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INVERTED RADIATIVE HIERARCHY OF QUARK MASSES

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We suggest that the mass hierarchy is first generated in a sector of heavy isosinglet fermions due to radiative effects and then projected in the inverted way to the usual quarks by means of a universal seesaw. The simple left-right symmetric gauge model is presented with the P- and CP-parities and the exact isotopical symmetry which are softly (or spontaneously) broken in the Higgs potential. This approach naturally explains the observed pattern of quark masses and mixing, providing the quantitatively correct formula for the Cabibbo angle. Top quark is predicted to be in the $90 - 150$ GeV range.

Recently [1], a new approach to the fermion mass problem was suggested: the mass hierarchy is radiatively generated in a hidden sector of the hypothetical heavy fermions and then transfered to the visible quarks and leptons by means of universal seesaw mechanism [2]. Providing a qualitatively correct picture of quark masses and mixing, this approach solves many principal problems of previous models [3,4] of radiative mass generation. In particular, the correct value of the Cabbibo angle can be accomodated without trouble for the perturbative expansion and, thus, for the idea of radiative mass generation itself, which was the generic problem [5] of previous approaches. Moreover, within the seesaw approach the effective low energy theory, after integrating out of the heavy sector, is simply the standard model with definite Yukawa couplings [2]. Thus, the dangerous flavour changing phenomena, characteristic [4] for the direct models of radiative mass generation, are naturally suppressed.

The key idea of the model [1] is to suppose the existence of weak isosinglet heavy fermions (Q-fermions) in one-to-one correspondence with the light ones. The model [1] has a field content such that only one family (namely the first) of Q-fermions becomes massive at the tree level, whereas the 2^{nd} family at the 1-loop level and the 3^{rd} only at 2 loops. Due to seesaw features [2] the mass of any usual quark or lepton appears to be inversely proportional to the mass of its heavy partner, so that the mass hierarchy between the families of light fermions is inverted with respect to the hierarchy of Q-fermion families. This pattern is attractive for the reason that we experimentaly observe the small mass splitting within the lightest quark family (u and d) and then increasing splitting from family to family, with the up-quark masses growing faster: $m_u/m_d \ll m_c/m_s \ll m_t/m_b$. The latter fact can be related with the difference of the perturbation theory expansion parameters in the up and down quark sectors.

In the present letter we show, that the simplest and most economical version of the model [1] provides a predictive ansatz for the quark mass matrices. We assume that the "isotopical" discrete symmetry I_{UD} between up and down quark sectors, as well as the left-right symmetry P_{LR} and CP-invariance, is violated only in the loop expansion, due to soft (or spontaneous) breaking in the Higgs potential. The appearance of both the mass splitting within the lightest family ($m_d/m_u = 1.5 - 2$) and the large compared to other mixing angles value of the Cabibbo angle ($V_{us} \simeq 0.22$) is related to the features of seesaw "projection", without the trouble for the perturbation theory. The model leads to certain predictions for the quark mass and mixing pattern, which we will discuss below.

Let us consider the simple left-right symmetric model based on the gauge group $G_{LR} = SU(2)_L \otimes SU(2)_R \otimes U(1)_L \otimes U(1)_R \otimes U(1)_{\bar{B}-\bar{L}}$, suggested in [1]. The left- and right-handed components of usual quarks $q_i = (u_i, d_i)$ and their heavy partners $Q_i = U_i, D_i$ are taken in the following representations:

$$q_{Li}(I_L = 1/2,\ \bar{B} - \bar{L} = 1/3), \qquad q_{Ri}(I_R = 1/2,\ \bar{B} - \bar{L} = 1/3)$$

$$\begin{aligned} &U_{Li}(Y_L = 1, \quad \bar{B} - \bar{L} = 1/3), \qquad && U_{Ri}(Y_R = 1, \quad \bar{B} - \bar{L} = 1/3) \\ &D_{Li}(Y_L = -1, \ \bar{B} - \bar{L} = 1/3), \qquad && D_{Ri}(Y_R = -1, \ \bar{B} - \bar{L} = 1/3) \end{aligned} \tag{1}$$

