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Devoted to memory of Prof. P. P. J. M. Schram

On Ionization in a Non-ideal Two-temperature Plasma

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For a classical two-temperature non-ideal plasma, in which temperature equilibration is slowed down due to the small mass ratio of electrons to heavy particles, the concentrations of electrons, ions, and atoms have been determined. The use of a generalized chemical equilibrium condition leads to a reduction in the ionization potential due to the Coulomb interaction between electrons and ions. Taking a hydrogen-like plasma as an example, it is shown that the effective ionization potential depends on both temperatures and decreases as the temperature difference increases.

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In many plasma systems of interest in both nature and technical applications, electron and ion temperatures can differ; this temperature difference can influence ionization processes. Examples of such systems include atmospheric plasma, discharge plasma, and thermonuclear plasma [1]. Standard equilibrium theory is inapplicable to two-temperature plasma. However, due to the slow rate of energy transfer from heated electrons to cooler heavy particles [2], particle densities in the plasma can be determined by the instantaneous temperatures and track their slow variations.

Given the non-equilibrium nature of this state, various methodological approaches have been employed to describe it [3–5], based on both non-equilibrium thermodynamics and kinetic theory. Mention should also be made of early works [6, 7] that did not yield the correct result but stimulated the development of the topic. In the works of A. Morro and M. Romeo [8] and M. van de Sanden and P. Schram [9], a general non-equilibrium approach (see, e.g., [10]) applicable to two-temperature plasma has been developed and applied to the two-temperature ideal plasma.

During plasma generation – whether in natural settings or laboratory experiments – energy is often transferred predominantly to either electrons or ions; this leads to the emergence of a quasi-equilibrium state in which the behavior of the two plasma components (electrons and ions) is described with high accuracy by two effective temperatures: T_e and T_i.

In the present work, we discuss an approach based on a chemical model for a two-temperature, moderately non-ideal plasma, restricting our consideration of non-ideality to the extended Debye–Huckel approximation for electron-ion interactions. A quantum-mechanical treatment will be presented in a separate publication.

The free energy F_{DH} for a non-degenerate two-temperature plasma $T_a \equiv 1/\beta_a$ in the Debye–Huckel approximation reads

$$F_{DH} = F_{id} - V\frac{2}{3}e^3\sqrt{\pi}\left(\sum_b z_b^2\beta_b n_{b0}\right)^{1/2}\sum_a z_a^2 n_{a0};$$
$$F_{id} = -\sum_a N_a T_a \ln\left[\frac{eg_aV}{N_a}\left(\frac{m_aT_a}{2\pi\hbar^2}\right)^{3/2}\right]. \qquad (1)$$

Let us use for the non-complete equilibrium condition $\sum_a \nu_a\beta_a\mu_a = 0$ [9] $(a = e; i; A)$. For the case of ionization reaction $e + A^+ = A$ and $\beta_i = \beta_A$ we arrive at equality

$$\beta_e\mu_e + \beta_i\mu_i = \beta_i\mu_A(\beta_i), \qquad (2)$$

where the chemical potentials $\mu_a = (\partial F/\partial N_a)_{T,V}$ are determined from the total free energy $F = F_{DH} + F_{A,id}$ of plasma (1) and ideal atomic subsystem $F_{A,id}$. To identify the primary effect, we shall restrict the subsequent discussion to this specific temperature range, where only ground-state atoms are present and the ions are singly charged $(Z \simeq \exp(-\varepsilon_0/T) = \exp(I/T_e))$.

