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QCD OBSERVABLES IN (e,e'p) SCATTERING ON NUCLEI IN THE COLOUR TRANSPARENCY REGIME

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I demonstrate that a transverse size of the ejectile state emerging from the quasielastic $e+p$ scattering on nuclei does not decrease with the momentum transfer Q. None the less, a strength of nuclear attenuation vanishes with Q, and I identify the QCD mechanism of vanishing attenuation (colour transparency). Strong correlation between the onset of colour transparency regime and the asymptotic behaviour of charge form factors is found. Colour transparency regime is elusive up to very large Q.

The quasielatsic $(e,e'p)$ scattering on nuclei is a much discussed candidate reaction for observation of colour transparency (CT) - vanishing final state interaction (FSI) at large momentum transfer Q. The standard treatment of this reaction focuses on the assertion [1,2] that the ejectile state is dominated by small size, $\rho \sim 1/Q$, quark configurations, which have small interaction cross section $\sigma(\rho) \propto \rho^2$ [3,4] and, henceforth, weak FSI.

The subject of this paper is an approach to CT regime. Firstly, I demonstrate that contrary to the reasoning of [1,2] the *true transverse size* of the ejectile state

$$|E\rangle = J_{em}|p\rangle = \sum_i |i\rangle\langle i|J_{em}|p\rangle = \sum_i F_{ip}(Q)|i\rangle\,, \tag{1}$$

defined as $\rho_E^2 = \langle E|\rho^2|E\rangle/\langle E|E\rangle$ does not decrease with Q^2. Secondly, I introduce the QCD observable $\Sigma_{ep} = \langle p|\hat{\sigma}|E\rangle/\langle p|E\rangle$ which quantifies the asymptotic strength of FSI, where $\hat{\sigma}$ is the diffraction scattering (cross section) operator. I show that although $\rho_E^2 = \mathrm{const}(Q)$, the strength of FSI Σ_{ep} does none the less vanish at large Q, and I identify the QCD mechanism of vanishing Σ_{ep}.

This observable Σ_{ep} controls an approach of the nuclear transmission coefficient (nuclear transparency) Tr_A to the CT regime of $\mathrm{Tr}_A = 1$:

$$\mathrm{Tr}_A = \frac{d\sigma_A}{A d\sigma_N} \approx 1 - \Sigma_{ep}\frac{1}{2A}\int d^2\mathrm{b}T(b)^2\,. \tag{2}$$

Here b is the impact parameter, $T(b) = \int dz n_A(\mathrm{b},z)$ is the optical thickness of the nucleus, $\int d^2\mathrm{b}T(b)^2 \propto A^2/R_A^2 \propto A^{4/3}$. Expansion (2) is valid at $1-\mathrm{Tr}_A \ll 1$, when FSI is weak and only the single rescattering of the ejectile is important. In generic case Tr_A contains the n-fold rescattering terms $\propto \langle p|\hat{\sigma}^n|E\rangle$ as well.

In order to elucidate which aspect of QCD is tested by measuring Σ_{ep}, I start with the electron-pion scattering in the non-relativistic quark model (NRQM), where (here $\mathrm{r} = (\vec{\rho}, z)$, the $\vec{\rho}$-plane is normal to the momentum transfer Q)

$$F_{em}(\mathrm{Q}) = \int d^2\vec{\rho}\int dz|\Psi(z,\vec{\rho})|^2 \exp\left(-\frac{i}{2}Qz\right) = \int \frac{d^3\mathrm{k}}{(2\pi)^3}\varphi^*(\mathrm{k}+\frac{1}{2}\mathrm{Q})\varphi(\mathrm{k})\,. \tag{3}$$

The proof of $\rho_E^2 = \text{const}(Q)$ is straightforward: In the NRQM $|E\rangle = \exp\left(\frac{i}{2}Qz\right)|p\rangle$ and $\langle E|E\rangle = 1$. Since the interaction cross section $\sigma(\rho)$ and ρ^2 do not depend on z, one readily finds $\langle E|\rho^2|E\rangle = \langle p|\rho^2|p\rangle$. Recall that by the nature of CT experiments the electroproduced ejectile state $|E\rangle$ is probed by its intranuclear FSI at very short proper time scales when the intrinsic motion of quarks in nucleons can be neglected, i.e., the initial transverse separation of quarks $\vec{\rho}$ is obviously retained (notice a close similarity to Migdal's celebrated shake-up approximation introduced in 1939 [5]). Consequently, predictions [6,7] of CT effects under the assumption of ejectile of vanishing initial size are quite erroneous.

