Письма в ЖЭТФ, том 55, вып.11, стр.605 - 609 10 июня 1992 г.

CALCULATING DECONFINEMENT TEMPERATURE THROUGH THE SCALE ANOMALY IN GLUODYNAMICS

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Submitted 14 April 1992

The deconfining temperature T_c is estimated from the minimum of the free energy of the vacuum. T_c is expressed via scale anomaly through the gluonic condensate to be around 200 MeV and does not depend on N_c at $N_c \to \infty$.

1. It is known [1] that the nonperturbative vacuum energy density ε is nonzero due to the scale anomaly [2], namely

$$\varepsilon = \frac{\beta(\alpha_s)}{16\alpha_s} < G^a_{\mu\nu} G^a_{\mu\nu} > \cong -\frac{11}{3} N_c \frac{\alpha_s}{32\pi} < G^2 > \tag{1}$$

One can associate with ε the zero-temperature limit of the free energy $F = E - TS$, and represent F in the form

$$F = \varepsilon V_3 + f(T) \tag{2}$$

where $f(T)$ is obtained in the usual way via statistical sum [3]. Here we calculate $f(T)$ representing gluonic field A_μ as a background field B_μ plus perturbative gluonic part a_μ. To the lowest order in a_μ the statistical sum Z_0 can be written as

$$Z_0 = < \exp \left\{ \int_0^\infty \xi(t) \frac{dt}{t} Sp \left(\frac{1}{2} e^{-tW} - e^{tD^2(B)} \right) \right\} >_B = e^{-f_0(T)/T} \tag{3}$$

where $\xi(t)$ is the regularizing factor, i.e. in the ζ- regularization it is $\xi(t) = \frac{d}{ds} \frac{(M^2 t)^s}{\Gamma(s)}$ with $s \to 0$, and M — regulator mass. In (3) W and $D^2(B)$ are the gluon and ghost propagators in the backrgound B_μ, and Sp is the trace in Lorentz, coordinate and color spaces [4]. Finally, $<>_B$ means averaging over fields B_μ.

In the limit $B \to 0$ one obtains for $f_0(T)$ the usual free-gluon-gas expression

$$f_0(T) = T(N_c^2 - 1) \int \sum_n ln \frac{M^2}{k_0^2 + k^2} V_3 \frac{d^3 k}{(2\pi)^3} = f_0(T = 0) - \frac{(N_c^2 - 1)}{45} \pi^2 V_3 T^4. \tag{4}$$

Here in $f_0(T=0)$ we have included all infinite constant terms, which we finally disregard since we normalize the free energy at $T=0$ by the term εV_3 (see (2)). We show below that for the confining background B_μ the free energy $f(T)$ contains contributions of two-gluon glueballs, three-gluon glueballs etc. and their interacting ensembles. From physical point of view $f_0(T)$ contains the most important contributions both in the confining and in the deconfined phases. In the first case those are glueballs, and in the second—free gluon gas, which gives dominant contribution to the free energy immediately beyond the phase-transition region [3].

In what follows we identity the main term in $f_0(T)$ and, imposing the properties of continuity and minimality on F in (2), calculate the deconfining temperature.

2. For the averaging procedure in (3) we can use the cluster expansion [5,6] and write

$$Z_0 = <\exp\varphi>_B = \exp\left\{<\varphi>_B + \frac{1}{2!}[<\varphi^2>_B - (<\varphi>_B)^2] + \ldots\right\} \tag{5}$$

where

$$\varphi \equiv \int_0^\infty \xi(t)\frac{dt}{t}\mathrm{Sp}e^{tD^2(B)} = tr\int_0^\infty \xi(t)\frac{dt}{t}Dz\frac{d^4xT}{V_3}\exp(-K)\Phi(x,x) \tag{6}$$

In (6) we disregard the spin interaction term (the difference between $-W$ and $D^2(B)$ in (3)) and use the Feynman-Schwinger (FS) representation [7] where

$$K = \frac{1}{4}\int_0^t \dot{z}^2 d\tau,\ \Phi(x,y) = P\exp ig\int_y^x B_\mu dz_\mu$$

Note that the path integral in (6) is over such trajectories $z(\tau)$ which return to the same point x. When $B^\mu = 0$, then $\Phi(x,x) = 1$ and $\varphi = -f_0/T$ corresponds to the closed trajectories of a free gluon at a given temperature T, yielding (4).

