Pis'ma v ZhETF, vol. 112, iss. 5, pp. 303 – 304

Atom-field correlations in the weak-excitation limit of absorptive optical bistability

Th. K. Mavrogordatos[1)]

Department of Physics, Stockholm University, SE-106 91 Stockholm, Sweden

Submitted 27 July 2020
Resubmitted 10 August 2020
Accepted 10 August 2020

DOI: 10.31857/S1234567820170036

In this Letter, we calculate the steady-state and first-order time varying atom-field correlation functions in the weak-excitation limit of absorptive optical bistability from a linearized theory of quantum fluctuations. We formulate a Fokker–Planck equation in the positive P representation following the phase-space analysis of [1] which does not resort to adiabatic elimination. Special emphasis is placed on the limit of collective strong coupling as attained from a vanishing photon-loss rate. We compare to the cavity-transmission spectrum with reference to experimental results obtained for macroscopic dissipative systems, discussing the role of anomalous correlations arising as distinct nonclassical features. We follow the notation of Ch.15 in [2]. The steady-state averages

$$\begin{aligned}\langle \bar{\tilde{\bar{J}}}_+ \tilde{\bar{a}} \rangle_{\rm ss} &= \langle \bar{\tilde{\bar{J}}}_+ \rangle_{\rm ss} \langle \tilde{\bar{a}} \rangle_{\rm ss} + \frac{1}{N} C_{\rm ss}^{\tilde{\nu}_* \tilde{z}}(0) \approx \\ &\approx -X^2 \left[1 - X^2 \frac{\xi 2C(2+\xi+2C)}{N(1+2C)^2(\xi+1)^2} \right] \end{aligned} \quad (1)$$

and

$$\begin{aligned}\langle \bar{\tilde{\bar{J}}}_+ \tilde{\bar{a}}^\dagger \rangle_{\rm ss} &= \langle \bar{\tilde{\bar{J}}}_+ \rangle_{\rm ss} \langle \tilde{\bar{a}}^\dagger \rangle_{\rm ss} + \frac{1}{N} C_{\rm ss}^{\tilde{\nu}_* \tilde{z}_*}(0) \approx \\ &\approx -X^2 \left[1 + \frac{\xi 2C}{N(1+2C)(\xi+1)} \right], \end{aligned} \quad (2)$$

demonstrate explicit corrections of order $N^{-1} \ll 1$. In the above expressions, X is the scaled intracavity amplitude, N is the number of atoms inside the cavity, $2C$ is the cooperativity parameter and $\xi \equiv 2\kappa/\gamma$ is the ratio of the photon loss rate to the spontaneous emission rate. From Equation (15.103b) of [2] we read that in the weak-excitation limit,

$$\langle \Delta\tilde{\bar{a}}^\dagger \Delta\tilde{\bar{a}} \rangle_{\rm ss} \approx N^{-1} X^4\, 2C \frac{\xi 2C(2+\xi+2C)}{(1+2C)^2(\xi+1)^2}. \quad (3)$$

Hence, $\langle \Delta\tilde{\bar{a}}^\dagger \Delta\tilde{\bar{a}} \rangle_{\rm ss} / \langle \Delta\bar{\tilde{\bar{J}}}_+ \Delta\tilde{\bar{a}} \rangle_{\rm ss} = 2C$, which reveals the role of atomic cooperativity along the lower branch of absorptive bistability. The largest deviation of the ratio $r(X,\xi) \equiv |\langle \tilde{\bar{a}}^\dagger \tilde{\bar{a}} \rangle_{\rm ss} / \langle \bar{\tilde{\bar{J}}}_+ \tilde{\bar{a}} \rangle_{\rm ss}|$ from unity occurs for $\xi = 1$ as a consequence of impedance matching for the two decoherence channels (see also Sec. V of [1]). The two cross-correlation components of order X^4 are

$$\begin{aligned} C_{\rm ss}^{\tilde{\nu}_* \tilde{z};1}(\bar{\tau}) &= \exp\left[-\frac{(\xi+1)}{2}\bar{\tau}\right] \left\{ C_{\rm ss}^{\tilde{\nu}_* \tilde{z}}(0) \cos(\bar{G}\bar{\tau}) + \right. \\ &\left. + \frac{\xi 2C\, C_{\rm ss}^{\tilde{\nu}_* \tilde{\nu}}(0)_{\rm ss} + [(1-\xi)/2] C_{\rm ss}^{\tilde{\nu}_* \tilde{z}}(0)}{\bar{G}} \sin(\bar{G}\bar{\tau}) \right\}, \end{aligned} \quad (4)$$

