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ANALYTIC DESCRIPTION OF ENERGY LOSS BY A BOUNDED INHOMOGENEOUS HOT PLASMA DUE TO THE EMISSION OF ELECTROMAGNETIC WAVES

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Universal formulae for total power losses by a bounded media due to the emission of electromagnetic (both transverse and longitudinal) waves in the regimes of nonlocal (non–diffusive) heat transport are proposed, which, in particular, generalize Trubnikov's formula for synchrotron losses to the case of arbitrary emission/absorption process and inhomogeneous non–stationary plasma. Their derivation is based on a non–diffusion method which generalizes the "escape probability method" in the theory of radiative transfer in resonance atomic lines (RTRAL). The results suggest a qualitative model for the global heat transport in a tokamak.

1.Introduction. The heat transport by electromagnetic (EM) waves, both transverse (i.e. photons) and longitudinal waves (i.e. plasmons), in the wide range of parameters (which includes, in particular, magnetically confined thermonuclear plasmas) is characterized by its nonlocal (non–diffusive) nature which manifests itself in nonlocal correlation of plasma temperatures and, correspondingly, non–diffusive law of heat propagation. Mathematically, the non–diffusive nature is expressed by the fact that the original equation for heat transfer which is, in general, an integral equation in space variables, cannot always be reduced to a differential diffusion–type equation in those variables. Such a reduction leads to an infinite diffusion coefficient in case of unbounded media or medium–size–dependent diffusion coefficient in case of bounded media, so that the very concept of diffusion coefficient appears to be meaningless.

2.Universal formula for total energy losses. Let us consider the transfer of EM energy which is described by the intensity $J(\phi, \vec{r}, t)$, differential with respect to EM wave (photon or plasmon) parameters ϕ,

$$\phi \equiv \{\omega, \vec{n}, \zeta\}, \tag{1}$$

where ω and $\vec{k}$ are the frequency and wave vector respectively, $\vec{n} \equiv \vec{k}/k$, parameter ζ describes polarization state of the wave. Space–time evolution of the intensity is guided by equation of the well-known form:

$$\left(\frac{1}{v_g}\frac{\partial}{\partial t} + \vec{n}_g \frac{\partial}{\partial \vec{r}}\right)\left(\frac{J(\phi, \vec{r}, t)}{(N_r)^2}\right) = -\kappa\left(\frac{J(\phi, \vec{r}, t)}{(N_r)^2}\right) + \frac{Q}{(N_r)^2}, \tag{2}$$

where $\kappa(\phi, \vec{r}, t)$ is absorption coefficient, $Q(\phi, \vec{r}, t)$ is the source function, N_r is ray refractive index, v_g and $\vec{n}_g$ are the group velocity and its direction. In a medium with dispersion the substitution of the dependence $k = k(\omega, \vec{n}, \zeta)$ from the dispersion relation is implied.

In case of radiative transfer by emission/absorption of EM waves by plasma electrons, the quantities Q and κ contain averaging over electron velocity distribution (EVD). Therefore the source function Q may implicitly depend on the intensity J via corresponding distortions of the EVD, caused by radiative transfer. If these distortions are small then Q doesn't depend on J, and eq.(2) appears to be a closed equation for energy carriers. The multiple reflection at the medium boundary (e.g., tokamak wall) prevents from straightforward use of the analytic solution of eq.(2) in the form of the integral over ray path.

Another example of the reduction of transport problem to a closed equation is the Biberman–Holstein equation for accumulators of energy, namely excited atoms, in the theory of radiative transfer in resonance atomic lines (RTRAL) in gases and plasmas in case of the so–called complete redistribution in photon frequencies in an individual act of photon's scattering by medium's atom (approximation of complete redistribution is valid for a wide range of astrophysical and laboratory plasmas [1]).

In both above–mentioned cases the energy transport appears to be characterized by the following general features.

(*) The dominant contribution to energy loss stems from those long–free–path energy carriers (photons or plasmons) whose long flights are of medium's size L_0 in length ,or greater, $L_0/(1-R)$, for the case of reflection at boundaries, R is reflection coefficient, (for the RTRAL, these are the photons in the so-called line wings).

(**) Each of the variables of the total phase space $\{\Gamma\} \equiv \{\phi, \vec{r}\}$ manifests one of two limiting forms of its evolution ("redistribution") along the trajectory of energy carrier from its birth to its death (by convertion into medium's heat); either no redistribution ("independent" variables) in which case for this variable the energy transport takes place independently (e.g., the variable ω for the case of absorption/emission by free electrons with fixed velocity distribution and the variable $\vec{r}$ for the case of bounded electrons, see below) or complete redistribution in which case for each of those variables the transport equation may be properly averaged, according to actual redistribution law.