In the confined phase $<\varphi>_B \sim <\Phi(x,x)>_B$ i.e. a Wilson loop which obeys the area law [6]:

$$<\Phi(x,x)>_B = \exp(-\sigma_a S(x,x)),$$

where $S(x,x)$ is the minimal area inside the trajectory $z(\tau)$, and $\sigma_a = \frac{9}{4}\sigma$, σ is the string tension. The area law confines trajectory to the size $R \sim \sigma^{-\frac{1}{2}}$. One can identically rewrite the integral (6) as that corresponding to two gluons, propagating from a point x to a point y, interacting via confining area law term

$$-\frac{f_0^{(1)}}{T} = <\varphi_B> = \int_0^\infty \xi(t)\frac{dt}{t}\int_0^t \frac{dt_1}{t}d^4y\frac{d^4x}{V_4}[Dz(\tau)Dz(\tau')\exp(-K-K'-\sigma_a S)]_n \tag{7}$$

with $K' = \frac{1}{4}\int_0^{t-t_1}\dot{z}^2(\tau')d\tau'$, and the simbol $[\ \ ]_n$ means that the Matsubara series for the center of mass time is taken.

Thus $<\varphi_B>$ is a two-gluon bound-state Green's function of which the zero's Matsubara frequency is kept due to the integration in $d(x_4 - y_4)$. It gives no contribution to the free energy whenever relative motion of gluons is confined. When however, $\sigma \to 0$, and relative size of the bound two- gluon system compares to $V_3^{1/3}$ one finally recovers the limit of free gluons (4) . One can visualize this picture as ensemble of pairs of gluons frozen by confinement forming condensate for $T < T_c$, which"evaporates" for $T > T_c$ and become a free gluon gas.

3. The free glueball gas contribution is associated with the second term in (5), $<\varphi^2>_B$, where φ is given by (6).

The confining dynamics in $<\varphi^2>_B$ is given by the term

$$<\mathrm{tr}\Phi(x,x)\mathrm{tr}\Phi(y,y)>\sim \exp(-\sigma_a S(z(\tau),z'(\tau'))) \tag{8}$$

where we have defined the minimal area surface S joining the trajectories of two gluons $z(\tau)$ and $z'(\tau')$.

In this way we obtain

$$<\varphi^2>_B=\int_0^\infty \xi(t)\frac{dt}{t}Dz\xi(u)\frac{du}{u}Dz'\frac{d^4x}{V_4}\frac{d^4y}{V_4}\exp(-K-K'-\sigma_a S) \tag{9}$$

Introducing the full set of bound states of two gluons (glueballs) $<xy\mid n>$ and integrating over relative coordinates we find finally the glueball contribution to f_0

$$f_0^{(2)}=-\frac{T}{2!}<\varphi^2>_B=-TV_3\int\frac{d^3p}{(2\pi)^3}e^{-\beta\sqrt{p^2+M_0^2}}\approx-\frac{V_3M_0^{3/2}T^{5/2}}{(2\pi)^{3/2}}e^{-M_0/T} \tag{10}$$

Here we kept only the lowest glueball mass M_0 and consider the case $T\ll M_0$.

Note that in the limit $B_\mu\to 0$ the whole term $<\varphi^2>_B-(<\varphi>_B)^2$ tends to zero, since it is a connected contribution. In the deconfining transition when $\sigma\to 0$ also $M_0\to 0$, and $f_0^{(2)}$ vanishes.

4. Combining (2),(4) and (10) we obtain the relation

$$\frac{F}{V_3}\cong-\frac{11}{3}N_c\frac{\alpha_s<G^2>}{32\pi}-\frac{(N_c^2-1)\pi^2}{45}T^4\tilde{\Theta}(T-T_c)-\frac{M_0^{3/2}T^{5/2}}{(2\pi)^{3/2}}e^{-M_0/T} \tag{11}$$

Here we have introduced the transition function $\tilde{\Theta}(T-T_c)$ which is very close to the step function and corresponds to eq.(7). Indeed the steepness of this function is due to the fact that it is zero for $\sigma>V_3^{-2/3}$ and only for σ as small as $V_3^{-2/3}$, $f_0^{(1)}$ grows to a value given in (4). Since we know that the free energy F should be a continuous function [8] of σ around $\sigma=0$ (or of T around T_c) the nearly step function $\tilde{\Theta}(T-T_c)$ should be cancelled by other terms in (11).

We shall now see that the first term on the r.h.s. in (11) indeed jumps at $\sigma=0$ (at $T=T_c$) in the opposite direction to that of $f_0^{(1)}$. To this end we decompose the nonlocal vacuum correlator as in [6,9]

$$<\mathrm{tr}E_i(x)E_j(y)>=D^E(x-y)\delta_{ij}+0(\partial D_1^E) \tag{12}$$

$$<\mathrm{tr}B_i(x)B_j(y)>=D^B(x-y)\delta_{ij}+0(\partial D_1^B) \tag{13}$$

where $0(\partial D_1)$ means terms proportional to derivatives of another independent function D_1. At $T=0$ we have $D^E=D^B, D_1^E=D_1^B$, which is not true for $T>0$.

