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CONFINEMENT OF ACOUSTICAL MODES DUE TO ELECTRON - PHONON INTERACTION WITHIN ELECTRON SHEET

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We study the confinement of acoustical modes within 2DEG due to only the electron–phonon interaction. The confined modes split out from the bulk phonons even at uniform lattice parameters.

One of the current topics of the semiconductor physics is the confinement of phonons in semiconductor heterostructures [1–3]. This phenomenon is interesting because of fundamental aspects, as well as its affect on the electron transport that is very important for applications. It is well known that the phonon confinement in the heterostructures is due to different lattice characteristics of the semiconductor compounds forming the heterostructure (various lattice constants, lattice forces, symmetry, etc.). On the other hand, existence of free carriers in these layers is not considered as the main reason for the confinement effect.

In this paper we predict and study the phonon confinement originating from the electron–phonon interaction. We show that confinement of acoustical modes appears due to only the electron–phonon interaction if there is an electron gas sheet ($3D$ or $2D$ electron layer). This effect exists even at the uniform lattice characteristics. Such a physical situation and the electron layers can be realized by modulation doping, for example under δ–doping [4].

The following two known phenomena can be the base for understanding of proposed mechanism of the phonon confinement. Firstly,the electrons bring about the renormalization of phonon spectrum and always lead to a reduction of the elastic modulus and a softening of the lattice. Secondly, the embedded layer characterized by decreased elastic modulus always splits the bulk acoustical spectrum into bulk–like modes and localized modes. The latter are confined into or near the embedded layer, and propagate along the layer. Therefore we can expect that the electron–phonon interaction under the localization of electrons within the electron sheet would lead to the phonon confinement effect.

We will describe the long–range acoustical vibrations of lattice by equation of the sound waves [5].

$$\rho\frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ik}}{\partial x_k}, \tag{1}$$

where u_i are components of displacement vector $\vec{u}$ of medium, ρ is its density, σ_{ik} is the stress tensor. For simplicity, we consider the isotropic elastic medium and assume that electrons are also characterized by isotropic energy law. Then, contribution of the lattice and the electrons to the stress tensor σ_{ik} is [5,6].

$$\sigma_{ik} = \sigma_{ik}^{(L)} + \sigma_{ik}^{(E)} = (\lambda + \frac{2}{3}\mu)u_{ll}\delta_{ik} + 2\mu(u_{ik} - \frac{1}{3}\delta_{ik}u_{ll}) + bn\delta_{ik}, \qquad (2)$$

where λ and μ are Lamé coefficients, and u_{ik} is the strain tensor. Then the electron–phonon interaction is described by only one constant of deformation potential b [6]; n is concentration of the electrons.

We assume the electrons are confined into a sheet of thickness d by a corresponding potential (for example, by electrostatic potential of the positive charge of donor sheet). The phonon wave vector q is restricted so that characteristic decay length of modes outside the sheet $\kappa_{ch}^{-1}(q)$ is much larger than the layer thickness d: $\kappa_{ch}(q)d \ll 1$.

In such a case it is possible to consider the electrons as confined in a plane (for example, in plane $z = 0$). Hence the concentration of the electrons can be written as: $n(\vec{r}, t) = n_s(x, y)\delta(z)$, where n_s is "surface" concentration of the electrons. We consider that the electrons follow adiabatically the vibrations of lattice and are redistributed in potential of the acoustic wave. Inequality $\bar{\epsilon} \gg \hbar\omega$ which is necessary condition of this adiabatic approximation always holds for the semiconductors ($\bar{\epsilon}$ is characteristic electron energy, ω is phonon frequency). The potential induced by acoustic wave is $h(\vec{r}) = bu_{ll} - e\varphi$. Here φ is electrostatic potential arised from the non–uniform redistribution of the electrons in space and it is governed by Poisson equation:

$$\nabla^2\varphi = \frac{4\pi e}{\epsilon_0}\delta n_s(x, y)\delta(z), \qquad (3)$$