The whole set of first-type variables we shall denote as Γ_{ind} and the second-type, Γ_{crd}. The properties (*) and (**) enable us to obtain finally the following general result for total power losses:

$$\frac{dE}{dt} = \int d\Gamma_{ind}\ \left(1 + \nu_{que}/\nu_{esc}\right)^{-1} \int d\Gamma_{crd}\ Q(\Gamma)\ , \tag{3}$$

Here, ν_{que} is the rate of such an absorption of EM energy by the medium, which converts transported EM energy into medium's heat (temperature), (this is the quenching of atom excitation by, e.g., medium particle's impact for the RTRAL, and the absorption of the photon(plasmon) by plasma electrons for the radiative transfer in continuous spectrum), and ν_{esc} is the rate of free escape of EM energy out of the medium. Both quantities ν_{que} and ν_{esc} are averaged over Γ_{crd} according to corresponding redistribution laws.

In case of the RTRAL, we have $\Gamma_{ind}=\vec{r}$ and $\nu_{esc} = < T(\vec{r}) > / t_{rad}$, where $< T(\vec{r}) >$ is Holstein function averaged over the angles of photon's escape, t_{rad} is spontaneous emission lifetime for excited atom, and we thus arrive at the well-known result of the RTRAL theory (for the accuracy of this result see the survey [1] and references therein).

It should be noted that the phenomenon of nonlocal, non-diffusive transport has been revealed and thoroughly investigated just in the RTRAL theory in the late forties and early fifties in the pioneer works by Biberman, Holstein and Sobolev. The derivation of eq.(3)-like formula and its further advances are known in literature as "escape probability method" (see, e.g.,[2]). Therefore formula (3) may be interpreted as a generalization of this methods to the case of heat transport via emission/absorption of (transverse and longitudinal) EM waves by free electrons with fixed velocity distribution (for comparison of eq.(3) with numerical calculations for heat transport by electron cyclotron radiation see Sec.3.).

Note that formula (3) covers both limiting regimes of energy loss, namely purely volumetric loss due to free escape from the whole phase space ($\nu_{que} \ll \nu_{esc}$) and purely surface loss ($\nu_{que} \gg \nu_{esc}$). In the latter case, the intensity of escaped EM field is close to the equilibrium Planck distribution with some effective, space-averaged temperature, the heat transport inside the medium being characterized in corresponding domain of phase space by diffusion–type regime of radiative thermoconduction. Nevertheless,the total losses in eq.(3) are determined dominantly by those part of the total phase space Γ in which the process of energy transport has essentially nonlocal character, namely the non–diffusion regime of free escape. The latter statement just constitutes the essence of a generalazed escape probability (GEP) method which enables us to obtain eq.(3) within the framework of principles (*) and (**).

3. Heat transport by plasma waves. Contrary to the RTRAL theory, nonlocal character of the transport by plasma longitudinal waves and its possible role have been realized much later starting from the paper by Rosenbluth and Liu [3]. For the case of transport by transverse waves (i.e. conventional electron cyclotron radiation) in inhomogeneous plasma, the concept of non-diffusive transport has been fruitfully exploited by Tamor [4].

The straightforward analogy between heat transport by plasma waves and the RTRAL was traced in [5] where the non-diffusive law of heat propagation by Bernstein modes was also obtained (approximately $t \propto r$ contrary to the standard diffusion law $t \propto r^2$).

The existing analytic descriptions have the character of the fit of numerical results and pertain to (a) specific mechanism of emission, namely cyclotron radiation, and (b) specific profiles of temperature and density: homogeneous, the well-known Trubnikov's formula [6], or tokamak-like, formula [7]. The required analytic description may be obtained from our general result (3) for a specific case of tokamak geometry under following, usually satisfied conditions, namely not too large aspect ratio, non–circular (in particular, elongated) cross–section and multiple reflection at plasma boundaries (for transverse waves the latter is assured by highly reflecting tokamak walls, $(1-R) \lesssim 0.1$, and for longitudinal waves it depends on edge plasma parameters). Neglecting the mixing of different modes in reflections at the boundaries, we have $\Gamma_{ind} = \{\omega, \zeta\}$ and eq.(3) reduces to the form:

$$\frac{dE}{dt} = \sum_{\zeta} \int d\omega \; \frac{\int dV \int d\Omega_n \; Q(\phi, \vec{r})}{1+\tau_{eff}} \; , \tau_{eff} = \frac{\int dV \; \int d\Omega_n \; Q(\phi, \vec{r})}{\int d\Omega_n \; \int(\vec{n}, d\vec{S}_s) \; (1 - R(\phi, S_s))}, \quad (4)$$