Since $\hat{\sigma} \propto \rho^2$, the alternate measure of the strength of FSI is $\langle \rho^2 \rangle = \langle p|\rho^2|E\rangle / \langle p|E\rangle$ which should not be confused with ρ_E^2 (the subsequent presentation follows, and supersedes, the authors preprint [8])

$$\begin{aligned}\langle \rho^2 \rangle &= \frac{1}{F_{em}(Q)} \int d^2\vec{\rho}\rho^2 \int dz |\Psi(z,\vec{\rho})|^2 \exp(-i\frac{1}{2}Qz) \\ &= \frac{1}{F_{em}(Q)} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\partial_{\mathbf{k}_\perp} \varphi^*(\mathbf{k} + \frac{1}{2}\mathbf{Q})\right] \left[\partial_{\mathbf{k}_\perp} \varphi(\mathbf{k})\right] . \end{aligned} \tag{4}$$

In the relativistic case of $Q^2 \gg m^2$ one must use the Drell-Yan light-cone wave functions (LCWF). Let $p_z \to \infty$ and let $\mathbf{Q}$ be along the y-axis. Then, ρ_x is the transverse size which does not change under the Lorentz transormation from the light-cone frame in which the momentum transfer $\mathbf{Q}$ is purely transversal to the laboratory frame in which $\mathbf{Q}$ is purely longitudinal, so that one must compute $\langle \rho^2 \rangle = 2\langle \rho_x^2 \rangle = 2\langle \partial_{k_x}^2 \rangle$:

$$F_{em}(Q) = \int \frac{d^2\mathbf{k}}{2\pi} \frac{dx}{x(1-x)} \varphi^*(M_f^2)\varphi(M_{in}^2) = \int_0^1 dx \int d^2\vec{\rho} \exp[-i(1-x)\vec{\rho}\mathbf{Q}] |\Psi(x,\vec{\rho})|^2 \tag{5}$$

$$\langle \rho_x^2 \rangle = \frac{1}{F_{em}(Q)} \int \frac{d^2\mathbf{k}}{2\pi} \frac{dx}{x(1-x)} \Psi^{*\prime}(M_f^2)\Psi'(M_{in}^2) \left[\frac{2k_x}{x(1-x)}\right]^2 \tag{6}$$

where x is a fraction of the (light-cone) momentum of the pion carried by the struck quark, and in (5) the invariant variables [9] of LCWF equal $M_{in}^2 = (m_q^2 + \mathbf{k}^2)/x(1-x)$ and $M_f^2 = [m_q^2 + (\mathbf{k} + (1-x)\mathbf{Q})^2]/x(1-x)$. Evidently, the ρ_E^2 retention property holds in the relativistic case too.

Consider now the Q dependence of the strength of FSI. With the Gaussian Ansatz for the NRQM wave function the $d^2\vec{\rho}$ and dz integrations in eqs.(3),(4) do factorize, so that $\langle \rho^2 \rangle = R^2$ and does not depend on Q^2. With the Gaussian LCWF $\varphi(M^2) = \varphi_0 \exp(-\frac{1}{2}R^2M^2)$ the form factor (5) is dominated by the end-point contribution from

$$1 - x \sim 2m_q/Q \tag{7}$$

and from (6) one finds $\langle \rho^2 \rangle \sim \langle R^2/(1-x) \rangle \propto R^2 Q/m_q$. The striking observation is that the CT law $\sigma(\rho) \propto \rho^2$ does not guarantee vanishing FSI, and neither Q independent nor rising with Q strength of FSI contradicts any general principles. Indeed, in the expansion over the intermediate states

$$\langle p|\hat{\sigma}|E\rangle = \sum_i \langle p|\hat{\sigma}|i\rangle\langle i|J_{em}(Q)|p\rangle \tag{8}$$

the matrix elements of the diffractive operator $\langle p|\hat{\sigma}|i\rangle$ do not depend on energy and/or Q^2. Furthermore, the larger is Q^2 the heavier are the electroproduced intermediate states $|i\rangle$, which have the ever growing radius and interaction cross section.

