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ON DICKE NARROWING IN PLASMA

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Ionic spectral line narrowing because of elastic ion-ion scattering is calculated for equilibrium plasma. The problem is solved for collision frequency much greater than Doppler width and in the opposite limit. It is shown that the velocity dependence of Coulomb collision frequency decreases the narrowing.

As Dicke [1] has shown, the Doppler absorption line becomes narrower when the frequency of elastic collisions ν increases. The physical mechanism of Dicke narrowing consists in interference of wave trains emitted by an ion. A phase correlation between the trains exists if the phase changes weakly over the train's duration ν^{-1}: $kv_T/\nu \ll 2\pi$. The condition entails small mean free path of a probe ion as compared with the light wavelength $2\pi/k$. Frequent collisions confine the region within reach of the excited ion. The ion becomes almost fixed like in solid state, therefore the Doppler broadening vanishes. The line narrows because the correlation function falls not as rapidly [2] as the Doppler one $\exp(-k^2v_T^2t^2/4)$. In this paper we study the effect in equilibrium plasma, in which scattering for small angles dominates, and the effective collision frequency depends strongly on velocity.

To calculate the narrowing in the Coulomb case include the scattering to the quantum kinetic equation for density matrix $\hat\rho$ [3]

$$\left(\frac{\partial}{\partial t}+\vec v\frac{\partial}{\partial \vec r}+\hat\Gamma\right)\hat\rho+i[\hat V,\hat\rho]=\hat Q-\frac{\partial \hat q_\alpha}{\partial v_\alpha},\quad \hat q_\alpha=\frac{F_\alpha}{m}\hat\rho-D_{\alpha\beta}\frac{\partial\hat\rho}{\partial v_\alpha}. \tag{1}$$

Here $\hat\Gamma$ is the relaxation matrix. The velocity dependence of dynamic friction force and of the diffusion tensor is

$$F_\alpha=-\nu m v_T\xi_\alpha\Phi_l(\xi),\quad D_{\alpha\beta}=\frac{\nu v_T^2}{2}\left[\Phi_l(\xi)\frac{\xi_\alpha\xi_\beta}{\xi^2}+\Phi_{\rm tr}(\xi)\left(\delta_{\alpha\beta}-\frac{\xi_\alpha\xi_\beta}{\xi^2}\right)\right], \tag{2}$$

where ν is effective transport collision frequency, $\xi_\alpha=v_\alpha/v_T$ is the dimensionless velocity of ions, m, v_T are their mass and thermal velocity. Functions $\Phi_l(\xi),\Phi_{\rm tr}(\xi)$ for Maxwellian distribution of field particles are expressed in terms of Chandrasekhar function $g(\xi)$ [4]:

$$\Phi_l=\frac{3\sqrt{\pi}}{2}\frac{g(\xi)}{\xi},\quad \Phi_{tr}=\frac{3\sqrt{\pi}}{4}\frac{\Phi(\xi)-g(\xi)}{\xi}. \tag{3}$$

Function g is a combination of the error function and its derivative

$$g(\xi)=\frac{\mathrm{erf}(\xi)-\xi\mathrm{erf}'(\xi)}{2\xi^2},\quad \mathrm{erf}(\xi)=\frac{2}{\sqrt{\pi}}\int_0^\xi e^{-t^2}\,dt. \tag{4}$$

The diffusion tensor is anisotropic but invariant under rotation around the velocity vector of probe ion. At small velocity $v\ll v_T$ the spherical symmetry is restored $\Phi_l(0)=\Phi_{tr}(0)=1$.

