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GLUON AND QUARK CONTRIBUTIONS TO THE HADRON MASS

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Trace anomaly in QCD and low-energy theorems are used to determine the gluon and quark contributions to the hadron mass and express it in terms of the string tension and current quark masses.

1. In QCD all hadrons consist of quarks and gluons, and in the limit when the current quark masses are much less then Λ_{QCD} the quark contribution to the hadron mass disappears. It means that e.g. nucleons and all objects around us are massive with good accuracy due to gluons inside of them. How one can reconcile this conclusion with the constituent quark model (CQM) where the nucleon mass is mostly due to the masses of constituent quarks?

In this letter we show that the dominant contribution to the mass of light flavour hadrons indeed comes from gluons in the strings connecting quarks. The strings also create constituent quark mass and the resulting picture agrees with CQM.

In QCD the hadron (meson or baryon) mass can be expressed through the energy density of the quark and gluon fields inside the hadron, e.g. in the c.m. system

$$M = < h \mid \Delta\Theta_{\mu\mu} \mid h >, \qquad \Delta\Theta_{\mu\mu} = \Theta_{\mu\mu} - < \Theta_{\mu\mu} >, \tag{1}$$

where [1]

$$\Theta_{\mu\mu} = \frac{\beta(\alpha_s)}{4\alpha_s} F^a_{\mu\nu} F^a_{\mu\nu} + \sum_q m_q \bar{\Psi}_q \Psi_q (1 + \gamma_q), \tag{2}$$

m_q is the current quark mass, and γ_q is anomalous dimension, the sum is going over all quark species $q = u, d, s, ..$ while $< \Theta_{\mu\mu} >$ is the vacuum average of $\Theta_{\mu\mu}$.

We include $1+\gamma_q$ into the definition of the quark mass normalized at the typical hadronic scale of 1 GeV.

The hadron wave function Ψ_h depends on the center-of-mass time y_4 and can be written through the evolution operator $\mid h >\equiv \exp(-Hy_4)\Psi_0$, where H is the full QCD Hamiltonian. Similary for $< h \mid$ we write

$$< h \mid\equiv \Psi_h^+ \sim \Psi_0(T)\exp H(T-y_4).$$

and obtain for the mass in (1)

$$M =< \int d^3y\Psi_h^+(y_4)\Delta\Theta_{\mu\mu}(y_i,y_4)\Psi_h(y_4) >, \tag{3}$$

or else

$$M = \lim_{T\to\infty} \frac{< \int G(T,0)\Delta\Theta_{\mu\mu}(y)d^3y >}{< G(T,0) >} = \lim_{T\to\infty} \frac{1}{T}\frac{\int d^4y < G(T,0)\Delta\Theta_{\mu\mu}(y) >}{< G(T,0) >} \tag{4}$$

where G is the hadron Green's function evolving from some initial state at the c.m. Euclidean time $\tau=0$ to a final state at the c.m. time T. The limit of large T separates the ground state hadron. Angular brackets imply averaging over all gluonic and quark fields via functional integral with the standard measure [2]. For $G(T,0)$ the Feynman-Schwinger representation [3,4] readily relates it to a path integral over Wilson loops with spin insertion $W_{\sum}(A)$

$$G(T,0) = \int D\rho(z,s)W_{\sum}(A) \tag{5}$$

where

$$W_{\sum}(A) = P\exp g\int_0^s \sigma_{\mu\nu}F_{\mu\nu}d\lambda \cdot P\exp ig\int A_\mu dz_\mu \tag{6}$$

The path-integral measure $D\rho$ depends on the proper times s_i, quark (antiquark) trajectories $z_i(\lambda)$, and contains also the quark determinant $\prod_q \det(m_q+\hat{D}(A))$, properly normalized and regularized.

2. First we consider the case when the spin interaction (the first factor on the r.h.s. of (6)) can be taken as a perturbation. Neglecting it altogether (for spinless quarks) we get for the hadron mass

$$M = \lim_{T\to\infty}\frac{1}{T}\frac{\int D\rho d^4y < W(A)\Delta\Theta_{\mu\mu}(y) >}{\int D\rho < W(A) >} \tag{7}$$

Here $W(A)$ is the usual Wilson loop, i.e. Eq.(6) without the first factor.

Consider now a hadron with light valent quarks, and take the limit $m_q \to 0$. Naively one expects that the light quark term in $\Theta_{\mu\mu}$ does not contribute to M in the limit $m_q\to 0$, and indeed one makes sure that Eq. (7) even with the determinantal term is nonsingular in this limit and $m_q\bar{\Psi}_q\Psi_q$ gives vanishing result. The heavy quark contribution from the sea is suppressed and can be disregarded in the first approximation [5]. Thus all contribution to the mass is coming from the gluon part of $\Delta\Theta_{\mu\mu}$ in case of spinless quarks, Eq. (7).