Out of four independent nonperturbative correlators D^E,D^B,D_1^E,D_1^B only the first one is responsible for confinement [6,7], providing nonzero string tension

$$\sigma=\frac{1}{2}\int_{-\infty}^{\infty}D^E(\sqrt{x^2+t^2})dxdt+... \tag{14}$$

where dots stand for the contribution of higher order cumulants.

On the other hand all four correlators enter as a sum in the vacuum condensate in (1)

$$<G^2>=D^E(0)+D_1^E(0)+D^B(0)+D_1^B(0) \tag{15}$$

At the transition point, $T = T_c$, D^E is vanishing according to (14), and this provides also a jump in $< G^2 >$ due to (15). This jump should be matched (and cancelled) in the total expression for the free energy (11) by the second term, $f_0^{(1)}$, thus making F continuous.

In addition to this continuity argument one can also use the princple of minimality of F [8], to argue that three other quantities $D_1^E(0), D^B(0)$ and $D_1^B(0)$ should stay unchanged at $T > 0$ [9]. This statement has been confirmed in Monte-Carlo computations in two ways. It was shown that only a part of the condensate $< G^2 >$ has a jump across $T = T_c$ but the rest stays nonzero for $T > T_c$ [11]. In space-like Wilson loops [12] the string tension σ^B was measured for $T > T_c$ (connected to D^B as in (14)) and found nonzero and close to σ at zero T.

Therefore we can define $D^E(0) = \eta < G^2 > [1 - \tilde{\Theta}(T - T_c)$, where η is weakly dependent on temperature and

i) $\eta(T = 0) = 1/2$ if $D_1^E(0) = D_1^B(0)$ is small as compared to $D^E(0)$.

ii) $\eta(T = 0) = 1/4$ if $D_1^E(0) = D^E(0)$

We note that η is a part of $< G^2 >$ which disappears during the deconfinement transition.

The correlator D_1 defines the nonperturbative tensor force in heavy quakonia [13] and computations [14] show that $D_1(0)$ is at most of the same order as $D(0)$. Moreover, recent lattice calculations of $D(0)$ and $D_1(0)$ [15] support this conclusion.

Now we can put the continuity condition near $T = T_c$ for the free energy F in (11):

$$\frac{11}{3} N_c \frac{\alpha_s \eta < G^2 >}{32\pi} + \frac{M_0^{3/2} T_c^{5/2}}{(2\pi)^{3/2}} e^{-M_0/T_c} = \frac{(N_c^2 - 1)\pi^2}{45} T_c^4 \tag{16}$$

Solving for T_c we obtain

$$T_c = \left(\frac{165 N_c \eta \alpha_s < G^2 >}{32\pi^3 (N_c^2 - 1)}\right)^{1/4} \left(1 + \frac{C(M_0)}{N_c^2}\right) \tag{17}$$

where $C(M_0)$ comes from the second term on the l.h.s. of (16),

$$C(M_0 = 1GeV) \cong 7.65 \cdot 10^{-4}$$

5. One can see in (17) that T_c is $0(1)$ at $N_c \to \infty$, since $\alpha_s < G^2 >$ is $0(N_c)$ [16]. For $N_c = 3$ we obtain

$$T_c = 220 MeV.(\eta \cdot \lambda)^{1/4} \tag{18}$$

where λ measures gluonic condensate in units of the standard value [1], $\frac{\alpha_s}{\pi} < G^2 >_{st} = 0.012 GeV^4, \lambda = \frac{< G^2 >}{< G^2 >_{st}}$. For the options $\lambda = 1, \eta = \frac{1}{2} \div \frac{1}{4}$ we obtain

$$T_c = 185 \div 156 MeV \tag{19}$$

This value should be compared with the lattice value for gluodynamics, $T_c = 197 \div 254 MeV$ [17] and implies that $\lambda \sim 2 \div 4$.

Also the fact that T_c is $0(1)$ for large N_c which is crucial for our mechanism of deconfinement. Note, that the glueball term $C(M_0)$ is small and suppressed at large N_c, which means that the Hagedorn-type mechanism is not operative in our case.

Summarizing, we have suggested a mechanism of deconfinement, in which a part of gluonic condensate evaporates into gluons in the deconfining transition.

Requirements of minimality and continuity of the free energy allows to estimate the deconfinement temperature within the uncertainty region (19), which is reasonably close to the lattice results.

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