Pis'ma v ZhETF, vol. 111, iss. 3, pp. 173 – 174

Joule–Thomson cooling in graphene

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Submitted 31 August 2019
Resubmitted 20 December 2019
Accepted 22 December 2019

DOI: 10.31857/S0370274X20030066

Collective effects of electron interactions may prevail over impurity scattering in a very clean sample. Electrons then flow like a viscous fluid [1], as observed in graphene in a certain range of temperatures [2–7]. Spectacular consequences of hydrodynamic electronic transport include negative local resistivity [2, 3, 4, 7], violation of ballistic bound on conductance [2, 6, 8, 9] (Gurzhi effect [1]), breakdown of Wiedemann–Franz law [5, 9, 10], and negative magnetoresistance [9, 11]. Another possible manifestation of hydrodynamic mode of transport, discussed here, is cooling of electron fluid that passes through a narrow constriction. In the solid state setting electric current normally generates heat, and cooling of electron flow may look counterintuitive, but in fluid mechanics this phenomenon, the Joule–Thomson (JT) effect [12], is well known and underlies a widely used method of gas refrigeration.

Consider two strips of graphene connected by a narrow bridge and subject to a constant voltage δU. Assuming that one strip is kept at temperature T_1 and denoting electron temperature in the other by T_2, the cooling/heating effect can be characterized by the temperature drop $\delta T = T_1 - T_2$ relative to the potential difference $\delta\mu = \mu_1 - \mu_2 = e\delta U$:

$$\delta T = \alpha \delta\mu, \tag{1}$$

where μ_1 and μ_2 are chemical potentials on the two sides of the bridge. The dimensionless coefficient α can take either sign and is defined such that $\alpha > 0$ corresponds to cooling.

Textbook derivations of the JT effect start with the enthalpy conservation: $\delta[(\epsilon + P)/n] = 0$, where ϵ and P are the energy density and pressure of the electron fluid and n is the charge carrier density. Thermodynamic relation $\epsilon + P = \mu n + Ts$, where s is the entropy density, then yields

$$\alpha = -\frac{A + T\,\frac{\partial \hat{s}}{\partial \mu}}{A\hat{s} + T\,\frac{\partial \hat{s}}{\partial T}}, \tag{2}$$

where $\hat{s} = s/n$ is specific entropy and $A = 1$. The rationale to introduce a special notation for a constant equal to one is that a more accurate calculation gives $A \approx 2/3$, while the overall functional form of the cooling coefficient remains the same.

Electrons in graphene form a 2d Fermi gas with linear dispersion relation, and their pressure is given by

$$P = 4T \int \frac{d^2p}{(2\pi\hbar)^2} \sum_{q=\pm} \ln\left(1 + \mathrm{e}^{\frac{q\mu - v_F|\mathbf{p}|}{T}}\right), \tag{3}$$

where v_F is the Fermi velocity, q labels particles/holes, and the overall factor of four takes into account valley and spin degeneracy. We tacitly assume that holes and electrons are in thermodynamic equilibrium, which is not a good approximation at the neutrality point, where our derivation is not applicable.

The rest of thermodynamic quantities can be calculated from $dP = nd\mu + sdT$. When applied to (2) the standard thermodynamic machinery gives

$$\frac{1}{\alpha} = \frac{3A\,\frac{\mathcal{F}}{\mathcal{F}'}}{A + 2 - 3\,\frac{\mathcal{F}\mathcal{F}''}{\mathcal{F}'^2}} - \xi, \qquad \xi = \frac{\mu}{T}, \tag{4}$$

where

$$\mathcal{F}(\xi) = \mathrm{Li}_3(-\,\mathrm{e}^{\,\xi}) + \mathrm{Li}_3(-\,\mathrm{e}^{\,-\xi}). \tag{5}$$

In the two limiting cases we get:

$$\alpha \overset{\mu \gg T}{\simeq} \frac{3A\mu}{2(1+A)\pi^2 T}, \qquad \alpha \overset{\mu \ll T}{\simeq} -\frac{T}{(1+A)\mu}. \tag{6}$$

The JT effect results in cooling in the Fermi liquid regime ($\mu \gg T$) and in heating in the Dirac fluid case ($\mu \ll T$). The sign of the effect changes at an inversion point, which for the physical case of $A = 2/3$ lies at $\mu_{\mathrm{inv}} = 3.32T$.

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The thermodynamic derivation of the JT effect rests on enthalpy conservation, but enthalpy production in the moving electron fluid cannot be neglected. The viscous heating turns out to be the prime source of enthalpy production and leads to order one effects, while Ohmic resistivity only gives small corrections. The derivation is based on the hydrodynamic theory of electron transport in the commonly used Stockes approximation:

$$\eta\partial^2 v_i - \frac{\eta}{\lambda^2}\, v_i = \partial_i P, \qquad \partial_i v^i = 0, \tag{7}$$

where v_i is the electron velocity, η is the shear viscosity of the electron fluid and λ is the momentum-relaxation length.

