Письма в ЖЭТФ, том 56, вып.10, стр.523 - 526 25 ноября 1992 г.

ONCE AGAIN ABOUT INTERCHAIN HOPPING

V.M.Yakovenko

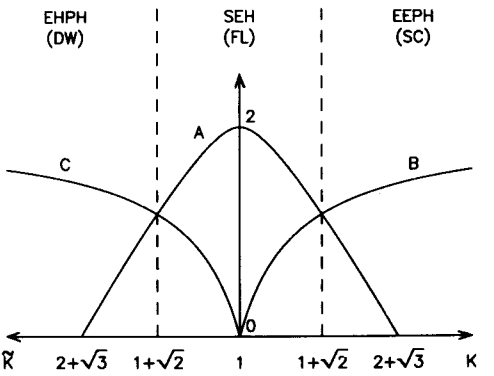
Department of Physics and Astronomy, Rutgers University,
P. O .Box 849, Piscataway, NJ 08855-0849, USA

L.D.Landau Institute for Theoretical Physics RAS,
117940, Moscow, Russia

Submitted 21 October 1992

Renormalization group equations and a phase diagram are derived for a system of two chains with a single-electron hopping between chains in order to correct the results of a recent paper by F. V. Kusmartsev, A. Luther, and A. Nersesyan, Pis'ma v Zh. Exp. Teor. Fiz. **55**, 692 (1992) [JETP Lett. **55**, 724 (1992)].

In a recent paper [1], the effect of a single-electron hopping (SEH) between chains in a system of two chains was studied by means of the bosonization technique and the Coulomb gas description. Possible phase transitions of the Berezinskii-Kosterlitz-Thouless type, which result in generation of pair hoppings between the chains, were considered. Unfortunately, despite the spirit of the paper is correct, Eq. (4) turns out to be incomplete and leads to a wrong phase diagram shown in Fig.



RG dimensions of t^2 (curve A): $4-K-\tilde{K}$; J (curve B): $2-2\tilde{K}$; and $\tilde{J}$ (curve C): $2-2K$ as functions of K (the right horizontal semi-axis) and $\tilde{K}=1/K$ (the left horizontal semi-axis). The difference in a physical behavior in the three regions, separated by the dashed lines and labeled by the abbreviations, is explained in the text

The considered problem is the old one which recently attracted attention again [2]. The generation of the pair hoppings in a gapless case was discovered for the first time in Ref. [3] via a perturbational theory. Later, these results were confirmed

by a renormalization group (RG) approach [4]. The aim of the present paper is to correct the RG equation (4) and the phase diagram of Ref. [1] and achieve in this way an agreement with the previously known results [3,4].

Due to the generation of the pair-hopping terms, the action of the model,

$$S_0 = \int d\tau\, dx \left[\frac{u}{2}((\partial_\tau \Phi)^2 + (\partial_x \Phi)^2) + \frac{2t_\perp}{\pi\alpha} \cos\frac{1}{2}\gamma\Phi \cos\frac{1}{2}\tilde{\gamma}\tilde{\Phi}\right], \tag{1}$$

has to supplemented with the following term:

$$S_1 = \int d\tau\, dx \left[2J \cos\gamma\Phi + 2\tilde{J} \cos\tilde{\gamma}\tilde{\Phi}\right]. \tag{2}$$

The notation of Ref. [1] is used everywhere. In the fermion representation, the coefficients J and $\tilde{J}$ in Eq. (2) are the amplitudes of the electron-electron (EEPH) and the electron-hole pair hoppings (EHPH), respectively:

$$H_1 = (2\pi\alpha)^2 \int dx (J\Psi^+_{1,1}\Psi^+_{2,1}\Psi_{2,-1}\Psi_{1,-1} + \tilde{J}\Psi^+_{1,1}\Psi^+_{2,-1}\Psi_{2,1}\Psi_{1,-1} + \text{h.c.}). \tag{3}$$

The RG equations for the considered system can be derived using the approach of Ref. [5]

$$dt_\perp/dl = (2 - 0.5K - 0.5\tilde{K})t_\perp, \tag{4}$$

$$dJ/dl = 2(1 - \tilde{K})J + (\tilde{K} - K)t_\perp^2/2\pi v_F, \tag{5}$$

$$d\tilde{J}/dl = 2(1 - K)\tilde{J} + (K - \tilde{K})t_\perp^2/2\pi v_F, \tag{6}$$

$$t_\perp(0) = t_0, \quad J(0) = \tilde{J}(0) = 0. \tag{7}$$

The second terms in the r.h.s. of Eq. (5) and (6) reflect the generation of the pair hopping terms by combining the SEH terms. The first terms in the r.h.s. of Eq. (4) – (6) reflect the RG dimensions of the appropriate operators. They are plotted in Fig. The renormalization of $K = 1/\tilde{K}$ and the generation of irrelevant terms are neglected. The value $K = 1$ corresponds to the case of non-interacting electrons. In the Hubbard model, there are certain limitations on the range of possible values of K, however, these limitations do not apply to a generic model, like an extended Hubbard model [6]. Eq. (4) – (6) are essentially the same as Eq. (84) of Ref. [4], although there are some differences in details.