$$\begin{aligned} C_{\rm ss}^{\tilde{\nu}_* \tilde{z};2}(\bar{\tau}) &= \frac{\xi 2C X^4}{(1+2C)(\xi+1)} \frac{1}{2\bar{G}} \exp\left[-\frac{(\xi+1)}{2}\bar{\tau}\right] \times \\ &\times \left\{ \frac{(\xi+1)(\xi-1-2C)}{2\bar{G}} \left[\frac{\sin(\bar{G}\bar{\tau})}{\bar{G}} - \bar{\tau}\cos(\bar{G}\bar{\tau}) \right] + \right. \\ &\left. + (1+\xi+2C)\bar{\tau}\sin(\bar{G}\bar{\tau}) \right\}. \end{aligned} \quad (5)$$

In the above expressions,

$$\bar{G} \equiv \sqrt{\xi 2C - \frac{1}{4}(\xi-1)^2}$$

is the many-atom effective coupling strength in which dissipation also plays a role, and $\bar{\tau} \equiv \gamma\tau/2$. The transmitted-light spectrum is $\bar{\mathcal{C}}_{\rm ss}^{\tilde{z}_* \tilde{z}}(\bar{s}) = \bar{\mathcal{C}}_{\rm ss}^{\tilde{z}_* \tilde{z};1}(\bar{s}) + \bar{\mathcal{C}}_{\rm ss}^{\tilde{z}_* \tilde{z};2}(\bar{s})$ for $\bar{s} = -i2(\omega-\omega_0)/\gamma$, where

$$\frac{\bar{\mathcal{C}}_{\rm ss}^{\tilde{z}_* \tilde{z};1}(\bar{s})}{X^4} = \frac{4C^2(2+\xi+2C)}{(1+2C)^2(\xi+1)^2} \frac{1+\xi+\bar{s}}{(\xi+\bar{s})(1+\bar{s})+\xi 2C} \quad (6)$$

and

$$\frac{\bar{\mathcal{C}}_{\rm ss}^{\tilde{z}_* \tilde{z};2}(\bar{s})}{X^4} = \frac{4C^2\xi}{(1+2C)(\xi+1)} \frac{\xi(\xi-2C+\bar{s})}{[(\xi+\bar{s})(1+\bar{s})+\xi 2C]^2}. \quad (7)$$

The corresponding contribution to the correlation function for the cavity field is

1) e-mail: th.mavrogordatos@gmail.com

$$C_{\rm ss}^{\tilde{z}_*\tilde{z};2}(\bar{\tau}) = X^4\frac{4C^2\xi}{(1+2C)(\xi+1)}\frac{1}{2\bar{G}}\exp\left[-\frac{(\xi+1)}{2}\bar{\tau}\right]\times$$
$$\times\left\{\frac{\xi(\xi-1-4C)}{2\bar{G}}\left[\frac{\sin(\bar{G}\bar{\tau})}{\bar{G}}-\bar{\tau}\cos(\bar{G}\bar{\tau})\right]+\right.$$
$$\left.+\,\xi\bar{\tau}\sin(\bar{G}\bar{\tau})\right\}. \quad (8)$$

The sum of two corresponding components of a light-matter correlation and the cavity-field autocorrelation, $[C_{\rm ss}^{\tilde{\nu}_*\tilde{z};2}(\bar{\tau})+C_{\rm ss}^{\tilde{z}_*\tilde{z};2}(\bar{\tau})]/X^4$ obtained from Eqs. (4) and (8), is compared to the component $C_{\rm ss}^{\tilde{\nu}_*\tilde{z};2}(\bar{\tau})/X^4$ alone in Fig. 1 as we approach the many-atom strong-coupling