The kinetic equation (1) for off-diagonal element can be solved in the limiting case of rare collisions as an expansion in parameter $\nu/kv_T \ll 1$ while the excitation term $\hat{Q}$ of levels has Maxwellian distribution in velocities. The work done by the field of travelling wave is also written as an expansion

$$\mathcal{P} = \mathcal{P}_0 \mathrm{Re}\left[i \sum_{n=0}^{\infty} \left(\frac{i\nu}{kv_T}\right)^n I_n(z)\right],$$

$$I_n = \frac{1}{\pi^2} \int d^3\xi \left(\frac{\hat{\mathcal{L}}}{z - \xi\cos\vartheta}\right)^n \frac{\exp(-\xi^2)}{z - \xi\cos\vartheta};$$

$$\mathcal{P}_0 = -2\hbar\omega |G|^2 \Delta N \frac{\sqrt{\pi}}{kv_T}, \quad \hat{\mathcal{L}} = \frac{\partial}{\partial\xi_\alpha}\left(-\frac{F_\alpha}{m} + D_{\alpha\beta}\frac{\partial}{\partial\xi_\beta}\right), \tag{5}$$

where $z = (\Omega + i\Gamma_{mn})/kv_T$, ΔN is the population difference, Γ_{mn} is the homogeneous width, Ω is the detuning between wave frequency ω and Bohr frequency of the transition $\omega_{mn} = (E_m - E_n)/\hbar$, $G = V_{mn}\exp(-i\vec{k}\vec{r} + i\Omega t)/2\hbar$ is the matrix element of interaction in frequency units.

While collisions are rare, consider only terms with $n = 0, 1$. In zero order obtain Voigt contour $I_0 = -iw(z)$ (w is the probability integral of complex argument). For remote wings of the line $|z| \gg 1$ integral I_1 gives the coefficient at z^{-4}:

$$\mathcal{P} = \mathcal{P}_0 \frac{kv_T}{\sqrt{\pi}}\left[\frac{\Gamma_{mn}}{\Omega^2} + \frac{k^2 v_T^2}{2}\frac{3\Gamma_{mn} + \nu/\sqrt{2}}{\Omega^4} + \cdots\right]. \tag{6}$$

Comparing (6) with results of weak collision approximation, calculate the change in the velocity of a probe ion: $\dot{\vec{u}} = -2\nu\Phi_l(u)\vec{u}$, i.e., at $\xi \ll 1$ the collision frequency $\bar{\nu}$ of the model is replaced by $2\nu\Phi_l$ in the Coulomb case. Averaging over velocities decreases value $\bar{\nu}$ by $2\sqrt{2}$ times. At the line centre ($\Omega = 0$) substitute $z = iy$ to (5) and calculate the integral in the Doppler limit $y = \Gamma_{mn}/kv_T \to 0$. The work done by the field somewhat increases

$$\mathcal{P} = \mathcal{P}_0\left(1 + b\frac{\nu}{kv_T}\right), \quad b = \frac{2}{\sqrt{\pi}}\left(\sqrt{2} - \ln(\sqrt{2} + 1)\right) \cong 0.601. \tag{7}$$

Coefficient b differs from that in the model ($4/3\sqrt{\pi} \cong 0.752$) by 25% also because of the velocity dependence. A gain at the line maximum is proportional to frequency ν. So the line narrows, since the area under the contour is independent of ν.

In order to analyze the opposite limiting case $\nu/kv_T \gg 1$ build up the perturbation theory using weak collisional model as the zero-order approximation. Equation (1) for off-diagonal elements of the density matrix $\rho_{mn}(\vec{k}, \vec{v}) = \exp(-i\Omega t + i\vec{k}\vec{r}) \int_0^\infty \sigma(\vec{k}, \vec{v}; t)\, dt$ it is convenient to rewrite in the integral form for auxiliary function σ

$$\sigma(\mathbf{k},\mathbf{v};t) = \sigma_0(\mathbf{k},\mathbf{v};t) + \int d^3v' \int_0^t dt'\, f_0(\mathbf{k},\mathbf{v},\mathbf{v}'; t-t')\, \hat{\delta}\, \sigma(\mathbf{k},\mathbf{v}';t'), \tag{8}$$

