Pis'ma v ZhETF, vol. 113, iss. 9, pp. 587 – 588

Using relativistic kinematics to generalize the series solution of Bethe stopping power obtained from Laplace–Adomian Decomposition method

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Submitted 28 March 2021
Resubmitted 28 March 2021
Accepted 2 April 2021

DOI: 10.31857/S1234567821090020

As the charged particle passes through matter, the charged particle losses kinetic energy due to inelastic Coulomb interactions with target atoms. Energy transferred to orbital electrons may result in excitation or ionization, depending on the relative magnitude of energy transfer to the electron binding energy. Stopping power is a quantity that describes the mean energy loss of a charged particle in multiple inelastic interactions. More generally, stopping power (dE/dx) is related to the differential cross section in terms of energy loss ($d\sigma/dW$) by:

$$-\frac{dE}{dx} = \rho N_A \frac{Z}{A} \int W \frac{d\sigma}{dW} dW,$$

where ρ is the mass density of the medium, N_A is the Avogadro's constant, Z/A is the atomic number to atomic mass ratio, and W is the energy loss.

The relativistic Bethe stopping power is a formulation of electronic stopping power based on plane-wave analysis under Born approximation. One of the main uses of Bethe stopping power is in radiation dosimetry in various applications such as radiotherapy, and radiation protection. Analytical solution provides certain advantages over numerical solution such as generalization of mathematical behavior. Laplace–Adomian decomposition method (LADM) was used in the study of Gonzalez–Gaxiola et al. to find an analytical series solution to the Bethe stopping power. However, their method presented approximations that are limited to the non-relativistic energy range. In this study, relativistic kinematics was utilized to generalize the Bethe stopping power to all kinetic energies prior to LADM implementation.

The general expression of the Bethe stopping power is given by:

$$-\frac{dE}{dx} = 4\pi\rho r_e^2 m_e c^2 \frac{N_A Z}{A} \frac{z^2}{\beta^2} \left[\ln\left(\frac{2m_e c^2 \beta^2}{I(1-\beta^2)} \right) - \beta^2 \right],$$

where r_e is the classical electron radius, $m_e c^2$ is the electron rest energy, β is the ratio of particle velocity to the speed of light in vacuum (v/c), and I is the mean excitation energy. The general relationship between particle relative velocity β and kinetic energy was found to be:

$$\beta^2 = \frac{E(E+2m_0c^2)}{(E+m_0c^2)^2} = \frac{\tau(\tau+2)}{(\tau+1)^2},$$

where $\tau = E/m_0c^2$, and m_0c^2 is the rest mass of the particle. A change of variable $u = \tau(\tau+1)$ was introduced, which simplifies the Bethe stopping power into the following:

$$-\frac{du}{dx} = 8\pi\rho r_e^2 \frac{m_e c^2}{m_0 c^2} \frac{N_A Z z^2}{A} \left[\frac{(u+1)^{\frac{3}{2}}}{u} \ln\left(\frac{2m_e c^2 u}{I} \right) - \right.$$
$$\left. - \sqrt{u+1} \right] = N(u).$$

As adopted from the work of Gonzalez–Gaxiola et al., the LADM framework was applied to the Bethe stopping power to yield:

$$u(x) = \sum_{n=0}^{\infty} u_n(x) = u_0 - \mathcal{L}^{-1}\left\{ \frac{1}{s} \mathcal{L}\left\{ N(u) \right\} \right\},$$

where $\mathcal{L}$ is the Laplace transform operator and u_0 is the initial condition associated to the variable u. By expanding $N(u)$ as a series of Adomian polynomials, the following recursion relation was obtained:

$$u_{n+1}(x) = -\mathcal{L}^{-1}\left\{ \frac{1}{s} \mathcal{L}\left\{ A_n(u_0, ..., u_n) \right\} \right\},$$

where $A_n(u_0, ..., u_n)$ is the Adomian polynomial of order n. This results in the derived series solution being a function of both path length x and initial energy E_0 associated to u_0.

The series solution obtained from LADM was compared to numerical solution for different initial energies, absorbing material, and charged particle type. Figure 1 illustrates the series and numerical solutions for a pro-

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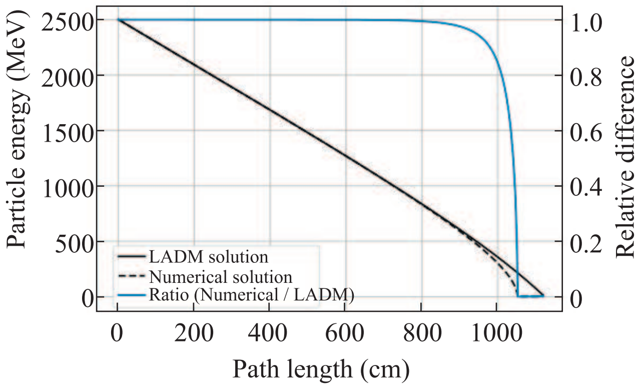

Fig. 1. (Color online) LADM series solution for $E_0 = 2.5$ GeV initial proton energy in water

ton particle with 5 GeV incident kinetic energy in water. Results showed that the derived series solution agrees with the numerical solution for a relatively large portion of the particle range. As the path length traversed increases, deviation between the two solution also increases due to exclusion of higher order terms in the series solution.

The study also demonstrates that the series solution derived is able to reasonably approximate the numerical solution for different absorbing medium and particle types. This generalization extends to both relativistic and non-relativistic incident kinetic energies. However, inaccuracies of the series solution at the end of the particle range also give rise to differences in continuous slowing down approximation (CSDA) range of the particle compared to that with numerical solution. Depending on the application, the series solution obtained can be utilized with sufficient accuracy. For instance, application to radiation shielding is viable since assumptions that slightly overestimate dose or particle penetrability are imposed.

Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364021090022