NONEQUILIBRIUM NEUTRINOS AND PRIMORDIAL NUCLEOSYNTHESIS

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We have found that the residual interactions between neutrinos and electrons, which have different temperatures after decoupling. result in the the appreciable spectral distortion of the order of 1% in the higher energy side of the distribution, when the temperature drops to below 1MeV. The resulting modification in the helium abundance, however, is small, and only of the order of $\Delta Y \approx 1.3 \times 10^{-4}$.

The correct prediction of the cosmic abundances of light elements has been regarded as a great success of the standard hot universe model [1-4]. The only free parameter, the baryon to photon ratio N_B/N_γ, deduced from a comparison between the prediction and the observation of the primordial abundances of d, ^{3}He, ^{4}He and ^{7}Li, now converges to quite a narrow range. It is interesting to ask, however, to what accuracy such an agreement holds, when more precise estimates become available for the primordial elemental abundances. For the primordial helium, for instance, the latest value from HII galaxies is $Y_{obs} = 0.229 \pm 0.004$ [5] with a relative error at a 2% level. Taking a small error seriously, this is marginally consistent with the standard calculation with three neutrino species and with N_B/N_γ determined from ^{3}He+d and ^{7}Li: $Y = 0.236 - 0.243$ [3].

After the freezing of the neutron to proton ratio, the calculation is quite accurately carried out with the standard code, and there seem also little uncertainties in the nuclear reaction rates used. In the calculation of the n/p ratio, however, all authors have assumed the equilibrium Fermi distribution for the electron neutrino

spectrum. We consider that this assumption is worth to be examined: neutrinos decouple from the primeval plasma at a temperature $T \sim 3\text{MeV}$ for ν_e and 5MeV for ν_μ and ν_τ. Around this epoch there is no doubt that the neutrino spectrum is described well by the Fermi distribution. After this epoch, however, the temperatures of neutrinos and of the $e^\pm$ and γ plasma become different because of annihilation of e^+e^- pairs that heats up the electromagnetic component of the plasma. The relative temperature difference is about 0.9×10^{-3} at 3MeV, about 1.6×10^{-2} at $T = 0.7\text{MeV}$, and reaches eventually the well-known value of 29% [4]. Although equilibrium ceases at a few MeV, some thermal contact between electrons and neutrinos remains, especially for a high energy tail of the neutrino spectrum due to stronger interactions between them at a higher energy. This would distort the equilibrium Fermi distribution. In fact we find that this distortion amounts to as large as 1% or more for the higher energy side of the spectrum. This motivates us to examine the change of the n/p ratio caused by this distortion.

Actually there have been a few authors who noticed the effect driven by the temperature difference between the photon and the neutrino components [6-8]. These authors, however, considered only average heating of the neutrino gas owing to residual interactions between electrons and neutrinos, and assumed that the effect is renormalized into the change of the effective neutrino temperature. What we really need to see is, however, the effect of the distorted spectrum, which cannot simply be absorbed into the temperature. In this paper we study the non-equilibrium effect on the n/p ratio by directly solving kinetic equations.

The kinetic equation that governs the ν_e phase space distribution in the expanding universe has the following form:

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right) n_\nu(t,p) = S \ , \tag{1}$$

where $H = 1/2t$ is the expansion parameter $\dot{a}/a$, and $p = E$ is the neutrino momentum with the mass of neutrinos assumed to be negligible. The collision term S is given, for example, for $\nu\bar{\nu} \leftrightarrow e^+e^-$ by the integral

$$S = \frac{(2\pi)^4}{2p} \int d\tau(e^-)d\tau(e^+)d\tau(\bar{\nu})\delta^4(p_+ + p_- - p - \bar{p}) \mid A(\nu\bar{\nu} \leftrightarrow e^+e^-) \mid^2$$
$$[n_{e^+}n_{e^-}(1 - n_\nu)(1 - n_{\bar{\nu}}) - n_\nu n_{\bar{\nu}}(1 - n_{e^+})(1 - n_{e^-})] \ , \tag{2}$$

where p_+, p_-, p and $\bar{p}$ are the momenta of e^+, e^-, ν and $\bar{\nu}$, and $d\tau(e^-) = d^3p_-/(2\pi)^3 2E_-$ etc. is the phase space volume element for the respective particles. The amplitude in the integrand is written

$$\mid A(\nu\bar{\nu} \leftrightarrow e^+e^-) \mid^2 = 128G_F^2[g_L^2(pp_+)^2 + g_R^2(pp_-)^2 + g_Lg_Rm^2(p\bar{p})] \ , \tag{3}$$

with $G_F = 1.03 \times 10^{-5}/m_N^2$ the Fermi coupling constant, $g_L = 1/2 + \sin^2\theta_W$, $g_R = \sin^2\theta_W$ ($\sin^2\theta_W = 0.23$) and m the electron mass. There are also contributions to S from elastic scattering $\nu e^\pm \leftrightarrow \nu e^\pm$ etc., which will be taken into account afterwards.

For the energy region that concerns us the number densities of neutrinos and electrons are small enough, so that we can approximate the Fermi distribution by the Boltzmann distribution, especially when we are interested in small correction terms. For electrons and positrons the Coulomb and Thomson scattering processes are fast enough, and their distribution is given by the equilibrium form,

$$n_e = \exp(-E_e/T_\gamma) \simeq \exp(-E_e/T) \left(1 + \frac{E_e}{T}\frac{\Delta T}{T}\right) \ . \tag{4}$$

Here the temperature of the $e\gamma$ plasma, T_γ, differs from the neutrino temperature T by $\Delta T = T_\gamma - T$. We write the neutrino distribution in the form,

$$n_\nu = \exp(-E_\nu/T)[1 + \delta(p,t)] \tag{5}$$

with $\delta(p,t)$ the spectral distortion due to neutrino heating by electrons and positrons.