where

$$\sigma_0(\mathbf{k},\mathbf{v};t) = iG\Delta N \int d^3\mathbf{v}'\, f_0(\mathbf{k},\mathbf{v},\mathbf{v}';t)\frac{\exp(-v'^2/v_T^2)}{(\pi v_T^2)^{3/2}} \tag{9}$$

is the unperturbed function σ, $\hat{\delta} = \hat{\mathcal{L}} - \hat{\overline{\mathcal{L}}}$ is the perturbation. Here $\hat{\overline{\mathcal{L}}}$ is Fokker-Plank operator and $f_0(\mathbf{k}, \mathbf{v}, \mathbf{v}'; t)$ is Green's function of the kinetic equation for weak collision model. In this model the coefficients of Fokker-Plank operator are independent of the velocity $\Phi_l = \Phi_{tr} = 1$. We denote collision frequency in thi- model as $\overline{\nu}$. Function f_0 is the Fourier transform of well-known Green's functioı [4] of weak collision model over relative coordinates $\vec{r} - \vec{r}'$.

$$f_0(\mathbf{k}, \mathbf{v}, \mathbf{v}'; t) = \frac{1}{\left[\pi v_T^2 \left(1 - \exp(-2\overline{\nu}t)\right)\right]^{3/2}} \exp\left\{ -\frac{\left[\mathbf{v} - i\mathbf{k}v_T^2/\overline{\nu} - (\mathbf{v}' - i\mathbf{k}v_T^2/\overline{\nu})\exp(-\overline{\nu}t)\right]^2}{v_T^2\left[1 - \exp(-2\overline{\nu}t)\right]} \right.$$
$$\left. - \left(\frac{kv_T}{\overline{\nu}}\right)^2 \frac{\overline{\nu}t}{2} - (\Gamma_{mn} - i\Omega)t - i\mathbf{k}(\mathbf{v} - \mathbf{v}')/\overline{\nu}) \right\}. \tag{10}$$

Express the work done by the field in terms of function σ

$$\mathcal{P} = -2\hbar\omega \mathrm{Re}\left(iG^* \int d^3v \int_0^\infty dt\, \sigma(\mathbf{k}, \mathbf{v}; t) \right). \tag{11}$$

Solve integral Fredholm equation as an expansion in small parameter kv_T/ν using the fact that $\sigma_0(\mathbf{k}, \mathbf{v}, t)$ is the eigenfunction of operator $\hat{\delta}$ at $\mathbf{k} = \mathbf{0}$ and arbitrary $\overline{\nu}$. Therefore, $\sigma_0(\mathbf{k}, \mathbf{v}, t)$ satisfies Eq.(8) in the highest order in kv_T/ν. Obtain formally the correction σ_1 of the first order replacing σ by σ_0 under the integral sign in (8). One can see that at $t < \nu/k^2v_T^2$ the correction obtained grows up linearly in time. So the correction increases by $\nu^2/k^2v_T^2 \gg 1$ times at long-time limit $t \gg \nu/k^2v_T^2$. The usual procedure to exclude growing secular terms in perturbation theory is renormalization of the frequency ν [5].

After the effective collision time ν^{-1} the probe particle forgets its previous history. As a result, Green's function can be factorized at $t \gg \nu^{-1}$

$$f_0(\mathbf{k}, \mathbf{v}, \mathbf{v}'; t) \approx \frac{1}{\left[\pi v_T^2\right]^{3/2}} \exp\left\{ -\frac{\left[\mathbf{v} - i\mathbf{k}v_T^2/2\overline{\nu}\right]^2}{v_T^2} \right\} \times$$
$$\exp\left[-\left(\frac{kv_T}{2\overline{\nu}}\right)^2 (2\overline{\nu}t - 3) - (\Gamma_{mn} - i\Omega)t - i\frac{\mathbf{k}\mathbf{v}'}{\overline{\nu}} \right]. \tag{12}$$