For the case of two temperatures and singly ionized atoms Eq. (2) can be represented in the universal dimensionless form useful for numerical calculation of the degree of ionization. Let us introduce particle concentrations $n_e + n_A = n_0 = \text{const}$. Therefore, $c_e = n_e/n_0 \equiv \alpha_\tau$, where α_τ is the ionization degree

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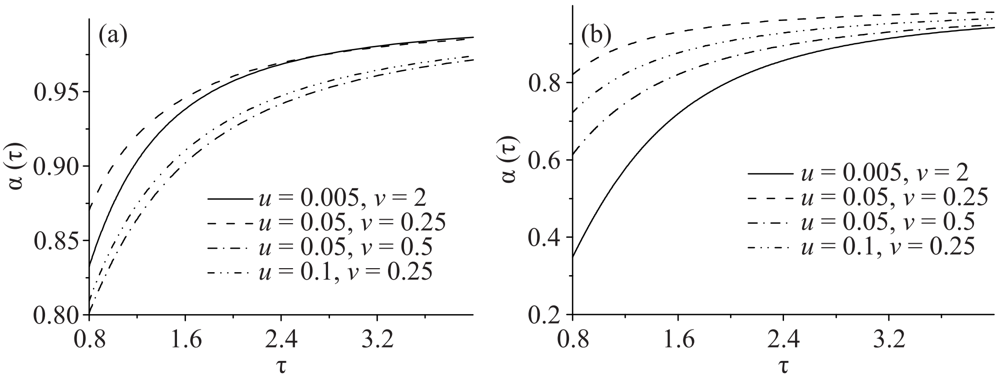

Fig. 1. (a) – The curve $\alpha_1(\tau)$ for various parameters $u = n_0 a_B^3$, $v = I/T$, $0.8 < \tau < 4$ for hydrogen $I = I_H$, accounted the Debye–Huckel reduction of ionization potential according Eq. (3); (b) – the same without the Debye–Huckel reduction of ionization potential

($c_i = c_e$, since $n_e = n_i$), $c_A = n_A/n_0 = 1 - \alpha$. For the degree of ionization we arrive at equation

$$\frac{1-\alpha_\tau}{\alpha_\tau^2} = n_0 a_B^3 \left(\frac{2I_H}{T_i}\right)^{3/2} \left(\frac{1}{\tau}\right)^{3/2} \times$$
$$\times \exp\left[I/(T_i\tau) - \frac{2\sqrt{\pi}}{3}\alpha_\tau^{1/2}(n_0 a_B)^{1/3}\left(\frac{2I_H}{T_i}\right)^{3/2}\frac{S(\tau)}{\tau^{3/2}}\right],$$
$$S = \frac{(1+\tau^2)}{(1+\tau)^{1/2}} + (1+\tau)^{3/2}. \tag{3}$$

In (3) we use the notifications $\tau = T_e/T_i = T_e/T_A$, I – the ionization potential of the ground state of an atom under consideration, $I_H = me^4/2\hbar^2$ – the hydrogen ionization potential and a_B – the Bohr radius. As $\tau \to \infty$, the obvious solution of the strongly nonlinear equation (3) is $\alpha = 1$, corresponding to fully ionised plasma. However, it should be noted that the values of $\tau \gg 1$, corresponding to a large disparity between electron and ion temperatures, require accounting for the heat flux between them [8, 9]. Furthermore, the approximation considered is valid for the interaction parameters $\gamma_i \ll 1, \gamma_e \ll 1$.

The calculations of the ionization degree for the hydrogen plasma $I = I_H$ based on Eq. (3) are presented below for various parameters.

In Figure 1 the curves for ionization degree α_τ are shown for various values of the parameters $u = n_0 a_B^3$ and $v = I_H/T$ with the Debaye—Huckel reduction of the ionization potential (a) and without it ((b) – standard Saha equation [11]).

The results presented above demonstrate that electron heating or ion cooling alters the instantaneous quasi-equilibrium degree of ionization in a weakly non-ideal two-temperature semiclassical plasma. These results can be generalized to moderately non-ideal semi-classical plasmas with the interaction parameter $\gamma_i \geq 1$, as well as to temperature and density regimes where a quantum-statistical description of the charged plasma components is required ($n_e\lambda_{ei}^3 \simeq 1$, where $\lambda_{ei} = (2\pi\hbar^2/m_e T_i)^{1/2}$ is the de Broglie wavelength of electrons with the ion temperature).

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