Substituting eqs. (4) and (5) into eq. (2) we get a kinetic equation for δ. It is linear integro-differential equation which is not easy to handle with, but we shall see in what follows that δ is a small quantity in the temperature range responsible to determine the neutron to proton ratio and the δ term which appears in the second term in the right hand side can be ignored as a first approximation.

We have to take into account also heating of neutrinos by elastic νe^- and νe^+ scattering. These processes conserve the neutrino numbers, but modify the spectrum. The kinetic equation is then

$$\left(\frac{\partial}{\partial t} - HE\frac{\partial}{\partial E}\right)\delta(E,t) \approx \frac{16G_F^2(g_L^2 + g_R^2)}{3\pi^3}\frac{\Delta T}{T}T^3 E[E + 4T + \frac{7}{4}(E - 4T)] \ , \tag{6}$$

where the terms proportional to δ are ignored in the right hand side. By noting that $\dot{T} = -HT$ we can easily integrate eq.(6) and obtain

$$\delta(E/T, T) \approx 0.031\frac{E}{T}\left(\frac{11}{4}\frac{E}{T} - 3\right)\int_\eta^{\eta_i} d\eta\eta^2\frac{\Delta\eta}{\eta} \tag{7}$$

with the use of $t = (45/32\pi^3 g)^{1/2}m_{pl}T^{-2}$ (g is the number of relativistic degrees of freedom and m_{pl} is the Planck mass). Here η is the temperature in units of MeV and η_i is its initial value corresponding to decoupling of ν_e from the plasma. $\Delta\eta/\eta \equiv \Delta T/T$ is given by ref.[4] We note that $(\Delta T/T)T^2 \simeq 0.60 \times 10^{-2}(\text{MeV})^2$ for wide range of T from 3 MeV down to 0.5 MeV (within 3%; the error is only 10 % even at 0.3MeV). With $T_i \approx 3 - 4$MeV (see below), this yields approximately

$$\delta \approx 6 \times 10^{-4}(E/T)(11E/4T - 3) \tag{8}$$

at $T \approx 0.6$MeV. The corrections due to the inverse processes and the dependence of the freezing temperature on the neutrino energy have been taken into account in our detailed calculations and proved to be small.

The helium abundance is basically determined by the neutron to proton ratio n/p, which is fixed by the competition of $n + \nu \leftrightarrow p + e^-$ and $n + e^+ \leftrightarrow p + \bar\nu$ with the expansion rate of the universe. The kinetic equation that governs the evolution of neutron number density with n_ν given by eq. (5) can be written as

$$\frac{dr_n}{dT} = -0.05T^2\int_0^\infty dx x^2\left(x + \frac{\Delta M}{T}\right)^2 e^{-x}\{e^{-\Delta M/T}[1 + \frac{1}{2}\delta(E + \Delta M)]$$

$$-r_n[1 + \frac{\delta(E)}{2} + e^{-\Delta M/T}\left(1 + \frac{\delta(E + \Delta M)}{2}\right)]\} \ , \tag{9}$$

131

where $r_n = r_n(t)$ is the fraction of the neutron number against the total number of baryons (so that $r_n + r_p = 1$), $\Delta M = 1.29$ MeV is the neutron-proton mass difference, $x = E/T$ and $\delta(x, T)$ is given by eq.(7) or (9) and $\Delta T/T$ is given by eq. (8). We integrated eq.(21) numerically with the fourth-order Runge-Kutta algorithm. Below 0.3 MeV the approximation $(\Delta T/T)T^2 = const$ is not valid, but the effect is small and it can be easily taken into account in the final answer. In this way, we find the deviation of r_n from the standard value $r_n(\varepsilon = 0)$ to be 0.9×10^{-4} at low enough temperatures. This is indeed a very small number, compared with what we naively expect from the deviation of the neutrino spectrum from the Fermi distribution.

Accordingly, the influence of the nonequilibrium distribution of neutrinos to the neutron to proton ratio is very small and it changes the helium abundance only by the amount of $\Delta Y = -1.3 \times 10^{-4}$. This value may nominally be compared with those obtained by Dicus et al. [6], $+3 \times 10^{-4}$, and by Herrera and Hacyan [7], -2×10^{-4} (note that signs do not agree with each other), and also by Rana and Mitra [8], -3×10^{-3}, which is certainly too large. All authors estimated the effect as a shift of the effective neutrino temperature and hence of the freezing temperature of beta equilibrium. Our emphasis here is on the point that one cannot absorb the effect into the shift of the freezing temperature. This may be demonstrated by the fact that the correction to the n/p ratio is temperature dependent. For instance, according to our calculations, ΔY would be $+1.1 \times 10^{-4}$ if we adopt r_n near the freezing temperature usually accepted for beta equilibrium, $T \approx 0.7$ MeV. Anyway the effect seems too small ($\Delta Y/Y = -0.05\%$) to be observationally relevant. The standard calculation assuming the equilibrium distribution is sufficiently accurate as a matter of fact for the purpose of estimating the helium abundance. If the discrepancy between the prediction and the observation were actually present for the primordial helium abundance, we must ask for the reason somewhere else.

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