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ANOMALOUS ORTHOGONALITY CATASTROPHE FOR LUTTINGER LIQUID WITH REPULSION

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The Orthogonality Catastrophe for Luttinger liquid is studied. Surprisingly, it is found, that the backscattering from external potential cause an enhancement ot the infrared divergence for repulsive internal interaction and changes the type of the singularity from usual logarithmic to a power one.

Interest in the properties of Luttinger liquid has recently been renewed, mainly in response to the proposal of non-Fermi-liquid behaviour for High-T_c Supercoductors [1] and in connection with the development of microfabrication technology for inorganic one-dimensional semiconductor compounds("Quantum wires"). In particular, the investigations of photoemission [2], Fermi-edge singularities [3] and the influence of the core hole dynamics on them have been recently done for Luttinger liquid.

As it is well known, the singularities in the X-ray response of metals [4], as well as some other phenomena, e.g. Kondo effect, have the common origin: the Orthogonality Catastrophe [5].

Thus, it seems to be interesting to investigate directly the Orthogonality Catastrophe phenomenon for Luttinger liquid. The Hamiltonian of the problem is:

$$\hat{H} = \hat{H}_{Lutt.} + \hat{V},$$

$$H_{Lutt.} = \sum_k \{k(a^+_{1k}a_{1k} - a^+_{2k}a_{2k}) + U(k)\rho_1(k)\rho_2(-k)\}, \qquad (1)$$

$$V = \theta(-t)e^{t/\tau}\sum_k \{V(k)[\rho_1(k) + \rho_2(k)] + V(2k_F + k)a^+_{1k}a_{2k} + h.c.\},$$

where $a_{1,2}$ are the operators for right and left fermions, $\rho_{1,2}$ are corresponding density operators, $U(k)$ is internal interaction and $V(k)$ is external potential, which *is adiabatically turned on during a time τ. One has to calculate the overlap $\langle 0|V\rangle$* between the wave-functions at $t = -\infty$ and $t = 0$, which correspond to the ground states with and without external potential in the limit $\tau \to \infty$.

The interaction with external potential (eq(1)) contains two processes: forward scattering from external potential governed by its zero-momentum Fourier component $V(0)$ and backward scattering governed by $V(2k_F)$. Neglecting the backscattering one arrives at the Hamiltonian, quadratic in the density operators, which satisfy a boson algebra [6]. This problem can be trivially solved by means of a Bogolubov transformation. In particular, the overlap integral is:

$$\log|\langle 0|V\rangle_{f.s.}| = -(\delta_{eff}/2\pi)^2\log(\epsilon\tau) \qquad (2)$$

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where $\epsilon \sim \epsilon_F$ is a high energy cutoff and δ_{eff} is the scattering phase, modified by the internal interaction(due to the renormalization of both Fermi velocity and strength of the potential): $\delta_{eff} = \delta_0/\sqrt{1-\alpha}; \delta_0 = V(0)/v_F$, $\alpha = U(0)/\pi v_F$.

The result eq(2) can be easily understood. Physically, the Orthogonality Catastrophe is caused by the creation of electron-hole excitations with small energies in the process of adaptation of the Fermi surface to an external potential. So, not the(single particle) non-Fermi-liquid behaviour itself, but the density of states of electron-hole excitations is relevant to the Orthogonality Catastrophe. In the absence of backscattering only the creation of electron-hole excitations with a total momentum close to zero is allowed. These excitations are sound waves and they remain well defined quasiparticles even if the internal interaction is taken into account. That's why, in this case, the response of the Luttinger liquid to an external potential is qualitatively the same as for free fermion gas. The above consideration can be regarded as a simple physical background for the results obtained in papers [3], where the backscattering was neglected.

Let's now consider the backscattering effects. It leads to the creation of electron-hole excitations with a total momentum close to $\pm 2k_F$. These excitations, on the contrary to the sound waves, are extremely sensitive to the internal interaction. The model eq(1) is no more exactly solvable. In order to proceed, one should expand the log of the oveplap integral in external potential(see Fig.1 and e.g. [4]):

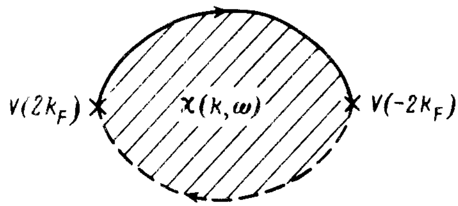

Fig.1.