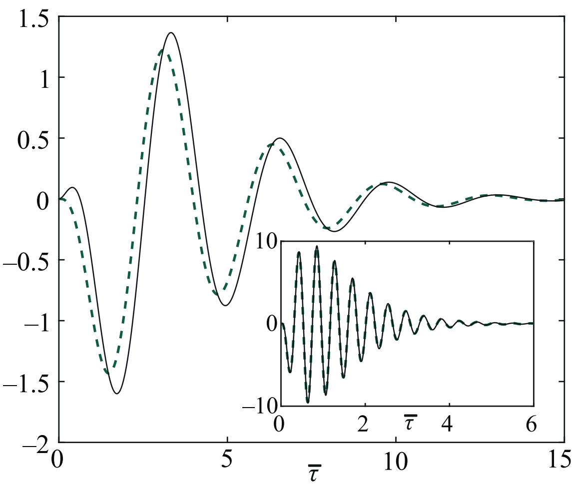

Fig. 1. (Color online) Correlations with a squared Lorentzian distribution. The sum of the two components: $C_{\rm ss}^{\tilde{\nu}_*\tilde{z};2}(\bar{\tau})/X^4$ and $C_{\rm ss}^{\tilde{z}_*\tilde{z};2}(\bar{\tau})/X^4$ is plotted in a solid black line, superimposed on $C_{\rm ss}^{\tilde{z}_*\tilde{z};2}(\bar{\tau})/X^4$ alone plotted with a dashed green line. Parameters: $\xi=0.05$, $C=40$. The inset depicts the same quantities, but for $\xi\approx1.9$, $C\approx58$ (see Fig. 4 of [3])

limit of absorptive bistability $[\xi 2C\gg(\xi+1)^2/4]$. Upon a further increase of the parameter $\xi 2C$, the two curves coincide. On the other hand, the correlation

$$C_{\rm ss}^{\tilde{\nu}_*\tilde{z}_*}(\bar{\tau}) = -X^2\frac{\xi 2C}{(\xi+1)(1+2C)}\exp\left[-\frac{(\xi+1)}{2}\bar{\tau}\right]\times$$
$$\times\left[\cos(\bar{G}\bar{\tau})+\frac{4C+\xi+3}{2\bar{G}}\sin(\bar{G}\bar{\tau})\right], \quad (9)$$

dominates at weak-excitation. In the limit $\xi\to0$, $C\to\infty$, with $\xi 2C\gg1$ remaining constant, the sum

$$C_{\rm ss}^{\tilde{z}_*\tilde{\nu}_*}(\bar{\tau})+C_{\rm ss}^{\tilde{\nu}_*\tilde{z}_*}(\bar{\tau})\approx-2X^2\xi\exp\left(-\frac{\bar{\tau}}{2}\right)\cos(\sqrt{\xi 2C}\bar{\tau}), \quad (10)$$

tends to zero as $\xi\to0$. Their difference, however, evaluating to

$$C_{\rm ss}^{\tilde{z}_*\tilde{\nu}_*}(\bar{\tau})-C_{\rm ss}^{\tilde{\nu}_*\tilde{z}_*}(\bar{\tau})\approx$$
$$\approx 2X^2\sqrt{\xi 2C}\exp\left(-\frac{\bar{\tau}}{2}\right)\sin(\sqrt{\xi 2C}\bar{\tau}), \quad (11)$$

does not vanish as long as atomic coherence is maintained; this is a sign of the competition between a restricted bandwidth in the communication channel across individual atoms, and the collective strong coupling of the atomic ensemble to the intracavity field. Monitoring a second channel, provided by an additional low-Q cavity coupled to the same atomic ensemble, via homodyne detection reveals a negative source-field spectrum of squeezing $S(\omega)\propto{\rm Re}[\bar{\mathcal{C}}_{\rm ss}^{\tilde{\nu}_*\tilde{\nu}_*}(-2i\omega/\gamma)]$ when the local oscillator is in phase with the mean collective atomic polarization, while combining the fields transmitted from the two cavities yields the cross-correlations of light-matter interaction in absorptive optical bistability.

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364020170014

1. H. J. Carmichael, Phys. Rev. A **33**, 3262 (1986).
2. H. J. Carmichael, *Statistical Methods in Quantum Optics 2* (Non-Classical Fields), Springer, Berlin (2008).
3. S. L. Mielke, G. T. Foster, and L. A. Orozco, Phys. Rev. Lett. **80**, 3948 (1998).