The solution is known explicitly when $\lambda = \infty$ [8]:

$$P = P_0 - \frac{4\eta u}{a}\,\mathrm{Im}\,\frac{z}{\sqrt{\frac{a^2}{4} - z^2}},$$

$$v_x = \frac{2uy}{a}\,\mathrm{Re}\,\frac{z}{\sqrt{\frac{a^2}{4} - z^2}},$$

$$v_y = \frac{2u}{a}\,\mathrm{Re}\sqrt{\frac{a^2}{4} - z^2} - \frac{2uy}{a}\,\mathrm{Im}\,\frac{z}{\sqrt{\frac{a^2}{4} - z^2}}. \tag{8}$$

Here $z = x+iy$, a is the width of the bridge, and u is the maximal velocity attained by the fluid. The square root is analytic on the complex plane with a semi-infinite cut representing the constriction.

The flow is sustained by the pressure drop:

$$\delta P = \frac{8\eta u}{a}. \tag{9}$$

The entropy production rate in the moving fluid is calculated according to

$$v^i\partial_i\hat{s} = \frac{\eta}{nT}\left(\frac{v^2}{\lambda^2} + \frac{\Pi^2}{2}\right), \qquad \Pi_{ij} = \partial_i v_j + \partial_j v_i. \tag{10}$$

Integrating the rate along the midflow (and setting $\lambda = \infty$) gives:

$$-\delta\hat{s} = \frac{2\eta u a}{nT}\int\limits_{-\infty}^{+\infty}\frac{dy\, y^2}{\left(\frac{a^2}{4} + y^2\right)^{\frac{5}{2}}} = \frac{16\eta u}{3anT} = \frac{2}{3nT}\,\delta P. \tag{11}$$

The relation $\delta P = n\delta\mu + s\delta T$ then results in the same formula (2) for the JT coefficient, but with $A = 2/3$.

When momentum relaxation is taken into account, the coefficient A starts to depend on the dimensionless ratio a/λ. Assuming that this ratio is small, $a \ll \lambda$, corrections due to momentum relaxation and Ohmic heating appear to be quadratic in the small parameter a/λ, but are logarithmically enhanced:

$$A = \frac{2}{3}\left(1 + \frac{a^2}{32\lambda^2}\ln\frac{L}{\lambda}\right), \tag{12}$$

where L is the total size of the system.

In conclusion, hydrodynamic nature of electron flow in biased graphene leads to JT cooling when the current is forced through a narrow constriction. Cooling occurs in the Fermi liquid regime, for sufficiently large charge imbalance or at sufficiently low temperatures. For lower chemical potentials the JT effect results in heating, which is most pronounced in the Dirac liquid regime at $\mu \ll T$. Although similar to conventional Joule heating the mechanics behind this effect is quite different, in particular the temperature increment is linear in applied voltage and not quadratic. It is necessary to keep in mind however that our derivation becomes invalid close to the neutrality point.

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364020030030

1. R. Gurzhi, Sov. Phys. JETP **17**, 521(1963).
2. I. Torre, A. Tomadin, A. K. Geim and M. Polini, Phys. Rev. B **92**, 165433 (2015); 1508.00363.
3. L. Levitov and G. Falkovich, Nat. Phys. **12**, 672 (2016); 1508.00836.
4. D. A. Bandurin, I. Torre, R. K. Kumar, M. Ben Shalom, A. Tomadin, A. Principi, G. H. Auton, E. Khestanova, K. S. Novoselov, I. V. Grigorieva, L. A. Ponomarenko, A. K. Geim, and M. Polini, Science **351**, 1055 (2016); 1509.04165.
5. J. Crossno, J. K. Shi, K. Wang, X. Liu, A. Harzheim, A. Lucas, S. Sachdev, P. Kim, T. Taniguchi, K. Watanabe, T. A. Ohki, and K. C. Fong, Science **351**, 1058 (2016); 1509.04713.
6. R. K. Kumar, D. A. Bandurin, F. M. D. Pellegrino et al. (Collaboration), Nat. Phys. **13**, 1182 (2017); 1703.06672.
7. D. A. Bandurin, A. V. Shytov, L. S. Levitov, R. K. Kumar, A. I. Berdyugin, M. Ben Shalom, I. V. Grigorieva, A. K. Geim, and G. Falkovich, Nat. Commun. **9**, 4533 (2018); 1806.03231.
8. H. Guo, E. Ilseven, G. Falkovich, and L. S. Levitov, Proc. Natl. Acad. Sci. USA **114**, 3068 (2017); 1607.07269.
9. J. Gooth, F. Menges, N. Kumar, V. Sü, C. Shekhar, Y. Sun, U. Drechsler, R. Zierold, C. Felser, and B. Gotsmann, Nat. Commun. **9**, 4093 (2018); 1706.05925.
10. A. Lucas, J. Crossno, K. C. Fong, P. Kim, and S. Sachdev, Phys. Rev. B **93**, 075426 (2016); 1510.01738.
11. A. Lucas, R. A. Davison, and S. Sachdev, Proc. Natl. Acad. Sci. USA **113**, 9463 (2016); 1604.08598.
12. W. Thomson and J. P. Joule, Philos. Trans. R. Soc. London **143**, 357 (1853).