POSSIBILITY OF NEW COHERENTLY PRECESSING SPIN STATES IN SUPERFLUID ^{3}He-B

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It is argued that in the case when the magnitude of spin polarization S of ^{3}He-B is considerably different from its equilibrium value $S_0 = (\chi_B/g)H_0$ new coherently precessing spin-orbit configurations are formed. Especially interesting dynamical regime is expected at $S = S_0/2$ or $S = 2S_0$.

The coherent spin-orbit interaction $\mathcal{F}_D$ (of dipole-dipole origin) plays a crucial role in the properties of ordered superfluid phases of liquid ^{3}He . This interaction lifts partially the degeneracy of the ground state with respect to the Goldstone degrees of freedom of the order parameter and an equilibrium spin-orbit configuration is established corresponding to the minimum of $\mathcal{F}_D$ (at given external conditions).

The spin-orbit coupling has a profound influence on the dynamical properties of superfluid A and B phases of ^{3}He . In the absence of dipole-dipole effects spin dynamics of the system subject to the action of a static magnetic field $\mathrm{H}_0 = H_0\hat{z}$ is reduced to purely precessional motion of the magnetization $\mathrm{M} = g\mathrm{S}$ about z -axis with the Larmor frequency $\omega_0 = |g|H_0$ ($g < 0$ being the gyromagnetic ratio for magnetic moments of ^{3}He nuclei) which is independent of the orientation of spins with respect to $\hat{z}$ (this orientation is characterized by $s_z = \cos\beta_s$ where β_s is the tipping angle). This simple but fundamental property is the manifestation of the fact that in the coordinate frame rotating with the Larmor angular velocity $\vec{\omega}_0 = \omega_0\hat{z}$ the magnetic field is eliminated and the spin-system restors isotropy. The coherent spin-orbit coupling perturbs, in general, the simple Larmor state and this was observed in variety of experimental situations (for a comprehensive review see [1]).

But even in the presence of the dipole-dipole potential $\mathcal{F}_D$ a purely Larmor regime of the spin motion can develope under special circomstences so that the spin system will precess with the frequency ω_0 being subject to the action of the external magnetic field only. The well known example of such a coherent spin precessing configuration is realized in the Brinkman-Smith regime when the orbital part of ^{3}He-B order parameter is at equilibrium and spins are precessing at the tipping angle β_s such that $\cos\beta_s > -1/4$. It is clear that such a situation corresponds to the case when the spin-orbit potential is at minimum and homogeneously precessing spin state is stable (only at the deviation from the Larmor state that the dipole-dipole torque is generated and the Leggett-Takagi spin relaxation mechanism is switched on).

Spin-orbit potential $\mathcal{F}_D$ depends on the variable s_z and the Larmor state at the minimum of $\mathcal{F}_D$ is realized for some particular stationary values of s_z, however in special cases the coherently precessing state shows up some degree of degeneracy (discret or contineouse) with respect to s_z. Such a situation takes place for the B phase of ^{3}He due to special properties of spin-orbit coupling in the supefluid state giving rise to a number of peculiarities of the coherent spin dynamics of ^{3}He-B, and in particular to the long-lived two domain structures in the presence of the magnetic field gradient [2-4].

It is well known that for the superfluid B phase

$$\mathcal{F}_D = \frac{2}{15}\chi_B(\Omega_B/g)^2(\mathrm{Sp}\hat{R} - 1/2)^2, \tag{1}$$

where χ_B is the magnetic susceptibility, Ω_B is the frequency of the linear longitudinal NMR characterizing the strength of the dipole-dipole interaction between magnetic moments of ^{3}He nuclei, and orthogonal matrix $\hat{R}$ describes the relative rotation of spin and orbital spaces:

$$\hat{R} = \hat{R}^{(S)} \cdot \hat{R}^{(L)-1}, \tag{2}$$

with $\hat{R}^{(S)}$ and $\hat{R}^{(L)}$ being matrices of three dimentional rotations of spin and orbital spaces, respectively. Parametrizing this rotations by triples of Eulerian angles $(\alpha_s, \beta_s, \gamma_s)$and $(\alpha_L, \beta_L, \gamma_L)$ we find that