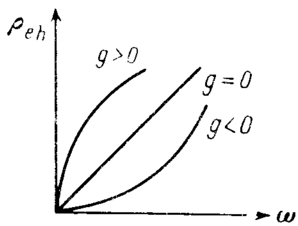

Fig.2.

Fig.1. The diagram for the overlap, which is in the secomd order in V and contains all internal interactions in the polarization loop. The full(dashed) curve presents right(left) fermions
Fig.2. The density of states $\rho_{eh} \sim \sum_{p\sim 2k_F} \mathrm{Im}\ \chi$ of the electron-hole excitations relevant to the Orthogonality Catastrophe(for different internal interactions g)

$$\log|\langle 0|V\rangle| = -\frac{1}{4\pi}\int \frac{d\omega}{\omega^2 + (1/\tau)^2}\int \frac{dp}{2\pi}|V(p)|^2 Im\chi(p,\omega), \tag{3}$$

where $\chi(x,t) = -i\langle T\{\rho(x,t)\rho(0)\} >$ is the *exact* correlation function of the *full* densities. Near $p = \pm 2k_F$ it can be either calculated by the bosonization method or extracted from the paper [7] using the relation $Im\chi = sign\omega Im\chi_R$, where the retarded function χ_R was found in [7]. It turnes out that

$$\mathrm{Im}\chi(2k_F + p, \omega) \propto \theta(\omega^2 - c^2p^2)(\omega^2 - c^2p^2)^{-g},$$

where c is the renormalized Fermi velocity, and $g = 1 - \sqrt{(1-\alpha)/(1+\alpha)}$. It means that the density of states of these electron-hole excitations is proportional to ω^{1-2g}, but not to ω, as in the usual case [4]. Thus, the backscattering contributes to the overlap integral an additional factor:

$$\log|\langle 0|V\rangle_{b.s.}| = -\gamma|V(2k_F)/2\pi v_F|^2(\epsilon\tau)^{2g}, \qquad (4)$$

where the constant $\gamma = 2\sqrt{\pi}(c/r)^{2g}\Gamma(3/2-g)/\Gamma^2(1-g)$, r being the range of the potential $U(k)$.

For the repulsive interaction ($g > 0$) the Orthogonality Catastrophe not only survives, but is even enhanced. The type of the singularity changes from logarithmic to a power one. In previous studies such a change was not obtained [8]; it is a special feature of internal correlations in one-dimensional Fermi system and it is caused by the enhancement of the density of states of electron-hole excitations near $\pm 2k_F$ (Fig.2). Note, that it is just the backscattering, that is crucial for the problem. On the contrary, for attraction ($g < 0$) the backscattering das not contribute to the Orthogonality Catastrophe at all and it remains logarithmic due to the forward scattering processes(eq(2)). Strictly speaking, the eq(4), being a perturbative result, makes sence as an intermediate asymptotic and it is a very difficult and amusing mathematical problem to compute the true limit $\tau \to \infty$ [9]. It is noteworthy, that the function Imχ is connected by the Kramers-Kronig relation with Reχ and, therefore, the anomalous Orthogonality Catastrophe(eq(4)) is "dual" to the anomalous Peierls susceptibility Re$\chi(2k_F,\omega) \propto \omega^{-2g}$, found in [7].

In application of the Orthogonality Catastrophe to the problem of motion of a core hole the fermionic system feels the change of the potential: $\Delta V = V(x) - V(x-a)$ (the hole tunnels from the point $x=0$ to the point $x=a$). The zero Fourier component of ΔV obviously vanishes, therefore, only the backscattering is responsible for the orthogonality effect and the overlap integral is:

$$\log|\langle 0|V\rangle| = -\gamma|V(2k_F)/2\pi v_F|^2 4\sin^2(k_F a)(\epsilon\tau)^{2g}.$$

It is interesting, that for attraction ($g < 0$) the core hole do not feel the orthogonality at all. For repulsion ($g > 0$) the orthogonality effect is generally very strong, but it disappears for $a = \pi n/k_F$. It looks like the core hole preferes to delocalize within some self-consistent "Peierls drop". This problem will be discussed in more details elsewhere [9].

The unusual power type of the singularity in the overlap integral, obtained above, requires reexamination of all connected phenomena: the X-ray response, the motion of a core hole and Kondo effect [9].

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