where i=1,2,3 is the family index (the indices of the colour $SU(3)_c$ are omitted).[1)] Only the nonzero quantum numbers are shown in the brackets: $I_{L,R}$ are the $SU(2)_{L,R}$ weak isospins and $Y_{L,R}$ are the $U(1)_{L,R}$ hypercharges. Let us introduce also one additional family of fermions with $\bar{B} - \bar{L} = 1/3$ and following hyprecharges:

$$\begin{aligned} &p_L(Y_L = -1/2,\ Y_R = 3/2), \qquad && p_R(Y_L = 3/2,\ Y_R = -1/2) \\ &n_L(Y_L = 1/2,\ Y_R = -3/2), \qquad && n_R(Y_L = -3/2,\ Y_R = 1/2) \end{aligned} \tag{2}$$

The scalar sector of the theory consists of

$$\begin{aligned} &H_L(I_L = 1/2, \quad Y_R = 1), \qquad && H_R(I_R = 1/2, \quad Y_L = 1) \\ &T_{uL}(Y_L = -2, \quad \bar{B} - \bar{L} = -2/3), \qquad && T_{uR}(Y_R = -2, \quad \bar{B} - \bar{L} = -2/3) \\ &T_{dL}(Y_L = 2, \quad \bar{B} - \bar{L} = -2/3), \qquad && T_{dR}(Y_R = 2, \quad \bar{B} - \bar{L} = -2/3) \end{aligned} \tag{3}$$

$$\Phi(Y_L = 2, Y_R = -2), \ \ \varphi(Y_L = 1/2, Y_R = -1/2), \ \ \Omega(Y_L, Y_R = 1/2, \ \ \bar{B} - \bar{L} = -1)$$

where T-scalars are supposed to be colour triplets. Let us impose also CP, P_{LR} and I_{UD} discrete symmetries. P_{LR}, [7] which is essentialy parity, and CP act in the usual way. The isotopical "up-down" symmetry I_{UD} is defined by

$$\begin{aligned} &U_{L,R} \leftrightarrow D_{L,R}, \qquad p_{L,R} \leftrightarrow n_{L,R}, \qquad H_{L,R} \leftrightarrow \tilde{H}_{L,R} = i\tau_2 H^*_{L,R}, \\ &T^u_{L,R} \leftrightarrow T^d_{L,R}, \qquad \Phi \leftrightarrow \Phi^*, \qquad \varphi \leftrightarrow \varphi^*, \qquad A^\mu_{L,R} \leftrightarrow -A^\mu_{L,R} \end{aligned} \tag{4}$$

where $A^\mu_{L,R}$ are the gauge bosons of $U(1)_{L,R}$. Then the most general Yukawa couplings consistent with gauge invariance, I_{UD}, P_{LR} and CP are

$$\begin{aligned} L_1 &= \Gamma_{ij}(\bar{q}_{Li}\, U_{Rj}\, \tilde{H}_L + \bar{q}_{Li}\, D_{Rj}\, H_L) + (L \leftrightarrow R) + h.c. \\ L_2 &= \lambda_{ij}(U_{Li} C U_{Lj} T_{uL} + D_{Li} C D_{Lj} T_{dL}) + (L \leftrightarrow R) + h.c. \\ L_3 &= h(\bar{p}_L p_R \Phi^* + \bar{n}_L n_R \Phi) + h_i(\bar{U}_{Li} p_R \varphi^* + \bar{D}_{Li} n_R \varphi) + (L \leftrightarrow R) + h.c. \end{aligned} \tag{5}$$

[1)] The inclusion of leptons in this model is strightforward and will be presented elsewhere. In fact $U(1)_{B-L}$ can be unified with $SU(3)_c$ within Pati-Salam[6] type SU(4). The $U(1)_L \otimes U(1)_R \otimes I_{UD}$ part can also be enlarged to $SU(2)'_L \otimes SU(2)'_R$, in which case the isotopical symmetry is obviously continuous.