$$\mathrm{Sp}\hat{R} = \cos\beta_L \cos\beta_s + \frac{1}{2}(1+\cos\beta_L)(1+\cos\beta_s)\cos(\alpha+\gamma)+$$

$$+\frac{1}{2}(1-\cos\beta_L)(1-\cos\beta_s)\cos(\alpha-\gamma) + \sin\beta_L \sin\beta_s(\cos\alpha + \cos\gamma), \tag{3}$$

where $\alpha = \alpha_s - \alpha_L$ and $\gamma = \gamma_s - \gamma_L$.

Considering in what follows the case of a strong magnetic field ($\omega_0 \gg \Omega_B$) we shall take into account that angular variables α and γ perform fast rotations (on the time scale of order Ω_B^{-1}). Substituting (3) into (1) and averaging over the fast oscillations we conclude that the spin-orbit potential (measured from now on in units of $(2/15)\chi_B(\Omega_B/g)^2$) is given by

$$\overline{\mathcal{U}}_D = \{[l_z s_z + \frac{1}{2}(1+l_z)(1+s_z)\cos\phi - 1/2]^2+$$

$$+\frac{1}{8}(1-l_z)^2(1-s_z)^2 + (1-l_z^2)(1-s_z^2)(1+\cos\phi)\}, \tag{4}$$

where $l_z = \cos\beta_L$ defines the orientation of the orbital momentum of the magnetized B phase with respect to the direction of the applyed field $\vec{H}_0 = H_0\hat{z}$, and the angular variable $\phi = \alpha + \gamma$ "survives" after averaging since $\dot{\alpha} \simeq -\dot{\gamma}$ (for the Larmor state under consideration ϕ is constant in time). Expression (4) is symmetric with respect to spin and orbital variables (as it should be for the B phase), and in somewhat different notations it was presented in [5]. Recently it was used to construct the phase diagram of ^{3}He-B subject to the action of the transvers field with pump frequency $\omega_p \neq \omega_0$ and the superfluid counterflows [6].

In order to construct the Larmor state (homogeneous spin precessing state with frequency ω_0) we have to explore the manyfold of variables (l_z, s_z, ϕ) realizing the minimum of (4). The equation defining this space of degeneracy is

$$\cos\phi = -\frac{(1-2l_z)(1-2s_z)}{(1+l_z)(1+s_z)} \tag{5}$$

and excluding ϕ from (4) we see that the ground state is realized at the minimum of

$$\overline{\mathcal{U}}_D^{(0)} = 3(1-l_z)(1-s_z)[1-\frac{5}{8}(1-l_z)(1-s_z)] \tag{6}$$

in the domain of the (l_z, s_z) plane where $|\cos\phi| \le 1$. The two boundaries of this domain are given by equations:

$$l_z + s_z = 2 + 5l_z s_z \qquad (\cos\phi = 1) \tag{7}$$

$$l_z + s_z = l_z s_z \qquad (\cos\phi = -1). \tag{8}$$

On crossing the curve (7) (with two branchs) we merge into the domain with $\cos\phi \equiv 1$ (in the case of (8) - into domain with $\cos\phi \equiv -1$) The minimum of (6) is attained at

$$s_z = 1, \qquad -1/4 \le l_z < 1 \tag{9}$$

$$l_z = 1, \qquad -1/4 \le s_z < 1 \tag{10}$$

and the residual degeneracy of the equilibrium spin state (9) with respect to l_z, and of the spin precessing state (10) with respect to s_z are lifted by external perturbations. In particular in the presence of the transvers rf field with pump frequency ω_p slitely larger than ω_0 the coherently precessing spin state with $s_z \simeq -1/4$ is stabilized at the boundary (7). In the presence of the magnetic field gradient a two domain state was first observed in [2], and the degeneracy of (9) with respect to l_z makes the structure of this two domain state very sensetive to the action of superfluid counterflows in the rotating ^{3}He - B [7-9].