ϵ_0 is the dielectric constant of crystal. We suppose that the dependence of all variables on plane coordinates (x, y) has the form $u_i, \varphi, h, \delta n_s \propto e^{i\vec{q}\cdot\vec{r}_{\parallel}}$, where $\vec{q}$ lies in plane of the electron sheet. Change in the electron concentration δn can be calculated by using the perturbation theory for the density matrix

$$\delta n_s(x, y) = h(x, y, z = 0)P(\vec{q}, T). \qquad (4)$$

Here $P(\vec{q}, T)$ is the static polarization of the electron subsystem including the quantization of the electron motion in z direction [7]. If the number of occupied $2D$ subbands is large, the electron motion within the sheet is almost 3 dimensional. In the contrary case, the polarization $P(\vec{q}, T)$ corresponds to the electron gas with reduced dimensionality. The set of the mentioned relationships is sufficient to consider the acoustical modes localized near the electron layer.

It is easy to see that only longitudinal acoustical waves interact with electrons in our model. Therefore it is convenient to consider the equation for relative volume change u_{ll} instead of several components of the displacement of the lattice. We seek the solutions which satisfy the following boundary conditions far away from the sheet: u_{ll}, $\phi \longrightarrow 0$, $z \longrightarrow \pm\infty$. The solutions can be written for both u_{ll} and $\phi = -e\varphi$ as

$$u_{ll} = Ae^{-\kappa|z|}, \qquad \kappa = \sqrt{q^2 - \frac{\omega^2}{c_l^2}} \qquad \phi = Be^{-q|z|}, \qquad (5)$$

Relationship between the magnitudes of the acoustic wave and the electrostatic potential is:

$$B = -A\, b\, \frac{4\pi e^2 P(\vec{q}, T)/\epsilon_0}{2q + 4\pi e^2 P(\vec{q}, T)/\epsilon_0}. \qquad (6)$$

Expression for κ is

$$\kappa = \sqrt{q^2 - \frac{\omega^2}{c_l^2}} = \frac{b^2 P(\vec{q},T)}{\lambda + 2\mu} \frac{q^3}{2q + 4\pi e^2 P(\vec{q},T)/\epsilon_0}. \tag{7}$$

The right hand side of eq.(7) is always positive. This means that the solutions decay exponentially far away from the layer. The same expression gives dispersion relation for the confined acoustical modes:

$$\omega^2 = q^2 c_l^2 \left(1 - \left(\frac{b^2 P(\vec{q},T)}{\lambda + 2\mu} \frac{q^2}{2q + 4\pi e^2 P(\vec{q},T)/\epsilon_0}\right)^2\right). \tag{8}$$

One can see from (8) that the frequencies of the confined phonons are always less than the frequencies of the bulk one. The splitting value of the frequencies depends on the fourth power of the coupling constant b. Distinction between the bulk phonons and the confined one also grows with increasing q: degree of the confinement becomes larger as it is seen from relation (7) and the dispersion relation falls of from the linear behavior.