Requiring at long-time limit $t, t' \gg \nu^{-1}$

$$\int d^3v'\, f_0(\mathbf{k}, \mathbf{v}, \mathbf{v}'; t)\, \frac{\hat{\delta}}{\nu}\, f_0(\mathbf{k}, \mathbf{v}', \mathbf{v}''; t') = \mathcal{O}\left(\frac{k^4v_T^4}{\nu^4}\right) \tag{13}$$

exclude secular terms and get expression for effective collision frequency

$$\overline{\nu} = \frac{\nu}{\sqrt{2}} + \mathcal{O}\left(\frac{kv_T}{\nu}\right). \tag{14}$$

As a result, within the accuracy up to $k^2v_T^2/\nu^2$ we have

$$\mathcal{P}(\Omega) = \mathcal{P}_0 \frac{kv_T}{\sqrt{\pi}} \mathrm{Re} \int_0^\infty dt\, \exp\left\{ -\frac{1}{2}\left(\frac{kv_T}{\overline{\nu}}\right)^2 \left[\overline{\nu}t - 1 + \exp(-\overline{\nu}t)\right] - (\Gamma_{mn} - i\Omega)t \right\}. \tag{15}$$

Hence, near the line centre $\Omega \ll \overline{\nu}$

$$\mathcal{P}(\Omega) = \mathcal{P}_0 \frac{kv_T/\overline{\nu}}{\sqrt{\pi}} \frac{\Gamma_{mn}/\overline{\nu} + \frac{1}{2}(kv_T/\overline{\nu})^2}{\left(\Gamma_{mn}/\overline{\nu} + \frac{1}{2}(kv_T/\overline{\nu})^2\right)^2 + (\Omega/\overline{\nu})^2} \left[1 + \mathcal{O}\left(\frac{k^2 v_T^2}{\nu^2}\right)\right]. \quad (16)$$

At line wings $\Omega \gg \nu$, omitting terms less than ν^4/Ω^4, obtain Eq.(6), which is valid at $|z| \gg \nu/kv_T$.

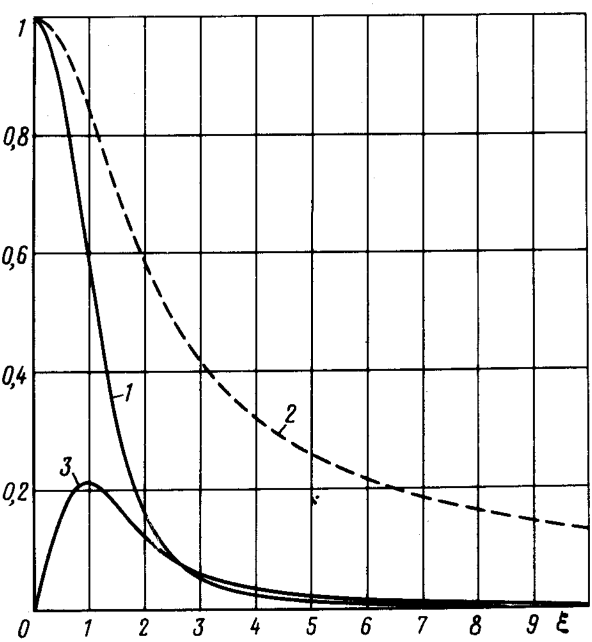

Fig.1. Normalized diffusion coefficients along (curve 1) and transverse (curve 2) to the velocity of probe ion, and Chandrasekhar function (curve 3) vs. dimensionless velocity

Note that we can hold terms of the order of $(kv_T/\nu)^2$ in the limit $\Gamma_{mn} \ll k^2 v_T^2/\nu$. In the opposite case they might be omitted. The work at the line center $\mathcal{P}(0)$ is shown in Fig.2 as a function of collision frequency. The ratio between homogeneous and Doppler width is $\Gamma_{mn}/kv_T = 0.1$. The general solution based on exact properties of the collision integral, obtained by Alekseyev and Malyugin [6], is expressed in terms of eigenvalues. It is difficult to find explicit spectrum of collision operator (1), nevertheless, qualitative behaviour of solution (16) is also Lorentzian.