where C is the charge conjugation matrix. The coupling constants $h, h_i, \lambda_{ij}, \Gamma_{ij} (i, j = 1,2,3)$ are real due to CP-invariance (λ_{ij} is antisymmetric, $\tilde{\lambda} = -\lambda$, since the T-scalars are colour triplets). In what follows we do not make any particular assumption on their structure. We only suppose that they are typically $O(1)$, as well as the gauge coupling constants. Without loss of generality, by suitable redefinition of the fermion basis we can always take $h_2, h_3 = 0$, $\lambda_{13} = 0$, $\Gamma_{12}, \Gamma_{13}, \Gamma_{23} = 0$, which we use in the following.

Let us suppose that the discrete symmetries CP, P_{LR} and I_{UD} are softly broken only by the bilinear and trilinear terms in the Higgs potential [2)] The latter are the following

$$\Lambda_u T^*_{uL} T_{uR} \Phi + \Lambda_d T^*_{dL} T_{dR} \Phi^* + h.c. \tag{6}$$

where the coupling constants $\Lambda_{u,d}$ are generaly complex, violating thereby both CP and P_{LR} invariances.

The VEVs $\langle\Phi\rangle = v_\Phi$ and $\langle\varphi\rangle = v_\varphi$, $v_\Phi \gg v_\varphi$, break $U(1)_L \otimes U(1)_R$ down to $U(1)_{L+R}$ (the VEV of Ω then breaks $U(1)_{L+R} \otimes U(1)_{\bar{B}-\bar{L}}$ to the usual $U(1)_{B-L}$: $B - L = Y_L + Y_R + \bar{B} - \bar{L}$). The fermions p and n become massive, $M_p = M_n = hv_\Phi$, and the Q-fermions of the first family, U_1 and D_1 get masses $M \cong h_1^2 v_\varphi^2 / h v_\Phi$ due to their seesaw mixing with the former ones (interactions L_3 in (5)). At the same time the coloured scalars $T_{uL} - T_{uR}$ and $T_{dL} - T_{dR}$ get mixed due to interaction terms (6). At this point the radiative mass generation proceeds following the chain $U_1 \to U_2 \to U_3$, $D_1 \to D_2 \to D_3$ and the Q-fermion mass matrices generated from the loop corrections due to L_2 in (5) can be presented in the following form:

$$M_{U,D} = M(\hat{P}_1 + e^{-i\omega_{u,d}} \xi_{u,d} \tilde{\lambda} \hat{P}_1 \lambda + C_{u,d} \xi^2_{u,d} \tilde{\lambda}^2 \hat{P}_1 \lambda^2 + \cdots) \tag{7}$$

where $\hat{P}_1 = diag(1,0,0)$ is a 1-dimensional projector and $\omega_{u,d} = -\arg\Lambda_{u,d}$. The loop expansion fa^tors are

$$\xi_q = \frac{1}{8\pi^2} sin2\alpha_q ln R_q, \qquad R_q = (M^q_+ / M^q_-)^2 \tag{8}$$

where M^q_+, M^q_- are the eigenvalues of mass matrices of the scalars $T_{qL} - T_{qR}$, $q = u,d$, and α_q are the corresponding mixing angles. In a "reasonable" range of parameters $(1 < R < 10)$ the 2-loop factor $C(R) = C(1/R)$ is practicaly constant:[4] $C_{u,d} \simeq 0.65$. Eq.(8) is valid in the natural regime $M < M^q_+, M^q_- < M_p$.

Apart from small $(\sim \varepsilon^2_{u,d})$ 1-3 mixing, the matrices $M_{U,D}$ are diagonal and the mass hierarchy between three families of Q-fermions is given by $1 : x^{-1}\varepsilon_{u,d} : \varepsilon^2_{u,d}$, where we denote $x = \sqrt{C}\lambda_{23}/\lambda_{12}$ and $\varepsilon_{u,d} = \sqrt{C}\lambda_{12}\lambda_{23}\xi_{u,d} \sim 10^{-2} - 10^{-1}$ are the effective perturbative expansion parameters for the up and down sectors, respectively.

