Pis'ma v ZhETF, vol. 119, iss. 3, pp. 253 – 254

Magnetic eigenmodes in chains of coupled φ_0-Josephson junctions with ferromagnetic weak links

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Submitted 17 November 2023
Resubmitted 2 December 2023
Accepted 9 December 2023

DOI: 10.31857/S1234567824030133, EDN: sszvaq

Introduction. By now it is well established that by hybridizing superconducting and ferromagnetic orders intriguing physics emerges and new device functionality can be achieved, which is inaccessible in conventional systems. One of actively studied types of interaction between the magnetic and superconducting subsystem can be realized under the presence of inversion symmetry breaking via the magnetoelectric effect, which provides a direct coupling between a supercurrent or a phase of superconducting condensate and a magnetic moment, thus opening up broad prospects for various options for controlling magnetization using supercurrent [1]. One of the most well-studied realizations of the magnetoelectric coupling is the so-called φ_0-superconductor/ferromagnet/superconductor (S/F/S) Josephson junction [1].

Further it was demonstrated [2, 3] that if the φ_0 - S/F/S JJs are coupled into a Josephson chain, there is a long-range interaction between them. It was also shown that such a chain manifests properties of n-level system, where the energies of the levels are only determined by projections of the total magnetic moment $\sum \mathbf{M}_i$, where $\mathbf{M}_i$ is a magnetic moment of i-th JJ, onto the easy magnetic axis. It resembles an atom in a Zeeman field, but the role of the field is played by the magnetoelectric coupling. It is interesting that, unlike the Zeeman effect in the atom, the relative order of energies of different states is controlled by phase difference between the external superconducting leads.

In this work we study eigen magnetic excitations in a system of magnets, which are weak link of coupled φ_0 - S/F/S JJs and interact via the superconducting condensate. It is shown that due to the long-range interaction between the magnets the energies of the eigenmodes can depend on the equilibrium magnetic configuration of all the magnets. The eigenfrequencies are controlled via the external superconducting phase. The eigenvectors of these modes, that is the distribution of oscillations of different magnets for a given mode, are also investigated.

System and model. We consider a linear chain of N coupled φ_0 – S/F/S JJs. The mutual orientation of the easy axis and the spin-orbit induced effective field (directed along the y-axis) determines behavior of the eigenmodes. For this reason in this work we consider two different orientations of the easy-axis: along the y-direction and along the z-direction. The current-phase relation (CPR) of a separate S/F/S junction takes the form $I = I_c \sin(\chi_i - \varphi_0)$. The energy of the system consists of Josephson energies of all junctions and easy axis anisotropy energies of all magnets:

$$E = \sum_{i=1}^{N} \Big[\frac{\hbar I_{c,i}}{2e} \big(1 - \cos(\psi_i - \psi_{i-1} - \varphi_{0,i})\big) - \frac{K_i V_{F,i}}{2} (\mathbf{m}_i \mathbf{e}_i)^2 \Big], \quad (1)$$

where the first term is the Josephson energy E_J and the second term is the magnetic anisotropy energy E_M. ψ_i is a phase of i-th superconductor.

Magnetic eigenmodes. The eigenfrequencies and eigenmodes of coupled system of magnetic moments $\mathbf{m}_i$ can be found in a quite standard way from the Landau–Lifshitz–Gilbert equation. The torque from the supercurrent can be accouted for via an additional contribution to $\mathbf{H}_{\text{eff}}$ as $\mathbf{H}_{\text{eff},J} = -(1/MV_F) dE_J/d\mathbf{m}$ [4–7].

If the easy axis is along the y-direction, the total effective field acting on each of the magnets $\mathbf{H}^i_{\text{eff}} = \mathbf{H}^i_{\text{eff},M} + \mathbf{H}^i_{\text{eff},J}$ is also along the y-direction and takes the form

$$\mathbf{H}^i_{\text{eff}} = \frac{K}{M} \left[m_{yi} + \frac{r E^0_J}{2 E^0_M} \sin \Phi \right] \mathbf{e}_y. \quad (2)$$

With this effective field we obtain the following eigenfrequencies:

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$$\omega_{\pm} = \gamma \frac{K}{M} (1 \pm \frac{r E_J^0}{2 E_M^0} \sin\Phi). \tag{3}$$

The eigenfrequencies $\omega_{\pm}$ are plotted in Fig. 1a as functions of ψ_N. As it was found in [3], we can change stable magnetic configuration of the system by varying ψ_N. With such a change, the eigenfrequencies will experience a jump, as it is seen in Fig. 1a. The eigenmodes, corresponding to a given eigenfrequency can be chosen as independent oscillations of separate magnets.