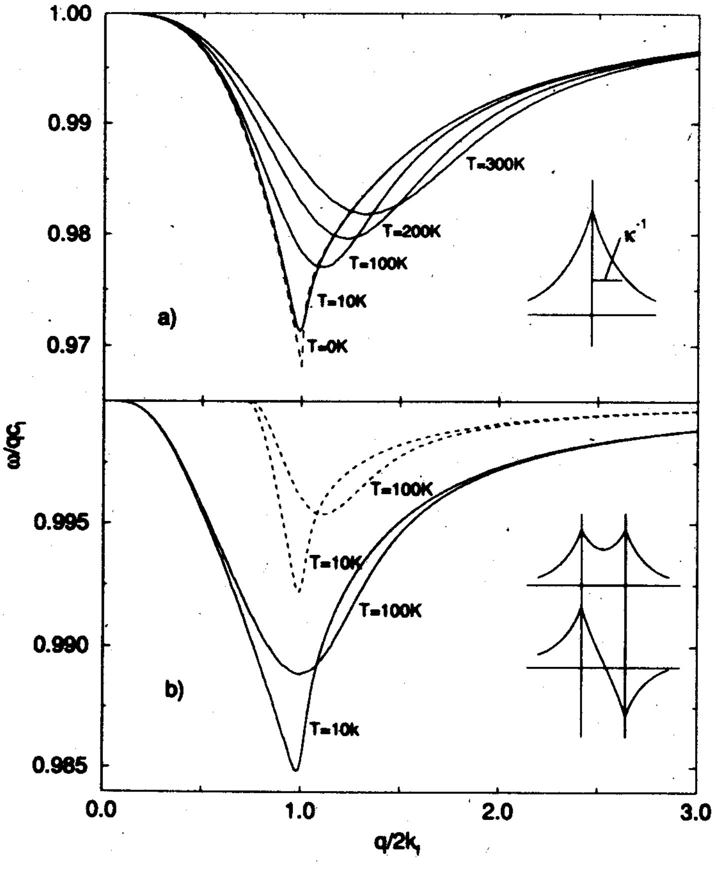
The term $4\pi e^2 P(\vec{q},T)/\epsilon_0$ in denominator of expressions (6),(7), evidently, describes screening effect of the electron charge which is redistributed in the electron sheet. We can introduce the characteristic wave vector q_{sc} by equality: $q_{sc} = 2\pi e^2 P(q_{sc})/\epsilon_0$. In the case of $q < q_{sc}$, the total potential $h(\vec{r})$ induced by the acoustical wave is small, because the change of the bottom of the conduction band bu_{ll} and the electrostatic energy $-e\varphi$ compensate each other. In this limiting case the dispersion relation takes a simple form $\omega^2 = q^2 c_l^2 \left(1 - \left(b^2 \epsilon_0 q^2 / 4\pi e^2 (\lambda + 2\mu)\right)^2\right)$ and does not depend on parameters of electron band, quantization into sheet, temperature. etc. Of course, this simple expression is valid under certain mentioned conditions. In this limit, the decay length of the acoustic mode outside the sheet κ^{-1} is proportional to q^{-3}.

In opposite case $q > q_{sc}$ the screening is not essential and the dispersion relation takes the form

$$\omega^2 = q^2 c_l^2 \left(1 - \left(\frac{b^2 P(\vec{q},T)}{2(\lambda + 2\mu)} q\right)^2\right). \tag{9}$$

In this case, the magnitude of the mode outside the electron sheet decays with $\kappa^{-1} \sim q^{-2}$. Analysis of the polarization $P(\vec{q},T)$ shows that the parameter q_{sc} and the confinement effect increase with decreasing temperature. Because of this we will consider the case of low temperature in detail. The following analysis will clarify the actual situation $q_{sc} < q$ better. The value q_{sc} is always small comparable with $k_F^{(n)}$ (i.e. Fermi vector of n^{th} subband) for semiconductors with large dielectric constant ϵ_0 (for example, IV–VI compounds). This means that at the region of $q \sim k_F$, where confinement effect is more pronounced the screening does not suppress the effect. In general, for the semiconductors with modest ϵ_0 the inequality $q_{sc} < q$ can be also true. In fact, maximum value of the q_{sc} is of the order of inverse Bohr radius a_B for semiconductors. Let the electron sheet is created by means of doping. It is necessary to dope the semiconductor up to such concentrations that $n_s a_B^2 > 1$ to achieve the free carriers and conductivity [4]. But this criterium is equivalent to the mentioned above inequality. When the criterium is hold, we can consider the case $q \sim k_F$ and $q_{sc} < q$ would be valid. The decay length of the acoustic wave outside the electron sheet $\kappa^{-1} = 2(\lambda + 2\mu)/b^2 P(\vec{q},T) q^2$