The VEVs $\langle H_L \rangle = (0, v_L)$ and $\langle H_R \rangle = (0, v_R)$, $v_R \gg v_L = (2\sqrt{2}\, G_F)^{-1/2} \approx 175$ GeV, break the intermediate $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetry down to $U(1)_{em}$. Then the ordinary quarks $q = u,d$ acquire masses due to their seesaw mixing with heavy fermions $Q = U, D$ (interactions L_1 in eq.(5)). The whole mass matrix written in the block form is

$$(\bar{u}, \bar{U})_L \begin{pmatrix} 0 & \Gamma v_L \\ \tilde{\Gamma} v_R & M_U \end{pmatrix} \begin{pmatrix} u \\ U \end{pmatrix}_R \tag{9}$$

[2)] Actualy, this symmetries can be spontaneously broken at the price of introduction of $P_{LR}-$ and $I_{UD}-$odd real scalars [1].

for up-type quarks and analagously for the down-type quarks. When $M_{U,D} \gg v_R, v_L$, the resulting mass matrix for the light states is given by seesaw formula

$$M^{u,d}_{light} = v_L v_R \Gamma M^{-1}_{U,D} \tilde{\Gamma}. \tag{10}$$

Substituting here eq.(7) we find in the explicit form

$$M_{light} = \frac{m}{\varepsilon^2} \begin{pmatrix} \varepsilon^2\gamma_{11}^2 & \varepsilon^2\gamma_{11}\gamma_{21} & \varepsilon^2\gamma_{11}\bar{\gamma}_{31} \\ \varepsilon^2\gamma_{11}\gamma_{21} & \varepsilon x e^{i\omega}\gamma_{22}^2 + \varepsilon^2\gamma_{21}^2 & \varepsilon x e^{i\omega}\gamma_{22}\gamma_{32} + \varepsilon^2\gamma_{21}\bar{\gamma}_{31} \\ \varepsilon^2\gamma_{11}\bar{\gamma}_{31} & \varepsilon x e^{i\omega}\gamma_{22}\gamma_{32} + \varepsilon^2\gamma_{21}\bar{\gamma}_{31} & 1 + \varepsilon x e^{i\omega}\gamma_{32}^2 + \varepsilon^2\bar{\gamma}_{31}^2 \end{pmatrix} \tag{11}$$

where $m = \Gamma_{33}^2 v_L v_R M^{-1}$, $\gamma_{ij} = \Gamma_{ij}/\Gamma_{33}$ and $\bar{\gamma}_{31} = \gamma_{31} + \sqrt{C}x^{-1}$; $\varepsilon = \varepsilon_{u,d}$, $\omega = \omega_{u,d}$ for the up and down quarks, respectively.

It is obvious from (11) that $\varepsilon_u \ll \varepsilon_d \ll 1$. The up quark mass matrix M^u_{light} is almost diagonal. Neglecting $\sim \varepsilon_u$ corrections we have $m_u = m\gamma_{11}^2$, $m_c = xm\gamma_{22}^2\varepsilon_u^{-1}$ and $m_t = m\varepsilon_u^{-2}$. Thereby, the quark mixing pattern is determined essentialy by the down quark mass matrix M^d_{light}, where $m_b \approx m\varepsilon_d^{-2}$. The contributions to the parameters of the CKM matrix from M^u_{light} are typically suppressed by the factor $\varepsilon_u/\varepsilon_d$ and we neglect them. After some algebra one can obtain:

$$V_{us} \approx \sqrt{\frac{m_d}{m_s}} \mid 1 - \frac{m_u}{m_d} e^{i\delta} \mid \tag{12}$$