where $\tau_{ef}(\omega, \zeta) = \nu_{que}/\nu_{esc}$ is the effective (dimensionless) optical length which describes the trapping (or, equivalently, imprisonment, according to RTRAL theory language) of plasmons (photons). Here V and S_s are plasma volume and surface, respectively, and $R(\phi, S_s)$ describes the dependence of reflection coefficient on wave parameters (first of all, frequency). The comparison of formula (4) with the results of numerical calculations, in particular, for homogeneous [6] and inhomogeneous [4] cases, and with formula [7] (and formula [6] in the region of successful approximation of numerical results, see [6,8]) shows good agreement (to an accuracy of about 30%) in the regions of applicability of these results. The GEP method allows also to obtain a universal analytic description for spatial profile of wave energy balance and for the case of arbitrary degree of mixing of different waves at plasma boundaries, both for stationary and non–stationary cases. Thus, the non–stationary counterpart of eq.(4) has the form:

$$\frac{dE(t)}{dt} = \sum_{\zeta} \int d\omega \int_0^t \frac{dt'}{t_{ef}} \int dV \int d\Omega_n \; Q(\phi, \vec{r}, t') \; \exp\left[-\int_{t'}^{t} \frac{dt''}{t_{ef}} \; (1+\tau_{eff}(t''))\right] , \quad (5)$$

where t_{ef} is the characteristic time of wave trapping exclusively due to reflecting by the boundaries,

$$t_{eff}(t) = \frac{\int dV \; \int d\Omega_n \; v_g^{-1}(\phi, \vec{r}, t)}{\int d\Omega_n \; \int(\vec{n}, d\vec{S}_s) \; (1 - R(\phi, S_s))}, \quad (6)$$

(eq. (5) assumes the smallness of retardation effects).

The formulae (4) and (5) generalize Trubnikov's formula[6] for synchrotron losses to the case of (a) arbitrary elementary process of radiation emission by high-temperature plasma with highly reflecting walls and fixed (e.g., maxwellian) velocity distribution and (b) inhomogeneous and non-stationary plasma parameters (density, temperature, magnetic field).

4. **On the global heat transport in a tokamak.** The application of the GEP concept (items (*) and (**) of Sec.(2)) to energy transport by plasma waves suggests a qualitative model for the global heat transport in a tokamak, which expoits the fact of a strong coupling of essentially nonlocal and local characteristics of a plasma in eqs. (4) and (5) for total power losses, namely, the coupling of space-averaged emission/absorption coefficients and the coefficient for the reflection of plasma waves at plasma boundary. In this model, the global energy transport is conducted by the long-free-path quanta of longitudinal EM waves (plasmons) which are responsible for such a strong coupling due to their multiple reflection at plasma boundary and appear to be the main carriers of energy whereas the particles are

invariably the main accumulators of plasma energy. Such a model suggests the possibility of an effective control of plasma global (nonlocal) parameters via a proper control of the reflection of plasma waves in edge plasma. From this viewpoint, the L-H transition may appear to be stimulated by a sharp change of the reflection coefficient due to, e.g., increased gradient of poloidal rotation of tokamak plasma or increased stability of magnetic surfaces which are responsible for the reflection of most significant energy carriers.

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1. V.A.Abramov, V.I.Kogan and V.S. Lisitsa, In: Reviews of Plasma Physics, Eds. Leontovich M.A. and Kadomtsev B.B., **12**, 151 (1987).
2. Methods in Radiative Transfer, Ed. W.Kalkofen, Cambridge Univ. press (1984).
3. M.N.Rosenbluth and C.S.Liu, Phys. Fluids **19**, 815 (1976).
4. S.Tamor, Fusion Technology **3**, 293 (1983); Nucl.Instr. and Meth.Phys.Res. **A 271**, 37 (1988).
5. A.B.Kukushkin, V.S.Lisitsa and Yu.A.Savel'ev, JETP Lett. **46**, 448 (1987); Proc. Int. Conf. Plasma Phys., New Delhi, India **3**, 897 (1989).
6. B.A.Trubnikov, In: Reviews of Plasma Physics, Ed. M.A.Leontovich, **7**, 345 (1979).
7. S.Atzeni, B.Coppi, and G.Rubinacci, Report PTP-81/7, MIT (1981).
8. M.Bornatici, R.Cano, O. De Barbieri, and F.Engelmann, Nucl.Fusion **23**, 1153 (1983).