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VORTEX MOTION IN FERMI-SUPERFLUIDS AND CALLAN-HARVEY EFFECT

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The spectral flow along the anomalous branches of the fermions, localized in the core of quantized vortex in fermi superfluids, defines at low temperature the parameter D' which characterizes the reactive force between the vortex and the system of normal fermions. This reversible momentum exchange between coherent condensate motion in three dimensions and the one-dimensional motion of the localized fermions is equivalent to the Callan-Harvey process of the anomaly cancelation. The number of the anomaloud branches is related to the vortex winding number.

1. *Introduction.* In axially symmetric quantized vortices in fermi-superfluids the energy levels $E(n_r, n_l, k_z)$ of the fermionic quasiparticles, localized in the vicinity of the vortex core, are characterized by the quantum numbers appropriate for the axial symmetry: linear momentum along the vortex axis, k_z; orbital angular momentum n_l and radial quantum number n_r. The distance between the levels with different n_r, and that between the levels with different n_l at given n_r are characterized by two energy scales, ω_r and ω_l correspondingly. In the singular vortex, ω_r is of order of the gap parameter $\Delta \sim T_c$, while $\omega_l \sim \Delta^2/E_F \ll \Delta$ [1]. The vortex dynamics and thermodynamics are essentially different in two regions of low temperatures: $\Delta \gg T \gg \omega_l$ and $T < \omega_l$.

We consider here the first region, which contains the anomalous (chiral) branches of the localized fermions, corresponding to $n_r = 0$. The spectrum $E(n_r = 0, n_l, k_z)$ forms the band with nearly equidistant levels, $E(n_r = 0, n_l, k_z) \approx \omega_l(k_z)n_l$, which as a function of (discrete) n_l crosses the zero energy and thus produces the finite density of states $N(\omega)$ if $\omega \gg \omega_l$. We show that in this region there are some general topological features in the vortex dynamics produced by the effect of spectral flow. This effect is defined by the winding number of the vortex and does not depend on behavior of the spectrum in the region below ω_l, where depending on the vortex core structure the spectrum $E(n_r, n_l, k_z)$ may or may not cross zero as a function of k_z [2,3].

Further we assume that either ω or the energy level width τ^{-1} is larger than the distance ω_l between the n_l levels, so the energy spectrum can be considered as continuous function of n_l. The main result is that the spectral flow along the chiral branch of n_l defines the reactive parameter D', which enters the force balance for the moving vortex:

$$\rho_s(\vec{v}_s - \vec{v}_L) \times \vec{\kappa} + D(\vec{v}_n - \vec{v}_L) - D'(\vec{v}_n - \vec{v}_L) \times \vec{\kappa} = 0 \ . \qquad (1.1)$$

Here $\vec{\kappa}$ is the circulation vector, $\kappa = n\pi\hbar/m$, where n is the vortex winding number (number of circulation quanta) and m is the bare mass of fermion.

The first term is the conventional Magnus force which arises when the velocity $\vec{v}_L$ of the vortex is different from the superfluid velocity $\vec{v}_s$; ρ_s is the superfluid density far from the vortex, which is close to the total density ρ since $T \ll T_c$. This force comes from the flux of the linear momentum from the vortex to infinity.

The second term describes the friction force acting on the vortex from the normal component, while the last term is the reactive force which arises when $\vec{v}_L$ deviates from the normal velocity $\vec{v}_n$. The parameter D' is naturally considered as proportional to the normal density ρ_n. However as was shown in Ref.4 for singular vortex, while ρ_n disappears at $T \ll T_c$, the parameter D' does not: it approaches the finite value, close to ρ providing that either $\omega \gg \omega_l$ or $\tau^{-1} \gg \omega_l$. The same result has been obtained for the continuous vortices in the superfluid ^{3}He-A [5,6]. Here we want to stress that for all the vortices, singular or continuous, the parameter D' at $\omega_l \ll T \ll T_c$ is defined by the same mechanism of the momentum transfer due to the level flow.

The process of the momentum transfer from the superfluid vacuum to the normal motion of fermions within the core is the realization of the Callan-Harvey effect [8] for vortices in condensed matter: the anomalies - nonconservation of linear momentum both in the one-domensional world of the vortex core fermions and in the three-domensional Bose-condensate outside the vortex core - compensate each other. This is the same kind of the Callan-Harvey effect which has been discussed earlier for the motion of arbitrary textures in ^{3}He-A [9,10]. ^{3}He-A is however very specific superfluid, since due to its internal topology it always contains the gap nodes in the spectrum [7]. The nodes lead to the momentum nonconservation, if one considers the superfluid condensate motion alone. This is the result of the transfer of the momentum to the normal fermionic system due to the level flow through the gap nodes. As distinct from ^{3}He-A, where the gap nodes are always present, the Callan-Harvey effect for vortices occurs in any fermi-superfluid: the anomalous fermionic n_l branch, which mediates the momentum exchange, always appears in the singular or continuous core, due to nontrivial topology of the quantized vortex.