Now I shall demonstrate that, none the less, QCD as a theory of strong interactions *predicts* that Σ_{ep} *vanishes* at large Q^2. Besides the law $\sigma(\rho) \propto \rho^2$, the origin of vanishing Σ_{ep} is the one-gluon exchange, Coulomb, interaction between the constituent (anti)quarks of the hadron, which is an indispensable property of QCD. I shall illustrate the basic idea of the proof starting with the Schrödinger equation in the momentum-representation:

$$\varphi(\mathbf{Q}) = \frac{1}{\varepsilon - \mathbf{Q}^2/2m} \int \frac{d^3\mathbf{k}}{(2\pi)^3} V(\mathbf{Q}-\mathbf{k})\varphi(\mathbf{k}). \quad (9)$$

The confining potential is the dominant one, and the short-range Coulomb interaction can be treated as weak perturbation. Then, the dominant (confining) part of $\varphi(\mathbf{k})$ is a steep function of k^2. On the other hand, the asymptotics of the $V(\mathbf{Q}-\mathbf{k})$ at large $\mathbf{Q}$ is dominated by the QCD one-gluon exchange: $V(\mathbf{Q}-\mathbf{k}) \propto 1/(\mathbf{Q}-\mathbf{k})^2$ (the logarithmic $\alpha_S(Q)$ factor is not significant for our purposes). As a result, asymptotically $\varphi(\mathbf{Q}) \propto 1/Q^4$, and with this Coulomb tail the asymptotics of the form factor (FF) will be dominated by contributions when one of the wave functions in eqs.(3,4) will be in the Gaussian-like (confining) regime and the second in Coulomb tail regime. Then, in (3) I can neglect $\mathbf{k}$ compared to $\mathbf{Q}$ in $\varphi(\mathbf{k}+\frac{1}{2}\mathbf{Q})$, factor out $\varphi(\frac{1}{2}\mathbf{Q})$, and will find the "QCD power-law" asymptotics $F_{em}(\mathbf{Q}) \propto \varphi(\frac{1}{2}\mathbf{Q}) \propto 1/Q^4$ (in the above analysis I have followed [10], in the relativistic case $V(\mathbf{Q})$ should be substituted for the relativistic $\sim 180^0$ Coulomb scattering amplitude, which leads to $\varphi(M^2) \propto 1/M^2$). In the same "QCD dominated" regime, the $\partial_{\mathbf{k}_\perp}$ differentiations in (4) lead to the extra factor $\sim R^2\mathbf{k}^2/\mathbf{Q}^2$ in the integrand, and I indeed find $\langle\rho^2\rangle \propto 1/Q^2$. Hence the major conclusion: both vanishing strength of FSI in quasielastic $(e,e'p)$ scattering and power-law asymptotics of the electromagnetic FF [10-12] originate from exactly the same short-distance QCD interaction in hadrons. In terms of expansions (1) and (8), the origin of vanishing FSI is not in a vanishing size ρ_E of the ejectile, rather weak FSI emerges after projecting the ejectile state onto the final state proton, and comes from cancellattions between the diagonal, $|i\rangle = |p\rangle$, and off-diagonal, $|i\rangle \neq |p\rangle$, intranuclear rescatterings (see also the CT sum rule [13]).

The most important effect of the one-gluon exchange QCD interaction in the relativistic case is that it eliminates a dominance of the end-point, eq.(7), contribution to the FF. The FF will be dominated by finite, weakly Q dependent, values of $1-x$. Nevertheless, a presence of extra $x^2(1-x)^2$ in the denominator of the integrand of (6) enhances a sensitivity of a strength of FSI to the end-point contribution. Consequently, the onset of vanishing Σ_{ep} is slower than the onset of the power asymptotics of the FF.

In Fig.1 I present the results from the light-cone toy model of the scalar pion with the scalar quarks, which incorporates the principal features of the QCD wave function:

$$\varphi(M^2) \propto \exp(-\frac{1}{2}R^2M^2) + \alpha\frac{1}{(1+R^2M^2/2)^n}. \quad (10)$$

For our purposes, the presence of $\alpha_S(M^2)^n$ factor in the 'Coulomb' correction is not essential. To the first order in the 'Coulomb' correction the $\propto 1/Q^2$ behaviour of the pion FF corresponds to $n = 1$.