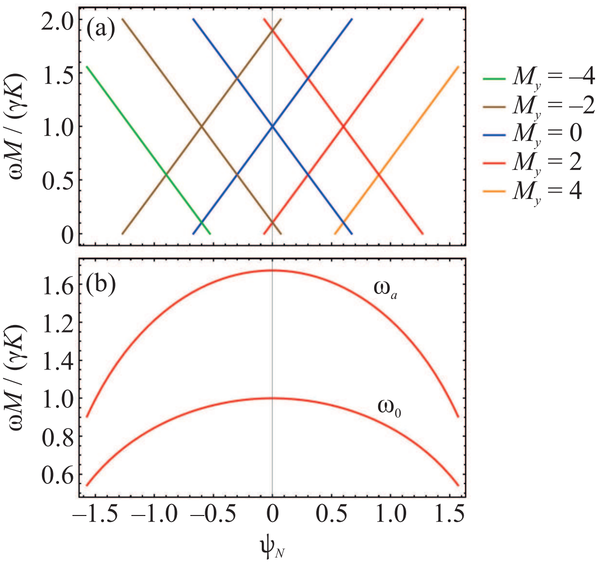

Fig. 1. (Color online) Eigenfrequencies of magnons as functions of the external phase ψ_N. (a) – Magnetic easy axis is along the y-axis. There are two eigenfrequencies $\omega_{\pm}$ for a magnetic configuration corresponding to a given y-projections M_y of the total magnetic moment. (b) – Magnetic easy axis is along the z-axis. There are only two eigenfrequencies, acoustic mode ω_a and optic mode ω_o, which are the same for all magnetic configurations. Parameters $N = 4$, $r = 0.3$, $E_M^0/E_J^0 = 0.025$ for the both panels

Now let us consider the case when the easy axis is along the z-axis. For each of the magnets we move to a local reference frame determined by three mutually orthogonal unit vectors $\mathbf{e}_x$, $\mathbf{e}_{\parallel} \equiv \mathbf{m}_i^0$, $\mathbf{e}_{\perp} = \mathbf{e}_{\parallel} \times \mathbf{e}_x$, where $\mathbf{m}_i^0$ is the unit vector along the equilibrium direction of a given magnet and $\delta\mathbf{m}_i$ is due to its excitation. We denote the angle between the vectors $\mathbf{e}_z$ and $\mathbf{e}_{\parallel}$ as α_i^0. The angle between $\mathbf{m}_i = \mathbf{m}_i^0 + \delta\mathbf{m}_i$ and $\mathbf{e}_z$ is $\alpha_i = \alpha_i^0 + \delta\alpha_i$. Then we can write $\mathbf{H}_{\rm eff}^i = H_{{\rm eff},\parallel}^i \mathbf{e}_{\parallel} + H_{{\rm eff},\perp}^i \mathbf{e}_{\perp}$ with

$$H_{{\rm eff},\parallel}^i \approx \frac{K}{M}\left[\cos^2\alpha_i^0 + \frac{r E_J^0}{2E_M}\sin\alpha_i^0 \sin\Phi\right] \tag{4}$$

and

$$H_{{\rm eff},\perp}^i \approx \lambda_0 \delta\alpha_i + \lambda_1 \cos\alpha_i^0 \sum_{j=1}^{N} \delta\alpha_j \cos\alpha_j^0, \tag{5}$$

where $\Phi = (1/N)[\psi_N - r\sum_{i=1}^{N}\sin\alpha_i^0]$, $\lambda_0 = (K/M)\sin^2\alpha_i^0$ and $\lambda_1 = -(r^2 E_J^0/V_F M N)\cos\Phi\cos^2\alpha_i^0$.

Since there are two stable solutions $\mathbf{m}_i^0 = (0, m_{y,st}, \pm\sqrt{1 - m_{y,st}^2})^T$, we can write that $\sin\alpha_i^0$ is the same for all magnets $\sin\alpha_i^0 \equiv \sin\alpha^0$ and $\cos\alpha_i^0 = \sigma_i\cos\alpha^0$, where $\sigma_i = \pm 1$. Then the effective field $H_{{\rm eff},\parallel}^i \equiv H_{{\rm eff},\parallel}$ is also the same for all the magnets. The resulting eigenfrequencies neglecting α take the form:

$$\omega_a = \gamma\sqrt{H_{{\rm eff},\parallel}(H_{{\rm eff},\parallel} - \lambda_0 - N\lambda)}, \tag{6}$$

$$\omega_o = \gamma\sqrt{H_{{\rm eff},\parallel}(H_{{\rm eff},\parallel} - \lambda_0)}. \tag{7}$$

The eigenfrequencies depend on the external phase difference ψ_N via α^0 and via the dependence on Φ in λ_1. The frequencies are plotted in Fig. 1b as functions of ψ_N. Since in the case when the easy axis is along the z-axis there is no splitting of the equilibrium spectra for different M_y, in contrast to the case of easy y-axis, here we can see only one branch for each of the frequencies.

Funding. The work has been supported by the Russian Science Foundation grant #22-42-04408, https://rscf.ru/project/22-42-04408/.

Conflict of interest. The authors declare no competing interests.

This is an excerpt of the article "Magnetic eigenmodes in chains of coupled φ_0-Josephson junctions with ferromagnetic weak links". Full text of the paper is published in JETP Letters journal. DOI: 10.1134/S0021364023604013

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