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Stochastization of long living spin-cyclotron excitations in a spin-unpolarised quantum Hall system

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We address the kinetics of long-lived cyclotron spin-flip exitations (CSFEs, actually magnetoexcitons) in a purely electronic quantum Hall system with filling factor $\nu = 2$. The initial coherent state of CSFEs with two-dimensional wave vector $\mathbf{q} = 0$ induced by laser pumping is stochastized over time due to emission of acoustic phonons. The elementary emission process requires participation of two magnetoexitons, so the effective rate of phonon emission is proportional to the excitation density squared, and the stochastization process occurs nonexponentially with time.

It is known that the lifetime of CSFEs (see [1, 2]) reaches a record magnitude, 1 ms, in a spin-unpolarised quantum Hall system [3]. In this paper we study the CSFE stochastization, i.e. decay of an initial coherent multi-excitonic state, where all excitations have equal 2D momenta $\mathbf{q} = 0$, into a diffusive incoherent state provided that the total number of excitations remains constant. When the "zero momentum" ensemble subsequently becomes stochastic, the main mass of excitons in the K-space diffuses to the vicinity of the CSFE energy dispersion minimum $\mathcal{E}_m$, where this energy counted from its value at the zero momentum is $\mathcal{E}_q \sim \mathcal{E}_m < 0$ at $q \sim q_m \approx 0.9/l_B$ (l_B is the magnetic length), and finally CSFEs completely relax/annihilate therefrom. The stochastization occurs without any change of the spin state, thus, certainly, it is much faster than the total CSFE-relaxation process. However, the stochastization is also associated, like relaxation, with emission of phonons and limited by the laws of conservation of energy and momentum. In the translationary invariant system, the one-exciton process associated with the emission of a phonon is kinematically forbidden: the energy and momentum preservation conditions are never fulfilled in the case.

We use an approach of "excitonic representation" (for more details see, e.g., [2] and [4]), which main idea is to abandon the basis of Fermi one-electron states and switch to the basis of so-called exciton states that diagonalize some essential part of the Coulomb interaction. Besides, in the case of CSFE, exciton states are eigen states of the Coulomb Hamiltonian to the first order in parameter representing ratio of the characteristic Coulomb energy to the cyclotron one, and allow to calculate the dispersion function $\mathcal{E}_q$. Exciton states, in contrast to single electron states, possess a natural quantum number, namely, the 2D momentum whose existence is the consequence of the translational invariance.

We consider the situation where the CSFE ensemble represents a rarefied gas: the number of excitons N is much smaller than $\mathcal{N}_\phi$ (number of magnetic flux quanta). Our two-excitonic process is transition from the initial coherent state $|N\rangle$, where all CSFEs are zero-momentum magnetoexcitons, to the state $|N; \mathbf{q}_1, \mathbf{q}_2\rangle$, where two of them acquire nonzero momenta $\mathbf{q}_1$ and $\mathbf{q}_2$. This transition obeys the momentum and energy conservation laws:

$$\mathbf{q}_1 + \mathbf{q}_2 - \mathbf{q} = 0 \tag{1}$$

and

$$\mathcal{E}_{q_1} + \mathcal{E}_{q_1} + \epsilon_{\rm ph}(\mathbf{k}) = 0, \tag{2}$$

where $\epsilon_{\rm ph}$ is the acoustic phonon energy, and $\mathbf{k} = (-\mathbf{q}, k_z)$ is the phonon wave vector.