It should be stressed that the preceding consideration of the coherently precessing Larmor state was based on the assumption that spin dynamics is developing along trajectories with the magnitude of spin polarization close to its equilibrium value ($S \simeq S_0 = (\chi_B/g)H_0$). In this case the angular variable $\phi = \alpha + \gamma$ is slow and we arrive to equation (4) for the dimensionless averaged spin-orbit potential $\overline{\mathcal{U}}_D$. In what follows we shall argue that when actual value of S is considerably different from S_0 (more precisely when $|\omega_s - \omega_0| \gg \Omega_D$ with $\omega_s = \omega_0(S/S_0)$) new coherent Larmor states are possible with specific spin-orbit configurations. In this situation two cases should be distinguished. In the first nonresonant regime ($S \neq (S_0/2, 2S_0)$) there are no slow angular variables and we can safely average expression (4) over ϕ (which is now fast since $|\dot\phi| \gg \Omega_D$). As a result we obtain new effective spin-orbit potential

$$\widetilde{\mathcal{U}}_D = \frac{3}{4}[1 + (1-l_z^2)(1-s_z^2) + 2l_z^2 s_z^2 \ ] \tag{11}$$

with degenerate minima at

$$s_z^2 = 1, \qquad l_z = 0 \tag{12}$$

$$s_z = 0, \qquad l_z^2 = 1. \tag{13}$$

The profile of $\widetilde{\mathcal{U}}_D(s_z, l_z)$ is shown on Fig.1. An interesting property of $\widetilde{\mathcal{U}}_D$ is that at $l_z^2 = 1/3$ it is independent of s_z (and vice versa). We note that the solution ($s_z = -1, l_z = 0$) belonging to (12) is similar to the reversed spin (RS)

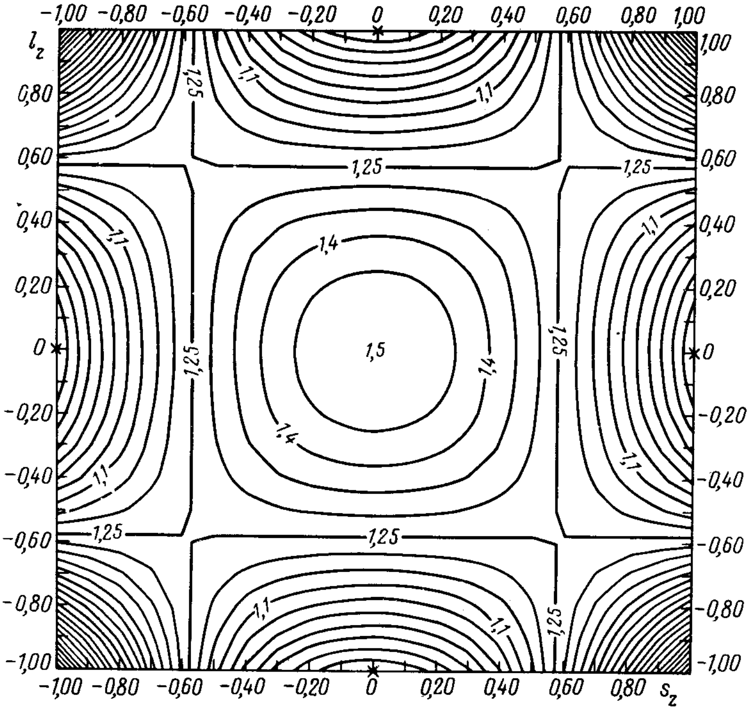

Fig.1. The topographic profile of averaged spin-orbit potential $\widetilde{\mathcal{U}}_D(s_z, l_z)$ (Eq.(11)). Four degenerate stationary points (shown by crosses) correspond to states (12) and (13)

state found in [6] for the case $S \simeq S_0$ in the presence of transvers rf field with pump frequency $\omega_p > \omega_0$ which stabilizes it. The RS state found by us as a stationary spin-orbit configuration of (11) at $\omega_p = \omega_0$ corresponds to the case with $|1 - S/S_0| \gg \Omega_D/\omega_0$. The transvers spin ($TS$) state (13) also belongs to the same class of solutions.