2. *Anomalous branch of localized fermions.* Since the result does not depend on details and is completely defined by topology, we consider here the simplest and well known case of axisymmetric singular vortex in superfluid or superconductor with s-wave pairing. The orbital number n_l is considered as the continuous variable, so one can use the quasiclassical approximation for the fermions localized in the vortex core. The Bogoliubov Hamiltonian for the fermions with given spin projection is 2×2 matrix

$$\mathrm{H} = \hat{\tau}_3 \vec{q} \cdot (-i\vec{\nabla})/m + \hat{\tau}_1 Re\Delta(\vec{r}) - \hat{\tau}_2 Im\Delta(\vec{r}) \ . \qquad (2.1)$$

Here $\vec{q}$ is the quasiparticle momentum in the transverse plane, $\Delta(\vec{r}) = e^{in\phi}|\Delta(r)|$ is the gap function in the axisymmetric vortex with winding number n.

The quantum numbers, which characterize the fermonic levels in this approximation, are (i) the magnitude of transverse momentum of quasiparticle q, which is related to the longitudinal projection of momentum $q^2 = k_F^2 - k_z^2$, (ii) the radial quantum number n_r and (iii) the continuous impact parameter $\rho = \hat{z} \cdot (\vec{r} \times \vec{q})/q$. It is related to the angular momentum $\hbar n_l$ as $\hbar n_l = q\rho$. Introducing the coordinate $x = \vec{r} \cdot \vec{q}/q$ along $\vec{q}$, such that $r^2 = \rho^2 + x^2$, and assuming that in the important regions one has $|\rho| \ll |x|$, one obtains the dependence of the gap function in the singly-quantized vortex ($n = 1$) on x and ρ:

$$\Delta(\vec{r}) \approx |\Delta(|x|)|(sign(x) - i\frac{\rho}{|x|}) \quad , \tag{2.2}$$

and the Hamiltonian:

$$\mathbf{H} = \mathbf{H}^{(0)} + \mathbf{H}^{(1)} \ , \quad \mathbf{H}^{(0)} = -i\hat{\tau}_3 \frac{q}{m}\nabla_x + \hat{\tau}_1|\Delta(|x|)|sign(x) \ , \quad \mathbf{H}^{(1)} = \hat{\tau}_2\rho\frac{|\Delta(|x|)|}{|x|} \ . \tag{2.3}$$

The Hamiltonian $\mathbf{H}^{(0)}$ is supersymmetric and has zero eigen value with the eigen function:

$$\Psi^{(0)} \propto (1 - \hat{\tau}_2)\exp -\frac{m}{q}\int_0^{|x|} dy \ |\Delta(y)| \ . \tag{2.4}$$

Using the first order in perturbation $\mathbf{H}^{(1)}$ one obtains the lowest energy levels:

$$E(n_r = 0, n_l, k_z) \approx < 0|\mathbf{H}^{(1)}|0 > = -\rho < \frac{|\Delta(|x|)|}{|x|} > = -n_l\omega_l(q) \quad ,$$

$$\omega_l(q) = \frac{1}{q}\frac{\int_0^\infty dx|\Psi^{(0)}(x)|^2\frac{|\Delta(x)|}{x}}{\int_0^\infty dx|\Psi^{(0)}(x)|^2} \quad . \tag{2.5}$$

This is the anomalous branch of the low-energy localized fermions obtained in Ref. [1]. If the energy spectrum is considered as continuous function of n_l, this anomalous branch crosses zero at $n_l = 0$. It is shown in Sec. 4. that the number of such anomalous branches, $N_{\rm zm}$, is completely defined by the number n of circulation quanta ($N_{\rm zm} = 2n$; for $n = 1$ two branches correspond to two spin projections). The similar relation between the number of fermionic zero modes in the core of the string and the string winding number n takes place in the relativistic field theories [11,12]. The difference is that in the core of the string the zero modes are exact, while in the condensed matter vortices they are approximate on the scale $\omega \gg \omega_l$.