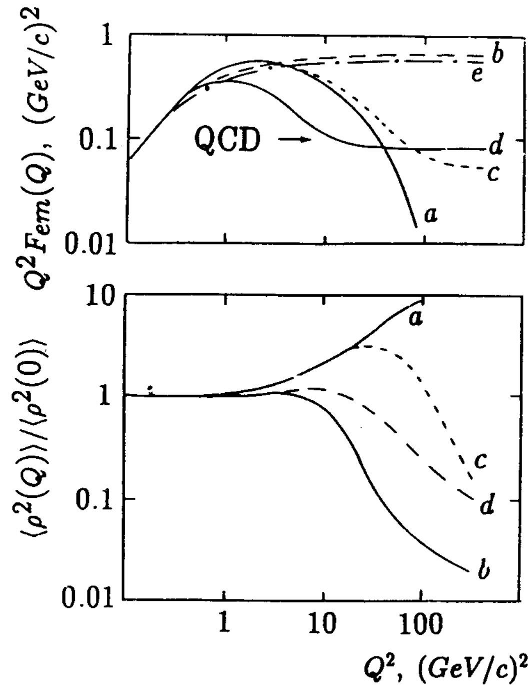

Рис.1. Correlation of the asymptotic behaviour of the electromagnetic form factor with the strength of final state interaction: top - $Q^2F_{em}(Q)$, bottom - the strength of final state interaction measured by $\langle\rho^2(Q)\rangle/\langle\rho^2(0)\rangle$ vs. Q^2. The order of magnitude of the asymptotic normalization suggested by perturbative QCD is indicated by the arrow. The curves correspond to ($m_q = 150\,(MeV/c)^2$):

(a) - the Gaussian wave function;
(b) - the Gaussian+Coulomb wave function, $m = 150\,(MeV/c)^2$, $\alpha = 0.1$;
(c) - the Gaussian+Coulomb wave function, $m = 150\,(MeV/c)^2$, $\alpha = 0.01$;
(d) - the Gaussian+Coulomb wave function, $m_q = 300\,(MeV/c)^2$, $\alpha = 0.01$;
(e) - the monopole form factor.

Experimentally, the pion FF follows the ρ^0-pole formula $F_{em}(Q) = 1/(1+Q^2/m_\rho^2)$ up to $Q^2 \sim 10\,(\mathrm{GeV/c})^2$ [14]. Therefore, in all cases I adjust R^2 to $\langle R_{ch}^2\rangle = 6/m_\rho^2$. The Coulomb-admixture parameter α controls the large-Q normalization $\Lambda^2 = Q^2F_\pi(Q)$, changing from $\Lambda^2 \to 0$ for the Gaussian LCWF, to $\Lambda^2 \approx m_\rho^2$ for the ρ^0-dominated monopole FF (Since neither the running QCD coupling, nor the the higher order QCD effects like the Sudakov FF [15], see below, are included, our toy model should not be extended up to very large Q^2). As a case in between I consider LCWF's giving $\Lambda^2 \sim 8\pi\alpha_S f_\pi^2 \sim 0.1(GeV/c)^2$, as suggested by the perturbative QCD [12]. In the problem of interest it is natural to use the effective mass of quarks m_q somewhere in between the spectroscopic constituent mass of $\sim 300\,MeV/c^2$ and the current mass $\sim 10\,MeV/c^2$. Here I use $m_q = 150\,MeV/c^2$.

The principal findings are:

1. With the purely Gaussian LCWF, $\alpha = 0$, $R = 1.90\,(GeV/c)^{-1}$, the FF $F_{em}(Q) \propto \exp(-R^2m_qQ)/Q^3$ follows closely the monopole FF up to $Q^2 \sim 5\,(GeV/c)^2$ and gives rising strength of FSI.

2. With $\alpha = 0.1$, $R = 2.23(GeV/c)^{-1}$ one reproduces the monopole FF. Notice,

that even in this case $\alpha \ll 1$. The strength of FSI stays approximately constant up to $Q^2 \sim 10-20(GeV/c)^2$, and then decreases rapidly with Q^2.

