THE $B \to D(D^*)e\nu$-DECAY AMPLITUDE FACTORIZATION AND THE ISGUR-WISE FUNCTION CALCULATION

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A new approach to the relativistic system of heavy and light quarks is formulated in which a light quark is supposed to be confined whereas a heavy quark behaves as usual Fermi particle with large mass. The form factors for the $B \to D(D^*)e\nu$-decay are calculated within this approach. The Isgur-Wise function is obtained in the heavy quark limit $M_Q \to \infty$.

Recently, the physics of hadrons containing one heavy quark has received more attention due to the development of new symmetry in the world of heavy quarks [1]. It was shown that in the limit of large heavy quark masses, first, the flavor of the heavy quarks becomes irrelevent, i.e. the heavy quarks may be rotated into each other. Second, the spin degrees of freedom of the heavy quark decouple from dynamics in this limit because of color hyperfine interaction scales inversly with the heavy quark mass. With the use of these symmetries, relationships among the form factors describing the $B \to D(D^*)l\nu$-decays were obtained. It was shown that all form factors may be expressed in terms of a single universal function being depended on product of four-velocities of initial and final heavy mesons and normalized to unity at zero recoil.

Here, we formulate a new approach to the relativistic system of heavy and light quarks in which a light quark is supposed to be confined whereas a heavy quark behaves as usual Fermi particle with large mass. We calculate the form factors for the $B \to D(D^*)e\nu$-decay within this approach. It is shown that the Isgur-Wise representation for the form factors are reproduced in the heavy quark limit and the universal function is obtained.

We will describe heavy B and $D(D^*)$ mesons as the bound states of light $q(x)$ and heavy $Q(x)$ quarks using the compositeness condition $Z_3 = 0$ in the quantum field theory (see, e.g., Ref. [2] and others therein). The starting point in this approach is the interaction Lagrangian describing a transition of a meson $H(x)$ into two quarks $q(x)\bar{Q}(x)$:

$$L_H(x) = g_H H(x)\bar{Q}(x)\Gamma_H q(x) + \text{h.c.}. \tag{1}$$

Here, Γ_H is a Dirac matrix: $i\gamma^5$ for pseudoscalar D, B, and γ^μ for vector D^* mesons, respectively. A meson H is assumed to be bound state of $q\bar{Q}$, which may be expressed in the *compositeness condition* that the renormalization constant Z_H for meson wave function $H(x)$ is equal to zero [2]:

$$Z_H = 1 + h_H \tilde{\Pi}'_H(m_H^2) = 0, \tag{2}$$

where $h_H = 3g_H^2/(2\pi)^2$ is the effective coupling constant and $\tilde{\Pi}'_H$ is the derivative of the (renormalized) meson mass operator. Physically, this condition means that the probability to find the meson H in a bare state is equal to zero. In other words the meson H is a bound state in the system of two quarks. It is important to remark that (i) the interaction Lagrangian (1) together with the compositeness condition (2) is equivalent to the heuristic QCD bozonization based on the ideas of the Nambu-Jona-Lasinio model [3], and (ii) the compositeness condition (2) allows one to determine the coupling constant h_H (or g_H) as a function of the physical meson mass. Such a procedure of hadronization was accepted in [4] as one of the crucial points of the quark confinement model (QCM). In the one-loop approximation physical processes are described by quark diagrams containing a convolution of quark propagators.

It is widely accepted that the behavior of quarks at large distances is defined by their interactions with the vacuum gluon background. There exist vacuum configurations with constant strengths [5-7] which provide a quark confinement, i.e. give a quark propagator to be an entire analytical function on the momentum plane. Physically it could be understood as the absence of a quark with a definite value of mass in the observable hadron spectrum. As it was shown in Ref. [4] the propagator of light quark in the confined gluon background may be represented in the form

$$G(\not{p}) = \int\limits_L \frac{d\mu\rho(\mu)}{\mu - \not{p}} = \frac{1}{\Lambda_{conf}}[a(-\frac{p^2}{\Lambda_{conf}^2}) + \frac{\not{p}}{\Lambda}b(-p^2\Lambda_{conf}^2)], \tag{3}$$

where the integration contour L, the density of the quark mass distribution $\rho(\mu)$ and whence the confinement functions $a(z)$, $b(z)$, and the parameter Λ_{conf} characterizing the scale of confinement region should be defined by solving an equation for the Green function of quark in the external gluon field. The representation (3) allows one to consider a confined quark propagator as a superposition of local quark propagators having a smeared constituent mass μ with a density $\rho(\mu)$. Here we will not restrict ourselves by using the concrete shapes for the confinement functions but investigate how are sensitive the final results for form factors to their different forms. The only requirement imposed on the confined propagator (3) is decreasing in the Euclidean direction $p^2 \to -\infty$ to provide a convergence of all Feynman integrals. It means that the propagator of confined quark is assumed to be localized near the origin $p \simeq 0$.

It is well-known (see, for example, [8]) that a heavy quarks weakly interact with vacuum gluon fields. Therefore, it seems quite reasonable to use local the Dirac propagator with large mass for describing the behavior of a heavy quark at large distances [9,10]:

$$S(\not p) = \frac{1}{M_Q - \not p}. \tag{4}$$

Here, M_Q is a constituent mass of a heavy quark. Especially, we will be interested in the heavy quark limit $M_Q \to \infty$.