$$V_{ub} \approx \frac{\bar{\gamma}_{31}}{\gamma_{11}}\frac{m_u}{m_b}, \qquad V_{cb} \approx \frac{m_d}{m_u}\left(\sqrt{\frac{m_s}{m_d}} V_{ub} + \frac{\gamma_{32}}{\gamma_{22}}\frac{m_s}{m_b} e^{i\omega_d}\right) \tag{13}$$

where $\delta = -\omega_d + arg(xe^{i\omega_d}\gamma_{22}^2 + \varepsilon_d\gamma_{21}^2) \approx -\omega_d + arg(1 + e^{i\omega_d})$ is a CP-violating phase. within uncertain (but supposed to be ~ 1) numerical factors the formulas (13) fit the experimental values of V_{ub} and V_{cb} (notice that for $\Gamma_{32} = 0$ one has $V_{ub}/V_{cb} = m_u/\sqrt{m_d m_s} = 0.11 - 0.15$). Their smallness implies that corresponding mixings cannot affect significantly the diagonal elements of M^d_{light}. As for 1-2 mixing, the situation is different. The mass splitting between u and d quarks requires some spread in Yukawa coupling constants (i.e. fluctuations around 1 within factor $2 - 3$), which is perfectly acceptable: $\Gamma_{21}/\Gamma_{11} \approx \sqrt{m_d m_s}/m_u = 7 - 9$, $\Gamma_{21}/\Gamma_{22} \approx \sqrt{x/\varepsilon_d}$ etc. This in turn automatically leads to the Cabibbo angle in the needed range. The comparison of (12) with experimental value $V_{us} \approx 0.22$ requires the large CP-phase, $\delta \sim 1$, in accord with the recent data.

From the mass matrices (11) one can also derive the relations

$$\frac{\varepsilon_d}{\varepsilon_u} = \frac{m_u m_c}{m_d m_s} = \sqrt{\frac{m_t}{m_b}} \tag{14}$$

which allow to calculate the Top quark mass. The large value of the latter requires that one has to account also for the "seesaw" corrections[9] to the formula (10), which implies for the physical top quark mass

$$m_t^* = m_t^0 \left[1 + \left(\frac{m_t^0}{\Gamma_{33} v_L}\right)^2\right]^{-1/2} \tag{15}$$

where m_t^0 is "would be" physical mass, calculated from eq.(14). Obviously, the analogous corrections are negligible for other quark masses since we demand all Γ's to be ~ 1. On the other hand, from (11) one can easily derive that $\Gamma_{21}/\Gamma_{33} \approx \varepsilon_d^{-1}\sqrt{m_d m_s/m_u m_b} \geq 0.17\varepsilon_d^{-1}$. In order to be consistent with perturbation theory (i.e. to avoide the appearance of Landau poles below the scale M_p) one has to assume that all Yukawa coupling constants, including Γ_{21} and λ's are less than 2, which implies that $\Gamma_{33} \leq 1$. Consequently, using the known values[8] of u, d, s, c and b quarks, from (14) and (15) we obtain $m_t^* = 50 - 150$ GeV. The large spread here is related mainly with the uncertainties in the light quark masses. Obviously, the lower limit is not interesting in view of the recent CDF limit $m_t^* > 90$ GeV. One can even turn the logic around and say that the experimental lower bound on the top quark mass favours the lower values of m_d/m_u and m_s from those allowed in[8].

Last but not least we wish to remark that in our approach the strong CP-problem can be automaticaly solved without axion. Owing to P and/or CP-invariances the initial $\Theta_{QCD} = 0$ and so called $\Theta_{QFD} = ArgDet\hat{M}$ arising from the whole mass matrix $\hat{M}$ of all fermions q, Q and p, n is also vanishing at tree level due to seesaw pattern[10]. The loop corrections can provide, however, $\bar{\Theta} = 10^{-9} - 10^{-10}$, which makes this scenario in principle accessible to the search of the $DEMON$ - dipole electric moment of neutron.

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