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Free Fock parafermions in the tight-binding model with dissipation

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Parafermions that generalize (Majorana or usual) fermions appear as interacting quasi-particles because of their nature. Although attempts to develop models with free (non-interacting) parafermions have been undertaken, existing proposals require unphysical conditions such as realizing purely non-Hermitian systems. Here we present a way for the realization of free Fock parafermions in the tight-binding model with controlled dissipation of a very simple form. Introducing dissipation transforms an originally non-integrable quantum model to an exactly solvable classical one.

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We consider a one-dimensional $\mathbb{Z}_n$ Fock-parafermion (FPF) tight-binding chain. FPFs generalize complex fermions by enlarging the on-site Hilbert space to n states $|m\rangle$ $(m = 0, \ldots, n-1)$, so that one may have up to $n-1$ identical particles in the same local state [1]. Their algebra includes nilpotency and generalized exchange statistics, which for $j \neq \ell$ can be summarized as

$$(F_j^\dagger)^n = F_j^n = 0, \tag{1}$$

$$\begin{aligned} F_j F_\ell &= \omega^{\mathrm{sgn}(\ell-j)} F_\ell F_j, \\ F_j^\dagger F_\ell &= \omega^{-\mathrm{sgn}(\ell-j)} F_\ell F_j^\dagger, \quad \omega = e^{2\pi \mathrm{i}/n}. \end{aligned} \tag{2}$$

Because Fourier modes do *not* satisfy the same algebra, even quadratic hopping Hamiltonians do not diagonalize in momentum space: different momenta remain coupled and the model is intrinsically interacting and non-integrable [2]. The central goal of this work is to show that free (non-interacting) *Fock* parafermions can nevertheless emerge in a physically realistic setting when one treats the system as open and engineers dissipation in a controlled way.

The coherent dynamics is described by an L-site open chain with multi-particle hopping processes ($1 \leq m \leq n-1$) and occupation-dependent chemical potentials:

$$H = -\sum_{m=1}^{n-1}\sum_{j=1}^{L-1}\Big[\alpha_m (F_j^\dagger)^m F_{j+1}^m + \mathrm{H.c.}\Big] - \\ -\sum_{m=1}^{n-1}\mu_m \sum_{j=1}^{L}(F_j^\dagger)^m F_j^m. \tag{3}$$

We couple the chain to a Markovian bath and use the Lindblad master equation $\partial_t \rho = \mathcal{L}\rho$, with the effective non-Hermitian Hamiltonian

$$H_{\mathrm{eff}} = H - \frac{\mathrm{i}}{2}\sum_{j,m}\gamma_m \big(L_j^{(m)}\big)^\dagger L_j^{(m)}, \tag{4}$$

and (quasi)local link-loss jump operators acting on bonds $(j, j+1)$,

$$L_j^{(m)} = F_j^m + \Delta_m F_{j+1}^m, \quad 1 \leq m \leq n-1, \tag{5}$$

where $\gamma_m > 0$ are dissipation rates and $\Delta_m \neq 0$ are complex control parameters. Such local dissipators are natural targets for reservoir-engineering methods developed in quantum optics and programmable quantum simulators [3, 4].

The key observation is that tuning the coherent couplings to match the dissipative ones,

$$\alpha_m = \frac{\mathrm{i}}{2}\gamma_m \Delta_m, \tag{6}$$

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produces a drastic simplification: in the natural Fock basis $\{|m_1,\ldots,m_L\rangle,\ 0\le m_j\le n-1\}$ the operator H_{eff} becomes lower-triangular. Physically, the dissipative contribution in Eq. (4) cancels the Hermitian-conjugate part of the Baxter-like hopping term, leaving only strictly off-diagonal hopping plus diagonal on-site terms. Therefore, the spectrum can be obtained in closed form. For any configuration $|m_1,\ldots,m_L\rangle$ one finds

$$H_{\mathrm{eff}}|m_1,\ldots,m_L\rangle = E_{m_1,\ldots,m_L}|m_1,\ldots,m_L\rangle, \qquad (7)$$

with

$$E_{m_1,\ldots,m_L} = -\sum_{j=1}^{L}\sum_{p=1}^{m_j}\left(\mu_p + \frac{\mathrm{i}}{2}\Gamma_j^{(p)}\right). \qquad (8)$$

Here $\Gamma_j^{(p)}$ are site-dependent decay coefficients fixed by γ_p and $|\Delta_p|^2$ (with boundary modifications). The energies are generally complex because H_{eff} is neither Hermitian nor $\mathcal{PT}$-symmetric [5]. Crucially, Eq. (8) is additive in single-particle contributions, i.e., it has the defining free-particle structure for Fock parafermions. This provides a physically consistent counterpart of earlier *purely* non-Hermitian constructions of free parafermions such as the Baxter $\mathbb{Z}_n$ Hamiltonian [6, 7].

Moreover, the same additivity persists for the *full* Liouvillian spectrum. The Hamiltonian (3) conserves total particle number $N=\sum_j N_j$, the jump operators (5) implement pure losses, and they satisfy the dynamical-algebra relation $[L_j^{(m)},N]=L_j^{(m)}$; under these conditions, the Liouvillian spectrum is completely determined by H_{eff} [8]. One obtains

$$\lambda_{m,p} = -\mathrm{i}\,(E_m - E_p^*), \qquad (9)$$

where E_m and E_p are given by Eq. (8). For generic $\gamma_1\neq 0$, all non-steady modes have strictly negative real parts and the unique steady state is the vacuum. In contrast, if single- particle hopping and loss are forbidden ($\alpha_1=0$ and $\gamma_1=0$), then all states with at most one FPF per site are annihilated by the remaining $m\ge 2$ loss operators and form a large decoherence-free (dark) subspace of dimension 2^L-1.

In summary, controlled dissipation acts here as a practical "solvability knob": it maps an interacting, non-integrable anyonic tight-binding model to an exactly solvable open system whose spectra, Eqs. (8) and (9), exhibit a free Fock-parafermion structure.

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