3. The FF with $\alpha = 0.01$, $R = 1.95\,(GeV/c)^{-1}$ follows the monopole FF up to $Q^2 \sim 10\,(GeV/c)^2$ and has the large-Q normalization close to the QCD prediction. The strength of FSI first increases up to a saturation at $Q^2 \sim 30-40\,(GeV/c)^2$ and steeply decreases beyond $\sim 100\,(GeV/c)^2$.

4. With the choice of $m_q = 300MeV/c^2$ ($\alpha = 0.01$ and $R = 2.72\,(GeV/c)^{-1}$) the strength of FSI stays approximately constant up to $Q^2 \sim 10\,(GeV/c)^2$ and then starts decreasing. However, the large-Q normalization of this FF drops to the perturbative QCD value too rapidly, in conflict with the experiment [14].

I conclude that the steeper is a decrease of the FF and the smaller is the large-Q normalization, the slower is an onset of CT regime. With realistic LCWF models FSI might remain strong up to very large Q^2. An interesting observation is that strength of FSI is very sensitive to the quark structure of the hadron. The QCD asymptotics $\Sigma_{ep} \propto 1/Q^2$ should be even more elusive in the electron-proton scattering [16].

Besides the short distance one-gluon exchange interaction between (anti)quarks, there is still another QCD mechanism which filters the small-size components of hadrons in ep scattering: the Sudakov FF [15]. Asking for the elastic scattering one asks for no-radiation of gluons. For the quark-antiquark system of size $\vec{\rho}$, hit by the electron with the momentum transfer Q, a mean number of the would-be radiated gluons N_g, which would have taken a fraction x_g of the hadrons momentum $x_{min} < x_g < 1$, equals [17]

$$N_g(\rho, Q^2, x_{min}) \approx \frac{16}{3}\int_{x_{min}}^{1} \frac{dx}{x}\int_0^{Q^2} \frac{d^2\mathbf{k}}{2\pi}\frac{\alpha_S(\mathbf{k}^2)}{2\pi}\frac{1-\exp(-i\mathbf{k}\vec{\rho})}{\mathbf{k}^2}. \tag{11}$$

Then, the probability of no-radiation, alias the Sudakov FF, can be estimated as (for the more rigorous derivation see [18]) $F_S(\rho, Q, x_{min}) \approx \exp\left[-N_g(\rho, Q^2, x_{min})\right]$ and the Sudakov-modified charge FF (5) takes the form

$$F_{em}(Q) = \int_0^1 dx \int d^2\vec{\rho}\exp[-i(1-x)\vec{\rho}\mathbf{Q}]|\Psi(x,\vec{\rho})|^2 F_S(\rho, Q, 1-x) \tag{12}$$

The relative significance of the Sudakov suppression of the large ρ contribution depends on the LCWF. With the Gaussian LCWF we have the end-point eq.(7) dominance, so that $2\log(1/x_{min}) = \log(Q^2/m^2)$ and the Sudakov FF takes on the standard form

$$F_S(\rho, Q, x_{min}) \approx \exp\left\{-\frac{8}{27}\log\left(\frac{Q^2}{m_q^2}\right)\log\left[\frac{\alpha_S(1/\rho^2)}{\alpha_S(Q^2)}\right]\right\} \tag{13}$$

Eq.(13) holds before the onset of Coulomb dominance regime, when x_{min} will become small but *finite* and will very weakly depend on Q. In this case the Sudakov suprression will be a very slow function of Q and the size ρ and its effect on the rate of vanishing of Σ_{ep} will be marginal (for the recent discussion of the Sudakov effects see also [19]).

Conclusions: I have shown that the onset of weak FSI is closely related to the onset of the power-law asymptotics of electromagnetic FF. The smaller is the large-Q normalization $Q^2F_{em}(Q^2)$, the larger Q^2 is needed for the onset of colour transparency. The end-point contribution is more significant in a strength of FSI than in the FF. With realistic LCWF's I find very late onset of the CT regime, which is consistent with the preliminary findings from the NE-18 experiment at SLAC [20].

Above I have focused on asymptotic Q^2, assuming always a complete set of intermediate excited states $|i\rangle$. At moderately large Q^2 there is still another reason for the elusive colour transparency regime: only finite number of intermediate states can be electroproduced and can contribute to the expansion (8) at finite energy [13].

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