Another regime is developed for $S = S_0/2$ (or $S = 2S_0$) which is a resonant case in the sence that the angular variable $\widetilde{\phi} = \alpha + 2\gamma$ (or $\widetilde{\phi}' = 2\alpha + \gamma$) turns to be slow and it "survives" the averaging procedure if one starts from general expressions (1) and (3). In this special conditions (analogous to the case considered in [10] for the A phase) instead of (11) we obtain the following averaged spin-orbit potential:

$$\widetilde{\mathcal{U}}_D^{(R)} = \frac{3}{4}[1 + (1 - l_z^2)(1 - s_z^2) + 2l_z^2 s_z^2 + + \frac{2}{3}\sqrt{1 - l_z^2}(1 + l_z)\sqrt{1 - s_z^2}(1 + s_z)\cos\widetilde{\phi}\ ]. \tag{14}$$

The stationary states realizing minima of (14) can be found numerically having in mind that the relevant stationary point $\widetilde{\phi}_s = \pi$. It turns out that there are two degenerate stable spin-orbit configurations

$$s_z = 0.75, \qquad l_z = 0.3 \tag{15}$$

$$s_z = 0.3, \qquad l_z = 0.75 \tag{16}$$

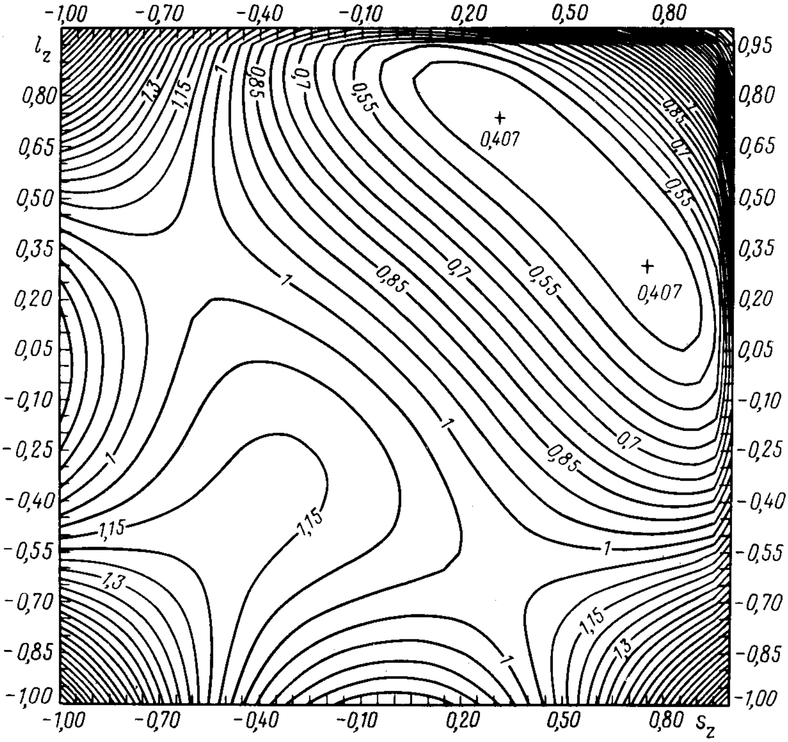

Fig.2. The topographic profile of averaged spin-orbit potential $\widetilde{\mathcal{U}}^{(R)}(s_z, l_z)$ (Eq.(14)). Two degenerate stationary points (depicted by crosses) correspond to states (15) and (16)

for which $\widetilde{\mathcal{U}}_D^{(R)}$ is at minimum. The profile of $\widetilde{\mathcal{U}}_D^{(R)}$ (for $\widetilde{\phi} = \pi$) is shown on Fig.2. We notice that two minima are located at opposite ends of an elongate "valley" and are sepatate by a tiny "hill".

