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Vielbein with mixed dimensions and gravitational global monopole in the planar phase of superfluid ^{3}He

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The planar phase is one of the possible superfluid phases of liquid ^{3}He [1]. It may exist in some region of the phase diagram of superfluid ^{3}He confined in aerogels [2]. The planar phase has two Dirac points in the quasiparticle spectrum, which are supported by combined action of topology and some special symmetry, see e.g. [3]. The quasiparticles in the planar phase with fixed spin behave as Weyl fermions. Similar to the chiral superfluid ^{3}He-A, they experience the effective gravity and gauge field produced by the deformation of the order parameter. But there is the following important difference. In ^{3}He- A, the spin-up and spin-down fermions have the same chirality, while in the planar phase the spin-up and spin-down fermions have the opposite chirality. As a result the Weyl fermions in planar phase form the massless Dirac fermions, see [4].

Here we study the planar phase fermions in the presence of the topological defect – the hedgehog. The effective gravity produced by the hedgehog appears to be similar to the gravitational effect of the global monopole in general relativity: it gives rise to the conical space [5–12]. Another consequence of the hedgehog is that the vielbein, which describes the effective gravity, is the 4×5 matrix, as distinct from the conventional 4×4 matrix in the tetrad formalism of general relativity.

In the general spin triplet p-wave pairing state the 2×2 matrix of the gap function is:

$$\hat{\Delta} = A^i_\alpha \sigma^\alpha p_i, \tag{1}$$

where $A_{\alpha i}$ is the 3×3 complex matrix [1]. In the planar phase the particular representative is:

$$A_{\alpha i} = c_\perp e^{i\Phi}\left(\delta^i_\alpha - \hat{l}_\alpha \hat{l}^i\right), \tag{2}$$

where Φ is the phase of the order parameter and $\hat{l}$ is the unit vector. All the other degenerate states of the planar phase are obtained by spin, orbital and phase rotations of the group $G = SO(3)_S \times SO(3)_L \times U(1)$ (here we ignore the discrete symmetry, since we are only interested in the global monopole).

The order parameter in Eq. (2) has the symmetry $H = SO(2)_J$ – the symmetry under the common spin and orbital rotations about the axis $\hat{l}$. As the result, the manifold of the degenerate states is $R = (SO(3)_S \times SO(3)_L \times U(1))/SO(2)_J$, which supports the monopoles (hedgehogs), described by the homotopy group $\pi_2(R) = Z$. The particular form of the monopole with the topological charge $N = 1$ is:

$$A_{\alpha i}(\mathbf{r}) = f(r)\left(\delta^i_\alpha - \hat{r}_\alpha \hat{r}^i\right), \tag{3}$$

where $\hat{r} = \mathbf{r}/r$, and $f(r\to\infty) = c_\perp$. The Bogoliubov–Nambu Hamiltonian for quasiparticles:

$$\begin{pmatrix} \epsilon(p) & \hat{\Delta} \\ \hat{\Delta}^\dagger & -\epsilon(p) \end{pmatrix}, \tag{4}$$

where $\epsilon(p) = c_\parallel(p - p_F)$, $c_\parallel = v_F$, and v_F and p_F are correspondingly the Fermi velocity and Fermi momentum of the normal Fermi liquid.

The planar phase has the Weyl–Dirac points at $\mathbf{p} = \pm p_F \hat{l}$. Near the Weyl–Dirac nodes the Hamiltonian is:

$$H = \sum_a \Gamma^a e^i_a (p_i - qA_i). \tag{5}$$

Here $\mathbf{A} = p_F \hat{l}$ is the vector potential of effective gauge field acting on the massless Dirac fermions; $q = \pm1$ is the corresponding electric charge; Γ^a with $a = 1,2,3,4$ are the Hermitian Γ-matrices with $\{\Gamma^a, \Gamma^b\} = 2\delta^{ab}$:

$$\Gamma^1 = \tau_1\sigma_x,\ \Gamma^2 = \tau_1\sigma_y,\ \Gamma^3 = \tau_1\sigma_z,\ \Gamma^4 = \tau_3; \tag{6}$$

e^i_a are the components of the spatial vielbein with $a = 1,2,3,4$ and $i = 1,2,3$:

$$e^i_a = c_\perp(\delta^i_a - \hat{l}_a \hat{l}^i) \text{ for } a = 1,2,3,\ e^i_4 = c_\parallel \hat{l}^i. \tag{7}$$

