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Thermodynamics of homogeneous Universes: de Sitter, Bonnor–Melvin and static Einstein

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In the theories, in which dynamic gravitational field emerges from the underlying matter fields, the gravitational field can be considered as a part of matter. Using this approach, we construct the thermodynamics of the homogeneous Universes – the de Sitter Universe, the Bonnor–Melvin-Λ Universe and the static Einstein Universe. It is demonstrated that although these three Universes have different types of matter fields (ordinary matter, magnetic field, gravitational field and vacuum energy), they have the same thermodynamic properties. Their energy densities obey the same equation, which contains the corresponding matter densities and the pairs of the thermodynamically conjugate variables. In Minkowski vacuum, where the ordinary matter and magnetic and gravitational fields are absent, this thermodynamic approach automatically leads to zero cosmological constant.

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In pre-geometric theories, gravitational field originates from the matter fields. For example, tetrads and metric fields may emerge as the non-linear combinations of the fermions fields [1–6], or from the scalar fields [7–10]. That is why the gravitational field also represents the matter in addition to the ordinary matter, which is represented by particles and fields. The same is in the Sakharov induced gravity, where metric becomes dynamical after integration over the fermionic and bosonic fields [11]. In a similar way, in the Zel'dovich approach, the electrodynamic field becomes dynamical after integration over the charged particles [12]. The dynamical effective gravitational and electromagnetic fields emerge also in condensed matter with Weyl fermionic quasiparticles [13].

All this suggests that the curvature term in the Einstein–Hilbert action should be considered in the same way as the matter terms. In particular, the energy-momentum tensor of the gravitational field should be treated in the same way as the energy-momentum tensor of the ordinary matter, and thus it can be obtained by varying the curvature term in the Einstein–Hilbert action with respect to the spacetime metric $g^{\mu\nu}$. This leads to the appearance of a generalized energy-momentum tensors, which are applicable to all types of matter including gravitational field. According to Einstein's equations, the total generalized energy-momentum tensor, which includes the contributions of all matter fields including gravity, is equal to zero. Being zero it is automatically conserved, $\partial_\mu T^{\mu\nu} = \nabla_\mu T^{\mu\nu} = 0$.

Here we apply this approach to the thermodynamics of the Universes that are homogeneous in space. This includes the de Sitter Universe, the Bonnor–Melvin-Λ Universe and the static Einstein Universe. We consider these Universes as equilibrium thermodynamic states, ignoring their possible instability.

The plan of the paper is the following.

In Section 2, the gravitational field is considered as the specific matter field. In this approach, the Einstein equations are obtained not from the functional derivative of the action, $\delta S/\delta g^{\mu\nu} = 0$, but from the conventional derivative of the Lagrangian, $\frac{d\mathcal{L}^{\rm M}}{dg^{\mu\nu}} = 0$. This also allows to introduce the generalized energy-momentum tensor, which is applicable to any kind of matter including the gravitational field. This approach looks somewhat similar to that developed by Chakraborty and Padmanabhan [14–16], see also the recent paper [17]. But we do not use the connection between thermodynamics and horizon.

In Section 3, this approach is applied to the thermodynamics of the de Sitter state. The de Sitter thermodynamics contains two components: the vacuum component (dark energy or cosmological constant) and the thermal gravitational component – the Ricci scalar R. This component is characterized by temperature $T = H/\pi$, where H is the Hubble parameter. The generalized energy density contains the pair of thermodynamically conjugate variables, the Ricci scalar curvature

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R and the gravitational coupling $K = 1/16\pi G$. In the $f(R)$ theory, $K = df/dR$ plays the role of the gravitational chemical potential. The KR pair is the result of the Kronecker anomaly introduced by Polyakov and Popov [18], which emerges in the homogeneous systems due to jump of the number of the degrees of freedom at $\mathbf{k} = 0$. Note also, that the local de Sitter thermodynamics with its local temperature $T = H/\pi$ allows to study the problem of stability of the regular black holes with de Sitter core [19, 20].

The thermal component of the de Sitter state has equation of state $w = 1$, and thus looks as the Zel'dovich stiff matter. This gives rise to the de Sitter massless graviton discussed in Section 7.

In Section 4, the thermodynamics of the Universe with constant magnetic field is discussed – the Bonnor–Melvin-Λ Universe with cosmological constant. Here the generalized energy density has the same form as in the de Sitter thermodynamics, but the energy density of the de Sitter thermal component is replaced by the energy density of the magnetic field $\frac{B^2}{8\pi}$, while the KR contribution remains the same.

In Section 5, the thermodynamics of the S^3 static Einstein Universe is discussed. Again, the generalized energy density has the same form as in the de Sitter thermodynamics, with the de Sitter thermal component replaced by the energy density of the ordinary matter.

In Section 6, the dynamical dark energy is considered using the 3-form gauge field introduced by Hawking. The generalized energy density has the same form, only another thermodynamic pair is added – the curvature q of the 3-form field and its conjugate. The final result of the paper is the equation for the total generalized energy density

$$\epsilon^{\text{gen}} = \epsilon_{\text{Matter}} + KR - \sum_a \mu^{(a)} q^{(a)} = 0. \qquad (1)$$

Here ϵ_{Matter} includes all the types of matter: ordinary matter, dark (vacuum) energy, gravitational field (matter of gravity), and all types of the gauge fields. It also contains the pairs of the thermodynamically conjugate variables, where we also took into account the possibility of several different 3-form fields, see Eq. (10) in [21].

When ordinary matter, magnetic and gravitational fields are absent, and only vacuum energy remains, the result is a Minkowski vacuum with an automatically zero cosmological constant:

$$\epsilon^{\text{gen}} = \epsilon_{\text{vac}}(q) - \mu q = \frac{\Lambda}{8\pi} = 0. \qquad (2)$$

This is similar to what takes place in the multi-component liquid with densities $n^{(a)}$, where in the absence of the external pressure and at zero temperature one has $\epsilon - \sum_a \mu^{(a)} n^{(a)} = 0$.

In conclusion, the thermodynamics of the homogeneous Universes is constructed, considering the gravitational field as a part of matter fields. It is demonstrated that the de Sitter Universe, the Bonnor–Melvin-Λ Universe and the static Einstein Universe, although they have different types of matter fields (ordinary matter, magnetic field, gravitational field and vacuum energy), they have the same thermodynamic properties. Their energy densities obey the same Eq. (1), which contains the generalized matter density and the pairs of the thermodynamically conjugate variables. In the absence of the ordinary matter, magnetic and gravitational fields, the Universe represents the Minkowski vacuum with automatically zero cosmological constant.

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