Using the excitonic representation approach, one can express any interaction in terms of special exciton operators. The interaction of electrons with 3D acoustic phonons is described by Hamiltonian proportional to the phonon annihilation operator $\hat{\mathcal{P}}_{\mathbf{k},s}$ (see [5–7]):

$$\hat{H}_{\text{e-ph}} = \sum_{\mathbf{q},k_z,s} \hat{\mathcal{P}}_{\mathbf{k},s} \mathcal{H}_{\rm e-ph}(\mathbf{q}, s) + \text{H. c.} \tag{3}$$

(index s denotes possible phonon polarizations), where $\mathcal{H}_{\rm e-ph}(\mathbf{q}, s)$ is the electron-phonon vertex averaged over transverse coordinate z in the quantum well. With the help of the excitonic representation we calculate the transition matrix element of operator (3) between the initial state $|N\rangle$ and the final one $\hat{\mathcal{P}}^\dagger_{\mathbf{k},s}|N; \mathbf{q}_1, \mathbf{q}_2\rangle$,

$$\mathcal{M}_{\mathbf{k},s,\mathbf{q}_1,\mathbf{q}_2} = \frac{\langle \mathbf{q}_2, \mathbf{q}_1; N|\hat{\mathcal{P}}_{\mathbf{k},s}\hat{H}_{\rm e-ph}|N\rangle}{\left(\langle N|N\rangle\langle \mathbf{q}_2, \mathbf{q}_1; N|N; \mathbf{q}_1, \mathbf{q}_2\rangle\right)^{1/2}}, \tag{4}$$

and, thus, find the probability of transition per unit of time according to the well-known formula

$$W_{\mathbf{k},s,\mathbf{q}_1,\mathbf{q}_2} = \frac{2\pi}{\hbar}\left|\mathcal{M}_{\mathbf{k},s,\mathbf{q}_1,\mathbf{q}_2}\right|^2 \delta[\mathcal{E}_{q_1} + \mathcal{E}_{q_2} + \epsilon_{\rm ph}(\mathbf{k})]. \tag{5}$$

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If we perform the summation

$$R_{\mathbf{p}} = \sum_{\mathbf{k},s,\mathbf{q}_2} W_{\mathbf{k},s,\mathbf{p},\mathbf{q}_2}, \tag{6}$$

we obtain the total probability of transition to a state, where one of "nonzero" magnetoexcitons has a fixed wave vector: $\mathbf{q}_1 = \mathbf{p}$ (or $\mathbf{q}_2 = \mathbf{p}$).

The value $R_{\mathbf{p}}$ (6) represents the rate of appearance of a magnetoexciton with momentum $\mathbf{p}$ due to the considered process of direct transition from the initial coherent state $|N\rangle$ to any state $|N;\mathbf{p},\mathbf{q}\rangle$ with unfixed number $\mathbf{q}$. When studying the problem kinetically, it will mean the rate of filling of a "one-particle" magnetoexcitonic state with specific momentum $\mathbf{p}$ (in fact, $R_{\mathbf{p}}$ does not depend on the $\mathbf{p}$ direction). It is obvious that the total rate induced by phonon-emission, $R = \sum_{\mathbf{p}} R_p$, is, on the one hand, the rate of the coherent state $|N\rangle$ decay/stochastization, and, on the other hand, the rate of appearance of nonzero magnetoexcitons in the system. The physical meaning of the value R allows us to consider the kinetic equation:

$$dN/dt = -R \equiv -\frac{N^2}{\mathcal{N}_\phi \mathcal{T}}. \tag{7}$$

For our specific case, when $B = 4.18$ T (cf. [8]), we get numerical value $\mathcal{T} \approx 0.88$ ns. Solving Eq. (7), we obtain the time law of change of the number of $q = 0$ excitations:

$$n(t) = n(0) \Big/ [1 + t\,n(0)/\mathcal{T}]\,, \tag{8}$$

where $n(t) = N/\mathcal{N}_\phi$ is the concentration of zero-momentum CSFEs, while $n(0) = N(0)/\mathcal{N}_\phi$ is the given total CSFE concentration in the system. When dividing the 'partial' rate R_p by the total one R we obviously obtain a "one-particle" nonzero magnetoexciton distribution function, $F_p = \mathcal{N}_\phi R_p/R$ (F_p is normalized as: $\int F_p p dp = 1$). Value F_p is the final distribution function when the stochastization is completed.

