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General scaling behavior of superconductors

V. R. Shaginyan[+*1)], *A. Z. Msezane*[*], *S. A. Artamonov*[+]

[+]*Petersburg Nuclear Physics Institute named by B. P. Konstantinov of National Research Centre Kurchatov Institute, 188300 Gatchina, Russia*

[*]*Department of Physics, Clark Atlanta University, Atlanta, GA 30314, USA*

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The physics of high-T_c superconductors, which has been a major topic in condensed matter physics for more than thirty years, reveals some features of conventional superconductors. We analyze the scaling of the condensation energy E_Δ divided by γ, $E_\Delta/\gamma \simeq N(0)\Delta_1^2/\gamma$, that equally applicable to both conventional and unconventional high-T_c superconductors. Here $N(0)$ is the density of states, Δ_1 is the maximum value of the superconducting gap and γ is the Sommerfeld coefficient. Basing on this observation, we analyze experimental facts that reveal the general scaling properties of both high-T_c and ordinary superconductors, and theoretically explain that the Homes' law $\rho_{s0} \propto T_c\sigma(T_c)$ is applicable to the both types of superconductors. Here σ is the conductivity, T is temperature and T_c is the temperature of superconduction phase transition, λ_D is the zero-T penetration depth, and ρ_{s0} is the superconducting electron density. For the first time, we also explain the reason of violation of the Homes' law. Our theoretical results agree well with experimental facts.

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The physics of high-temperature superconductors, which has been one of the main topics of condensed matter physics for more than thirty years, has revealed some common features of all superconductors. Experimentally observed scaling relationships lead to the identification of fundamental laws of nature and expose the essence of superconductor physics. We analyze the scaling of the condensation energy E_Δ divided by γ, $E_\Delta/\gamma \simeq N(0)\Delta_1^2/\gamma$, that equally applicable to both conventional and unconventional high-T_c superconductors [1]. Here $N(0)$ is the density of states, Δ_1 is the maximum value of the superconducting gap and γ is the Sommerfeld coefficient. Our consideration is based on both facts: Bogoliubov quasiparticles act in conventional and unconventional superconductors, and the corresponding flat band is deformed by the unconventional superconducting state [2–6]. These observations allow us to conclude that the Bardeen–Cooper–Schrieffer (BCS) physics is applicable in some cases in unconventional superconductors, as predicted [8, 4]. However, for example, in the case of high-T_c superconductors with flat bands the critical temperature [4, 9–15]

$$T_c \propto \Delta_1 \propto \lambda_0, \tag{1}$$

rather than being $T_c \propto \exp(-1/gN(0))$, where λ_0 is the superconducting coupling constant [7]. In the normal state, high-T_c superconductors exhibit the linear temperature resistivity, $\rho(T) \propto T$, explained within the Fermion condensation theory, see, e.g., [16, 17].

Now let us turn to Homes' scaling law [18–23]

$$\rho_{s0} = C_0 T_c \sigma(T_c). \tag{2}$$

Based on the observation that some part of the BCS physics applies to all types of superconductors, we analyze experimental facts that reveal general scaling properties of both high-T_c and ordinary superconductors, and theoretically explain that the Homes' law $\rho_{s0} \propto T_c\sigma(T_c)$ is applicable to all types of superconductors represented by high-temperature and conventional superconductors. Here σ is the conductivity, T is temperature and T_c is the temperature of superconduction phase transition, and ρ_{s0} is the superconducting electron density at $T = 0$. It is also shown that in the BCS model, Homes scaling is applicable in both the dirty and clean limits [20, 21]

$$\rho_{s0} = C_1 \Delta \sigma(T_c), \tag{3}$$

where $C_1 \simeq \alpha(k_B/h)$, $\Delta \simeq 1.7T_c$, Δ is the BCS gap at $T = 0$, and α is a factor. As a result, we conclude that $C_0 \simeq C_1$ and $\alpha \sim 1$, as in the case of BCS [20]. This possible deviation from unity of α does not have a significant effect on the logarithmic scale of Fig. 1 [18]. Equation (2) is supported by experimental facts collected on various superconductors [18, 21, 22], as seen from Fig. 1. Thus, Equation (2) shows the general behavior of both conventional and unconventional superconductors.

[1)]e-mail: vrshag@thd.pnpi.spb.ru

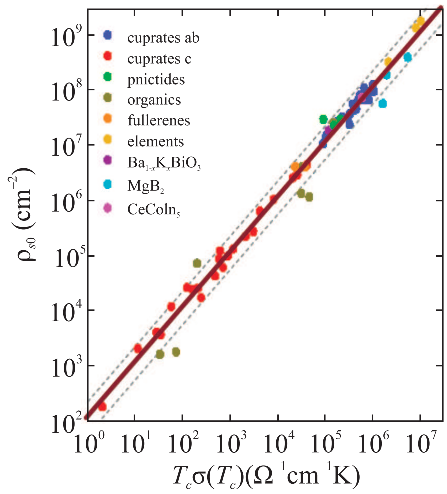

Fig. 1. (Color online) Double logarithm plot of ρ_{s0} as a function of $T_c\sigma(T_c)$ for various superconductors [18]. The solid wine line represents the theoretical calculations, see Eq. (2)

Several points need to be made here: It has been shown in [24] that at $T = 0$ the superfluid density n_s equals to the carriers n_{FC} in the FC state only. Note that n_{FC} is a free parameter, therefore it can be that $n_s \equiv n_{FC} \ll n_{\rm el}$, where $n_{\rm el}$ is the total density of electrons. These observations are in a good agreement with experimental results obtained on quasi-two-dimensional overdoped single-crystal films of $La_{2-x}Sr_xCuO_4$ [25]. For $n_s \ll n_{\rm el}$, Homes' law given by Eq. (2), is also violated [25]. Note that this violation is due to non-BCS behavior, since $\rho_{s0} \ll n_{\rm el}$ [24]. This observation agrees well with experimental facts [25].

We analyzed the experimental Homes' law, see Eq. (2), and theoretically explained that Homes' law is applicable to both conventional and non-conventional superconductors on equal grounds. Thus, superconductors exhibit the BCS-like properties and the corresponding general scaling behavior. For the first time, we have explained the case of violation of the Homes' law. Our studies of important experimental facts convincingly indicate that the topological fermion condensation quantum phase transition (FCQPT) is an integral feature of many strongly correlated Fermi systems and can be considered as a universal cause of their exotic non-Fermi-liquid properties, which cannot be observed in metals described by the Landau Fermi-liquid theory [4]. Moreover, the Fermion condensation theory based on topological FCQPT is able to explain the complex behavior of strongly correlated Fermi systems represented by unconventional superconductors and their connection with conventional superconductors, manifested in the scaling behavior of superconductors.

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