First, let us calculate the mass operator of pseudoscalar heavy mesons which is defined by the self-energy diagram. We have

$$\Pi_{HP}(p^2) = -\int \frac{d^4k}{4\pi^2 i} \mathrm{tr}\{\gamma^5 G(\not k)\gamma^5 \frac{1}{M_Q - (\not k + \not p)}\}\Pi_{HP}(p^2) = -\Lambda^2_{conf} I_{HP}(p^2), \tag{5}$$

where

$$I_{HP}(p^2) = \frac{1}{2}\int\limits_0^\infty du u b(u) + \int\limits_0^\infty du C(u, p^2, M_Q^2)\{M_Q a(u) + \frac{1}{2}(p^2 - M_Q^2 + u) b(u)\},$$

$$C(u,x,z) = \frac{\sqrt{(u+z-x)^2 + 4ux} - (u+z-x)]}{2x}.$$

Here and further, we assume for simplicity that all momenta and masses are given in the units of Λ_{conf}.

Substituting (6) into the compositeness condition (2) we get the following expression for the coupling constant g_{HP} of a pseudoscalar heavy meson:

$$g_{HP} = \frac{2\pi}{\sqrt{3}} \frac{1}{\sqrt{I'_{HP}(m_H^2)}}. \tag{6}$$

The coupling constant of a vector heavy meson is calculated by analogy.

In the heavy quark limit $m_H = M_Q \to \infty$, one can get

$$g_{HP} = g_{HV} \to \frac{2\pi}{\sqrt{3}}\sqrt{\frac{2M_Q}{G_0}}, \tag{7}$$

where

$$G_0 = 2\int\limits_0^\infty dE_4 E_4\{\mathrm{Re}G(iE_4) + \mathrm{Im}G(iE_4)\} = \int\limits_0^\infty du a(u) + \int\limits_0^\infty du\sqrt{u} b(u).$$

The amplitudes of the $B \to D(D^*)e\nu$ decay are written in the form:

$$M^\mu(p,p') = g_B g_D \frac{3}{(2\pi)^2}\int \frac{d^4k}{4\pi^2 i}\mathrm{tr}[\frac{1}{M_c - \not k - \not p'} O^\mu \frac{1}{M_b - \not k - \not p'}\gamma^5 G(\not k)\gamma^5],$$

$$M^{\mu\nu}(p,p') = g_B g_D \cdot \frac{3}{(2\pi)^2}\int \frac{d^4k}{4\pi^2 i}\mathrm{tr}[\frac{1}{M_c - \not k - \not p'} O^\mu \frac{1}{M_b - \not k - \not p}\gamma^5 G(\not k)\gamma^\nu], \tag{8}$$

where p and p' are momenta of intital and final states. Here we consider the heavy quark limit $m_H = M_Q \to \infty$. It is easily to get the asymptotic of the amlitudes in the leading order over $1/M_Q$:

$$M^{\mu}(p,p') = \sqrt{M_b M_c}\xi(w)(v+v')^{\mu},$$

$$M^{\mu\nu}(p,p') = \sqrt{M_b M_c}\xi(w)\{-g^{\mu\nu}(1+vv') + (v')^{\mu}v^{\nu} - i\epsilon^{\mu\nu\alpha\beta}(v')^{\alpha}v^{\beta}\}. \tag{9}$$

where w is a dot-product of four-velocities of initial and final particles:

$$v = \frac{p}{M_b} \qquad v' = \frac{p'}{M_c} \qquad w = vv' = \frac{M_b^2 + M_c^2 - (p-p')^2}{2M_b M_c}.$$

The function $\Phi(w)$ is equal to

$$\Phi(w) = \frac{1}{\sqrt{w^2-1}} \ln\left[w + \sqrt{w^2-1}\right]. \tag{10}$$

It exactly coincides with the Isgur-Wise representation. The Isgur-Wise function turnes out to be equal to

$$\xi(w) = \frac{\Phi(w) + \frac{2}{(1+w)}R}{1+R}, \tag{11}$$

where

$$R = \frac{\int\limits_0^\infty dE_4 E_4 \mathrm{Im}G(iE_4)}{\int\limits_0^\infty dE_4 E_4 \mathrm{Re}G(iE_4)} = \frac{\int\limits_0^\infty du\sqrt{u}b(u)}{\int\limits_0^\infty du a(u)}. \tag{12}$$

It may be seen that the only model parameter R comes to the expression for $\xi(w)$. Its value is defined by the integral of the light quark propagator $G(z)$ over the Euclidean direction.

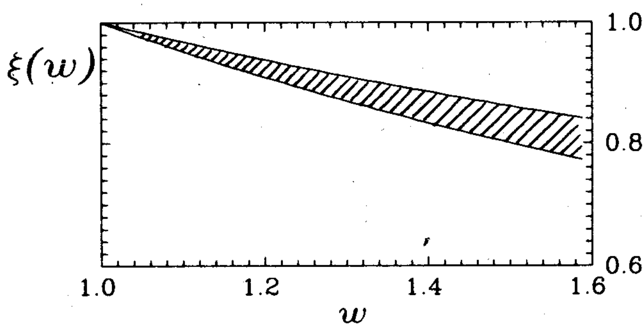

Fig.1. The Isgur-Wise function: $R=0$ - the lower bound; $R=\infty$ - the upper bound

The behavior of $\xi(w)$ for different values of model parameter R in (12) $(0 \leq R \leq \infty)$ is shown on the Fig. One can see that the behavior of $\xi(w)$ depends very slowly on this parameter. It means that form factors of $B \to D(D^*)e\nu$-decay depend very slowly on the behavior of light quark at large distances.

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