It has been assumed that the coherent ensemble of zero-momentum excitons is the only generator of nonzero magnetoexcitons with neglecting any subsequent evolution of the emerging nonzero magnetoexcitonic ensemble. This approach should be suitable if the relative nonzero CSFE concentration is small and, in addition, the temperature is sufficiently low to ignore any phonon-absorption processes. Thermalization is a much longer process than the stochastization considered. We compare the distribution function established due to stochastization with a thermodynamically equilibrium distribution corresponding to some temperature. The latter should be Boltzmann due to the rarefaction of magnetoexiton gas $[N(0) \ll \mathcal{N}_\phi]$

$$F_p^{(T)} = e^{-\mathcal{E}_p/T} \Big/ \int_0^\infty e^{-\mathcal{E}_p/T} p dp\,. \tag{9}$$

In Figure 1 we demonstrate both distributions equally normalized. In the $T \to 0$ limit $F_p^{(T)} \propto \delta(p - q_m)$.

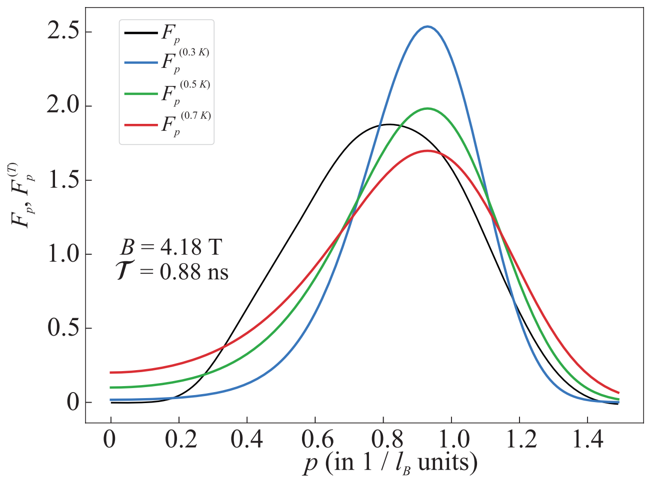

Fig. 1. (Color online) The result of calculating the distribution function F_p of CSFEs emerging due to the stochastization process (the black line; see text), and the thermodynamically equilibrium distribution functions $F_p^{(T)}$ at different temperatures (9). All graphs correspond to $B = 4.18$ T

At temperature $T \sim 0.5$ K, the stochastization distribution F_p becomes qualitatively similar to the thermodynamically equilibrium one $F_p^{(T)}$.

So, the presented model results in a nonexponential decay of the initial coherent CSFE ensemble. The time dependence of the decay (8) is parameterized by time $\mathcal{T}$ (Fig. 1), e.g., a tenfold decrease will take time $\approx 10\mathcal{T}/n(0)$, therefore, for $n(0) \leqslant 0.01$ it occurs during $\gtrsim 1\,\mu$s (cf. the 3D characteristic electron-phonon scattering time [9] which is ~ 0.1 ps if $q \sim k_z \sim 1/l_B \sim 10^6$ cm^{-1}, and the CSFE lifetime which is $> 50\,\mu$s at $B = 4.18$ T if estimated on the basis of the recent data [8]).

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1. C. Kallin and B.I. Halperin, Phys. Rev. B **30**, 5655 (1984).
2. S. Dickmann and I. V. Kukushkin, Phys. Rev. B **71**, 241310(R) (2005).
3. L. V. Kulik et al., Phys. Usp. **62**, 865 (2019).
4. S. Dickmann, L. V. Kulik, and V. A. Kuznetsov, Phys. Rev. B **100**, 155304 (2019).
5. S. V. Iordanskii and B. A. Muzykantskii, JETP **69**, 1006 (1989).
6. S. Dickmann and S. V. Iordanskii, JETP **83**, 128 (1996).
7. S. Dickmann, Phys. Rev. B **61**, 5461 (2000).
8. A. S. Zhuravlev et al., JETP Lett. **110**, 284 (2019).
9. V. F. Gantmakher and Y. B. Levinson, *Carrier Scattaring in Metals and Semiconductors*, North-Holland, Amsterdam (1987).