Such vielbein is the 3×4 matrix, instead of the conventional 3×3 matrix of the dreibein. Nevertheless, this asymmetric vielbein provides the correct expression for the elements of the effective metric:

$$g^{ik} = \sum_{a,b} \delta^{ab} e^i_a e^k_b,\ a,b = 1,2,3,4,\ i,k = 1,2,3, \tag{8}$$

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$$g^{ik} = c_\parallel^2 \hat{l}^i \hat{l}^k + c_\perp^2 (\delta^{ik} - \hat{l}^i \hat{l}^k). \tag{9}$$

This metric coincides with the effective metric in ^{3}He-A with Weyl nodes, see [4]. The $3+1$ effective metric is expressed in terms of the 4×5 vielbein,

$$g^{\mu\nu} = \sum_{a,b} \eta^{ab} e_a^\mu e_b^\nu, \; a,b = 0,1,2,3,4, \; \mu,\nu = 0,1,2,3. \tag{10}$$

In spite of the asymmetric non-invertible 4×5 vielbein, the effective metric is well defined and is invertible:

$$g_{ik} = \frac{1}{c_\parallel^2} \hat{l}_i \hat{l}_k + \frac{1}{c_\perp^2} (\delta_{ik} - \hat{l}_i \hat{l}_k), \; g_{00} = -1. \tag{11}$$

For the monopole one has:

$$g^{ik}(\mathbf{r}) = c_\perp^2 \delta^{ik} + (c_\parallel^2 - c_\perp^2) \hat{r}^i \hat{r}^k, \tag{12}$$

$$ds^2 = -dt^2 + \frac{1}{c_\perp^2} r^2 d\Omega^2 + \frac{1}{c_\parallel^2} dr^2. \tag{13}$$

Equation (13) represents conical spacetime, which in GR is produced by the global monopoles (monopoles without gauge fields, see [5–12]). This spacetime has the nonzero scalar curvature:

$$R = 2\frac{1-\alpha^2}{r^2}, \; \alpha^2 = \frac{c_\parallel^2}{c_\perp^2}. \tag{14}$$

The analog of the global monopole was considered in ^{3}He-A [8, 13], where it has the tail – the doubly quantized vortex. This is the analog of the Nambu monopole [14] terminating cosmic string (see classification of such composite objects in [15]). In the planar phase the monopole is topologically stable: the Dirac string of the monopole in the orbital vector $\hat{l}^i(\mathbf{r}) = \hat{r}^i$ in Eq. (3) is cancelled by the Dirac string from the monopole in the spin vector $\hat{l}_\alpha(\mathbf{r}) = \hat{r}_\alpha$.

Since in superfluid ^{3}He $c_\parallel^2 > c_\perp^2$, the metric corresponds to the spacetime with the solid angle excess [8, 13], $\alpha^2 > 1$, instead of the angle deficit discussed for the global cosmic monopoles with $\alpha^2 < 1$. For the cosmic monopole in the scalar field η with $\alpha^2 = 1 - 8\pi G \eta^2$, the angle excess corresponds to the repulsive gravity, $G < 0$, and super-Planckian field, $\eta^2 > 1/|G|$.

For $c_\parallel^2 = c_\perp^2 \equiv c^2$ the metric far from the monopole is flat, $g^{ik} = c^2 \delta^{ik}$. In cosmology this corresponds to the absence of the cosmic global monopole, or the absence of the scalar field in the vacuum, $\eta = 0$. However, the planar phase monopole does not disappear: the singularity remains in the vielbein field, while the metric has only the localized bump in the curvature and is flat (not conical) at infinity. The tetrad field monopole in the 4D Euclidean space (torsional instanton) with the localized bump in the curvature and the flat metric at infinity was considered in [16–18].

The planar phase provides an example, when the gravity for fermions and bosons can be essentially different. While the fermions are described by the 4×5 vielbein matrix e_a^μ, the bosons are described by the conventional 4D metric $g_{\mu\nu}$. The vielbein with non-quadratic matrix e_a^μ may exist in other superfluid phases, including the ultracold fermionic gases. In the presence of topological objects, they may give rise to exotic effective spaces and spacetimes, which are different for fermions and bosons. One may expect the similar effects in general relativity with degenerate metric. Exotic monopole in gravity with degenerate tetrads was discussed for example in [19]. It would be interesting to consider the transition from the planar phase to the ^{3}He-B with massive Dirac fermions.

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