, Int. J. Mod. Phys. D **21**, 1242014 (2012).
12. C. Rovelli and E. Wilson-Ewing, Phys. Rev. D **86**, 064002 (2012).
13. L. Boyle, K. Finn, and N. Turok, Phys. Rev. Lett. **121**, 251301 (2018).
14. G.,E. Volovik, Pis'ma v ZhETF **109**, 10 (2019).
15. G. E. Volovik, arXiv:1902.07584.
16. J. T. Mäkinen, V. V. Dmitriev, J. Nissinen, J. Rysti, G. E. Volovik, A. N. Yudin, K. Zhang, and V. B. Eltsov, Nat. Comm. **10**, 237 (2019).
17. T. W. B. Kibble, G. Lazarides, and Q. Shafi, Phys. Rev. D **26**, 435 (1982).
18. S. Weinberg, *The Quantum Theory of Fields*, Cambridge Univ., Cambridge (1996), Section 5.4.
19. G. E. Volovik, JETP Lett. **91**, 55 (2010).
20. D. Vollhardt and P. Wölfle, *The Superfluid Phases of Helium 3*, Taylor and Francis, London (1990).
21. V. V. Dmitriev, A. A. Senin, A. A. Soldatov, and A. N. Yudin, Phys. Rev. Lett. **115**, 165304 (2015).
22. S. Autti, V. V. Dmitriev, J. T. Mäkinen, A. A. Soldatov, G. E. Volovik, A. N. Yudin, V. V. Zavjalov, and V. B. Eltsov, Phys. Rev. Lett. **117**, 255301 (2016).
23. A. Schwarz, Nucl. Phys. B **208**, 141 (1982).