We now use the so-called low- energy theorem [6] and reduce Eq.(7) to an indentity. Indeed differentiating the quantity $ln < W(A) >$ in the bare coupling constant g_0^2 we obtain a theorem [6]

$$\int d^4y < W(A)\Delta\Theta_{\mu\mu} >= 2\sigma S < W(A) > \tag{8}$$

Here we assumed the area law for $< W(A) >$ with the minimal area S and the string tension σ. With that assumption we also have $\sigma S < W(A) >= = -\sigma \frac{\partial}{\partial\sigma} < W(A) >$. For simplicity we neglect the perimeter term in $< W(A) >$ and perturbative exchanges, producing color Coulomb interaction. This approximation is valid at large distances and therefore our results below apply to hadrons of large size $R, R \gg T_g$, where T_g is the gluon correlation length in vacuum. Recent Monte-Carlo calculations [7] yield $T_g \sim 0,2 fm.$, therfore our consideration should be reasonably good already for ground states of light hadrons and excited states of heavy quarkonia.

Hence Eq.(7) can be rewritten as

$$M = \lim \frac{1}{T}(-2\sigma\frac{\partial}{\partial\sigma}) \ln < G(T,0) > \tag{9}$$

The asymptotics of $< G(T,0) >$ is $\exp(-M_0T)$, where M_0 is the lowest hadron mass without spin interaction of quarks. As a result we obtain a simple relation

$$M = 2\sigma \frac{\partial}{\partial\sigma} M_0(\sigma) \tag{10}$$

which is readily satisfied since in absence of quark masses σ is the only mass parameter and $M_0 = \text{const}\sqrt{\sigma}$. Thus in this spinless case all the hadron mass is due to the strings connecting quarks (viz. the term $< W(A) >$) and $\Delta\Theta_{\mu\mu}$ exactly measures how much of energy-density (and hence the mass) is contained in the strings. This picture does not exclude the notion of the constituent quarks, on the contrary, in [8] it was shown for mesons that $M_0 = 4\mu(\sigma)$, where $\mu(\sigma)$ is the constituent quark mass depending on the hadron state. This means that a quark with an adjacent piece of the string makes a constituent quark, and the constituent quark mass here is due to confinement. The same picture occurs also for baryons [9].

3. We now consider a heavy quark system, and still neglect the spin interaction of quarks which is small in this case, $\Delta M_s \sim (1/m_q^2)$. Eq.(7) is still valid where $\Delta\Theta_{\mu\mu}$ can be replaced by

$$\Delta\Theta_{\mu\mu} \to 2\sigma S - \sum_q m_q \frac{\partial}{\partial m_q}. \tag{11}$$

Here we have used (8) and substituted $\bar{\Psi}_q\Psi_q$ by the derivative $\frac{\partial}{\partial m_q}$ of the action in the weight of averaging in (7).

Eq.(10) is replaced by

$$M = (2\sigma\frac{\partial}{\partial\sigma} + \sum m_q \frac{\partial}{\partial m_q})M_0(\sigma, m_q) \tag{12}$$

In the nonrelativistic approximation M_0 can be represented as

$$M_0 = \sum m_q + \sum < \frac{p^2}{2m_q} > + < V > \tag{13}$$

where the linear potential $< V >$ is proportinal to σ. From (12),(13) we have

$$M = 2 < V > + \sum m_q - \sum < \frac{p^2}{2m_q} > \tag{14}$$

Since V contains string interaction linear in distances between quarks the virial theorem applies [10]

$$< V > = 2 \sum < \frac{p^2}{2m_q} > \tag{15}$$

and we come to an identity $M = M_0$.

Hence in the case of heavy quark systems the hadron mass consists of quark masses plus gluon energy condensed in the form of strings between quarks.

4. We have disregarded hitherto quark spin interactions. Considering it as a perturbation and taking as an example the hyperfine term for the $q\bar{q}$ system we have to add to M_0 in (10) or (12) a term [4]

$$M_{ss} = \frac{8\alpha_s \pi}{9\mu_1\mu_2} \varphi^2(0) \vec{\sigma_1}\vec{\sigma_2} = \frac{4\alpha_s \sigma}{9(\mu_1 + \mu_2)} \vec{\sigma_1}\vec{\sigma_2} \tag{16}$$

where we have introduced the value of eigenfunction at origin $\varphi(0)$ for the linear interaction. For massless quarks $\mu_1 = \mu_2 = \mu(\sigma)$ is the constituent quark mass depending only on $\sqrt{\sigma}$.

Therefore the sum $M_0(\sigma) + M_{..}(\sigma)$ when substitued on the r.h.s. of (10) yields again an identity. The same reasoning applies to other spin-dependent terms. We thus obtain that the light hadron (meson or baryon) mass can be expressed in terms of constituent quark masses. It is generated by the confinement and disappears when σ tends to zero.

In conclusion we have shown in the framework of the Feynman- Schwinger representation that confinement and the notion of constituent quark mass provide a natural explanation of a hadron mass through its gluonic and quark contents.

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