Estimation for high-current discharge [3] (N_i = 3×10^{14} cm^{-3}, T_i = 1 eV, λ = 488 nm ArII) gives $\nu \sim 3 \times 10^7$ s^{-1}, whereas Doppler width is about $kv_T \sim 3 \times 10^{10}$ s^{-1}, so the effect is small $\nu/kv_T \sim 10^{-3}$. It seems that to make the narrowing more pronounced one should have higher charged particle density. However, the homogeneous width also increases with N_e because of level deactivation and Stark broadening. E.g., for dense plasma of z-pinch [7] ($N_i = 2 \times 10^{18}$ cm^{-3}, $T_i = 3$ eV, $\lambda = 164$ nm HeII) $\nu/kv_T \sim 1$, however $kv_T \ll \Gamma_{mn}$. It is difficult to observe the narrowing for hydrogen-like lines because of strong natural broadening ($\propto Z^4$) and linear Stark effect. In recent studies of neon-like

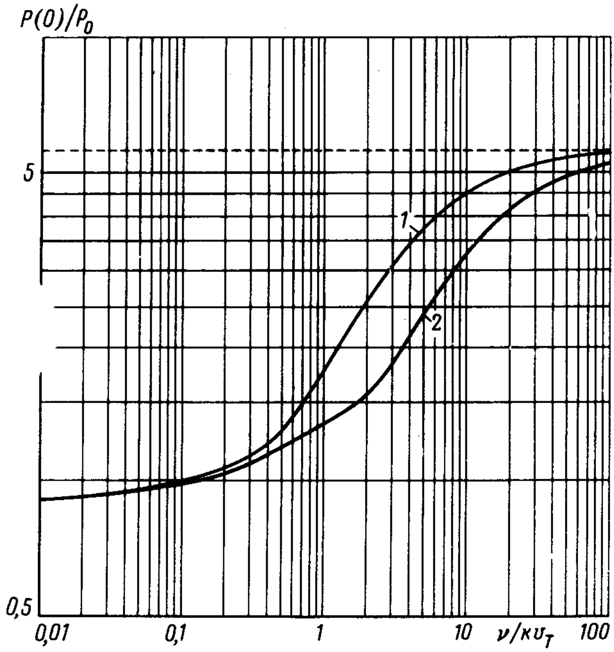

Fig.2. The work done by field at the line centre as a function of collision frequency. Curve 1 corresponds to the weak collision model with $\overline{\nu} = 2\nu\Phi_l(0)$; curve 2 describes the case of Coulomb scattering (1)

spectral line profiles of X-ray laser (charge of an ion Z=24) [8] the importance of ion-ion collisions is shown. Taking experimental values from [8] (T_i = 400 eV, $\lambda = 20.6$ nm SeXXV) and supposing solid state density ($N_i \sim 10^{20}$ cm^{-3}), we have $\nu \gg kv_T \sim 10^{13}$ s^{-1}. The Dicke effect might be significant for transitions with small homogeneous width ($\Gamma_{mn} \ll kv_T$).

Thus, the narrowing is strongest provided that $\nu \gg kv_T \gg \Gamma_{mn}$. In order to emphasize the effect one must use lines of multi-electron spectrum of multiple ions, since $\nu \propto Z^2$, and long wave transitions to diminish Γ_{mn} and to increase ν/kv_T. Plasma should be comparatively dense and cold. High ion density and low temperature lead to increase in $\nu \propto N_i v_T^{-3}$. However, under excessively low ionic temperature or high charged particle density the Doppler width can become less than homogeneous one.

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