On the Fig.3 coherently precessing states of ^{3}He-B with different spin-orbit configurations are depicted on the plane $(S_z, S_\perp)$ where the longitudinal component of spin $S_z = S s_z$ and its transvers component $S_\perp = S\sqrt{1 - s_z^2}$. We remind that for a given spin-orbit state the orientation of the relative rotation axis $\hat{n}$ can be established using the relation $\hat{l}_i = \hat{s}_\mu R_{\mu i}(\hat{n}, \theta_0)$ where θ_0 is the Leggett angle ($\cos\theta_0 = -1/4$). Among other coherently precessing states we have shown the possibility of realizing new stable spin-orbit configurations appropriate to $S = S_0/2$ (half spin (HS) mode) and to $S = 2S_0$ (double spin (DS) mode).

From the experimental point of view it is important to find out a proper route to HS and DS modes. A brute force way is to increase (decrease) abruptly the strength of the applyed magnetic field to the value $H = 2H_0$ ($H = H_0/2$) but as was shown by Volovik [11] by means of computer simulation (using the LESTER package elaborated by Golo and Leman) the efficient way to reach HS state is to tip first the magnetization by angle π, to let the spin system to relax along the trajectory with $S_\perp = 0$ to the value $S_z = -1/2$ and then to follow the route at $S = 1/2$ and increasing s_z.

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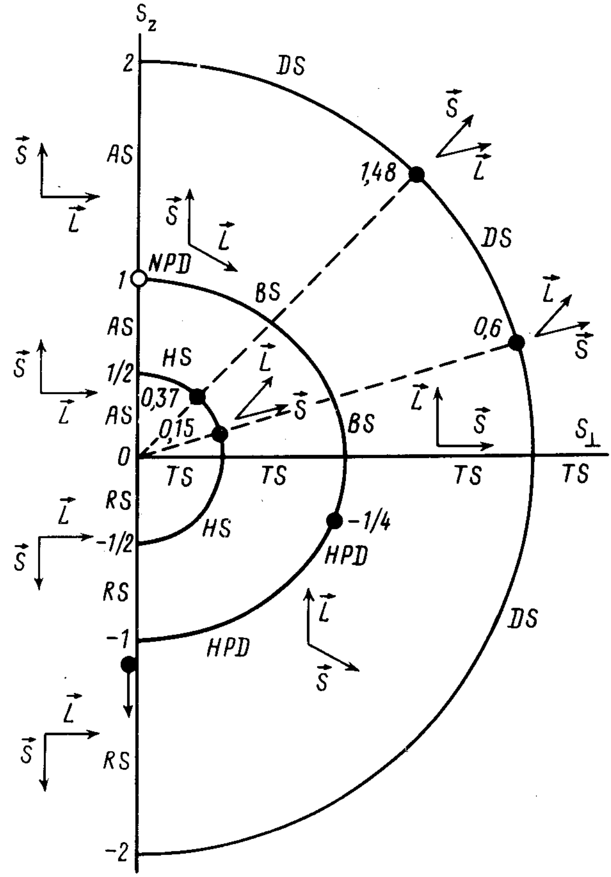

Fig.3. Coherently precessing states of ^{3}He-B. Two pairs of HS and DS states with $S_z = (0.15; 0.6)$ and $S_z = (0.37; 1.48)$ correspond to degenerate minima of spin-orbit potential $\tilde{\mathcal{U}}_D^{(R)}$ shown on Fig.2. AS – aligned spin mode, RS – reversed spin mode, TS – transverse spin mode, HPD – homogeneously precessing domain, HS – half spin mode, DS – double spin mode, BS – Brinkman–Smith mode, NPD – nonprecessing (equilibrium) state, • – Stationary states at ω close to Larmor frequency

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