Eq. (4) – (6) with the initial conditions (7) have the following solution (see Eq. (87) of Ref. [4]):

$$t_\perp = t_0 e^{(2-0.5K-0.5\tilde{K})l}, \tag{8}$$

$$J = \frac{t_0^2}{2\pi v_F} \frac{\tilde{K} - K}{(2 - K - \tilde{K})} \left(e^{(4-K-\tilde{K})l} - e^{2(1-\tilde{K})l}\right). \tag{9}$$

The formula for $\tilde{J}$ can be obtained from Eq. (9) by exchanging K and $\tilde{K}$.

Eq. (8) shows that the SEH is relevant (increases upon renormalization) if the condition

$$2 - 0.5K - 0.5\tilde{K} > 0 \quad \Leftrightarrow \quad 2 - \sqrt{3} < K < 2 + \sqrt{3} \tag{10}$$

is fulfilled. Considering the behavior of the EEPH amplitude J, the two different regimes can be distinguished. If

$$4 - K - \tilde{K} > 2 - 2\tilde{K} \quad \Leftrightarrow \quad K < 1 + \sqrt{2}, \tag{11}$$

then the first term in Eq. (9) dominates, and $J(l)$ grows essentially as $t^2(l)$. If the opposite condition

$$K > 1 + \sqrt{2} \tag{12}$$

is fulfilled, then the second term in Eq. (9) dominates, and $J(l)$ grows *faster* than $t^2(l)$. Analogously, the EHPH amplitude $\tilde{J}$ grows faster than $t^2(l)$ if the condition

$$4 - K - \tilde{K} < 2 - 2K \quad \Leftrightarrow \quad \tilde{K} > 1 + \sqrt{2} \tag{13}$$

is fulfilled. The mutual position of regions (10) – (13) is illustrated in Fig.

Upon renormalization, an amplitude $t(l)$, $\alpha J(l)$, or $\alpha\tilde{J}(l)$ may become of the order of the Fermi energy $\varepsilon_F \sim v_F/\alpha$, and a crossover to a different physical regime will take place at the corresponding temperature $T = \varepsilon_F e^{-l}$. In the region, labeled as SEH in Fig., the single-electron hopping amplitude t becomes of the order of ε_F first of all. In the case of two chains, the characteristic temperature represents a renormalized energy splitting of the two chains. In the case of an infinite array of chains, this temperature marks a crossover from a 1D Luttinger liquid to a 2D or 3D standard Fermi liquid (FL). There may be phase transitions to ordered states at lower temperatures.

On the other hand, in the region marked as EEPH, the electron-electron pair hopping amplitude $\alpha J(l)$ becomes of the order of ε_F when $t(l)$ is still small. In the case of two chains, the superconducting phases of the two chains become strongly bound below the corresponding temperature. In the case of an infinite array of chains, a phase transition to a superconducting (SC) state takes place at this temperature. It is essential that in the EEPH region, unlike in the SEH region, the phase transition takes place directly from the Luttinger liquid regime without an intermediate FL regime, and the transition is driven by the pair interchain coupling. This scenario is reminiscent of one suggested in Ref. [7] for the high T_c superconductors. The same consideration applies to the EHPH region in Fig. where a phase transition to a density-wave (DW) state is expected.

The boundaries between the single-electron and the pair hopping regimes (Eq. (12) and (13)) were found for the first time in Ref. [3]. In contradiction with the text of their paper, the authors of Ref. [1] neglect Eq. (5) and (6), thus their phase diagram reflects only condition (10). For this reason, they essentially repeat a wrong conclusion, made in Ref. [8], that a Luttinger liquid regime can exist at zero temperature.

The author thanks E. Abrahams for the support of this work via the NSF Grant No. DMR 89-06958 and the Aspen Center for Physics for the opportunity to have discussions with A. Luther.

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1. F.V.Kusmartsev, A.Luther, and A.Nersesyan, Pis'ma v Zh. Exp. Teor. Fiz. **55**, 692 (1992) [JETP Lett. **55**, 724 (1992)].

2. P.W.Anderson, Phys. Rev. Lett. **67**, 3844 (1991); C.Castellani, C.Di Castro, and W.Metzner, Phys. Rev. Lett. **69**, 1703 (1992); M.Fabrizio, A.Parola, and E.Tosatti, Phys. Rev. B **46**, 3159 (1992); A.M.Finkel'stein and A.I.Larkin, submitted to Phys. Rev. B.
3. S.A.Brazovskii and V.M.Yakovenko, J. Physique Lett. (France) **46**, L-111 (1985); Zh. Exp. Teor. Fiz. **89**, 2318 (1985) [Sov. Phys. JETP **62**, 1340 (1935)].
4. C.Bourbonnais and L.G.Caron, Int. J. Mod. Phys. B **5**, 1033 (1991).
5. P.B.Wiegmann, J. Phys. C **11**, 1583 (1978).
6. H.J.Schulz, Int. J. Mod. Phys. B **5**, 57 (1991).
7. P.W.Anderson, Lectures at the NATO Advanced Studies Institute, Cargese, 1990 (unpublished).
8. X.G.Wen, Phys. Rev. B **42**, 6623 (1990).