is much larger than the wavelength $2\pi/q$ even for $q \sim k_F$ and $q \sim 1/d$ for actual semiconductor parameters (see estimates below). Since the inequalities $\kappa_{ch}(q)d \ll 1, q_{sc} < q$, and $q_{sc} < k_F$ are compatible, that means that the expression for the dispersion relation (7) holds in the region $q \sim k_F$, the electrostatic potential is not essential and the splitting of the confined acoustical mode has maximum at $q = 2k_F$. At $q > 2k_F$ the magnitude of $P(\vec{q})q$,which determines ω, is proportional to q^{-1}, so the splitting decreases with increasing q. The behavior of $\omega(q)$ for the confined acoustical mode is shown in figure 1.a.



Phase velocity ω/qc_l of confined modes as a function of wave vector q at different temperatures for a) single electron sheet, b) two electron sheets. Insets show the form of solutions.

It is known that not only the heterostructures with one electron sheet but many layered system can be fabricated [4]. Distance between these electron sheets $2L$ can be varied artificially. If L is of the order of the characteristic scale κ_{ch}^{-1}, the effect of interaction of these sheets is appears. On the example of two electron sheets structure, we show that the interaction of these sheets leads to a splitting out of additional acoustic waves characterized by other features. Namely, the case shows two sorts of confined modes (symmetric and antisymmetric). The symmetrical one although differs from the single electron sleet solutions by magnitude of splitting, degree of confinement etc.,but shows the same physical trends. However, antisymmetrical modes are considerably different at small κ and q. This solution is splited out of the bulk one after a finite value of $q = q_c$. Since $\kappa_{antisym}$ is small for $q \sim q_c$, we can find the equation for q_c:

$$\frac{1}{L} = R(q_c) = \frac{b^2 P(\vec{q_c}, T)}{\lambda + 2\mu} \frac{q_c^3(1 + \coth q_c L)}{2q_c(1 + \coth q_c L) + 4\pi e^2 P(\vec{q_c}, T)/\epsilon_0}. \tag{10}$$

This equation has always single root. Near q_c, dispersion relation for the antisymmetrical modes is

$$\omega^2 = c_l^2 q^2 \left(1 - \frac{(R'(q_c))^2}{c_l^2 q_c^2}(q - q_c)^2\right) \quad , \; q > q_c. \tag{11}$$

Analysis shows that the splitting between bulk phonons and antisymmetrical confined modes increases when q increases up to $2k_F$, then the splitting falls down. The antisymmetrical branch is always between the bulk phonons and the symmetrical modes. The phonon spectrum for two electron sheet is illustrated by fig.1,b

In conclusion, we have showed that the electron sheet, in particular $2D$ electron gas, localizes the acoustic modes due to only electron–phonon interaction even at uniform lattice characteristics. The confined modes propagate along the sheet and decay far away from it. The acoustic waves are accompanied with the charge waves. The splitting of the modes out the bulk phonons increases when wave vector q increases and reaches a maximum at $q = 2k_F$, then the splitted modes are converged to the bulk phonons with increasing q. Additional features of the confined modes arise for the case of several electron sheets.

From the above results it follows that the confinement effect can be significant for the media with strong electron–phonon interaction, large effective mass of electrons, high concentration and low temperature.

We use the following typical parameters of semiconductors to estimate the order of value of the confinement effect: $\lambda + 2\mu = 10^{12}$ gr/cmsec^2, $b = 15$ eV, $m = 0.5m_0$ (p–material), $\epsilon_0 = 15$. Then for typical electron concentration for δ–doping layer $n = 6, 7.10^{12}$ cm^{-2} ($k_F = 6, 488.10^6$ cm^{-1} [4]), we find the results,presented on fig.1 a,b. For semiconductors with large dielectric constant (IV–VI compounds) the splitting increases as much as 3 times.

Thus, the studied confinement effect of the acoustical modes by the electron sheet can be considerable. It may be investigated by acoustical measurements of the semiconductors with δ–doping. The scattering on the confined modes can affect the electron transport in these materials.

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