3. *Spectral flow and mutual friction parameter D'.* Now let us consider the vortex moving with the velocity $\vec{v}_L$ relative to the heat bath. In this case the coordinate $\vec{r}$ is replaced by the $\vec{r} - \vec{v}_L t$ and the parameter ρ which enters the quasiparticle energy in Eq.(2.5) is also shifted with time:

$$E(n_r = 0, n_l, k_z, t) = -(\rho - \frac{\epsilon(\vec{q})}{q}t)q\omega_l(q) = -(n_l - \epsilon(\vec{q})t) \ \omega_l(q) \ , \tag{3.1}$$

where $\epsilon(\vec{q}) = \hat{z}\cdot(\vec{v}_L \times \vec{q})$ acts on fermions localized in the core in the similar way as the electric field acts on the fermions localized on string in the relativistic quantum theory. Under this field the number of the fermionic levels, which cross zero per unit time, is

$$\partial_t n_l = \epsilon(\vec{q}) = \hat{z}\cdot(\vec{v}_L \times \vec{q}) \ . \tag{3.2}$$

The rate of the quasiparticle momentum transferred from the vacuum (from the levels below zero) along the anomalous branch is thus

$$\partial_t\vec{P} = \sum \vec{q} \ \partial_t n_l(\vec{q}) = \frac{1}{2}N_{\rm zm}\int_{-k_F}^{k_F}\frac{dk_z}{2\pi}\int_0^{2\pi}\frac{d\phi}{2\pi} \ \vec{q}\epsilon(\vec{q}) = \pi n\frac{k_F^3}{3\pi^2}\hat{z}\times\vec{v}_L. \tag{3.3}$$

The factor $\frac{1}{2}$ compensates the double counting of particles and holes. This gives the D' parameter in the force acting from the normal component if the vortex moves with respect to the normal heat bath:

$$D' = m\frac{k_F^3}{3\pi^2} \ . \tag{3.4}$$

Here it is implied that all the quasiparticles, created from the negative levels of the vacuum state, finally become the part of the normal component, i.e. there is nearly reversible transfer of the linear momentum from fermions to the heat bath. This should be valid in the limit of large scattering rate: $\omega_l\tau \ll 1$. The small retardation in this process leads to the effective friction force $D \propto \omega_l\tau D'$ (see Refs. 3 and 6).

Note also that D', though is very close to the density ρ, nevertheless is not exactly equal to ρ. The mass density equals $m\frac{k_F^3}{3\pi^2}$ only in the normal fermi-liquid, while in superfluids $\rho \neq m\frac{k_F^3}{3\pi^2}$ but close to this value if $\Delta \ll E_F$. For the singular vortex in s-paired superfluids D' corresponds thus to the density at the vortex axis, where the gap disappears: $D' = \rho(r=0)$. In continuous ^{3}He-A vortex, D' coincides with the C_0 parameter, which is responsible for the chiral anomaly, $D' = C_0 = m\frac{k_F^3}{3\pi^2}$ (see Ref.5).

4. *Number of anomalous branches and vortex winding number.* The general relation between N_{zm} and n can be found using the Green's functions (in the spirit of Refs. 2 and 7). Here we consider the simplified derivation, which uses 2×2 Bogoliubov matrix (2.1). The number of the anomalous branches of the spectrum $E(\rho)$, which cross zero as a fuction of ρ, coincides with the number of the topological zeroes of the classical energy $E(\rho, x, p_x)$ [2,7]. The classical limit of Hamiltonian (2.1) is expressed in terms of the vector function $\vec{m}(\vec{s})$ in the 3-d space of parameters $\vec{s} = (\rho, x, p_x)$:

$$\hat{H}_{\rm class}(\vec{s}) = \vec{\tau}\cdot\vec{m}(\vec{s}) \ . \tag{4.1}$$

The components of $\vec{m}(\vec{s})$ are

$$m_3(\vec{s}) = qp_x/m \ , \quad m_1(\vec{s}) = Re\Delta(\rho, x) \ , \quad m_2(\vec{s}) = -Im\Delta(\rho, x) \ . \tag{4.2}$$

The number of zeroes of this vector function (points $\vec{s}_0$ where $\vec{m}(\vec{s}_0) = 0$ and therefore $E(\vec{s}_0) = 0$) and thus the number of anomalous branches is given by the topological invariant[7]:

$$N_{zm} = \frac{1}{8\pi}\int_\sigma dS^i e_{ikl} \mid \vec{m}(\vec{s}) \mid^{-3} \left(\vec{m}\cdot\frac{\partial\vec{m}}{\partial s_k}\times\frac{\partial\vec{m}}{\partial s_l}\right) , \tag{4.3}$$

where the integral is over the closed surface σ about zeroes. For the gap function $\Delta(\vec{r}) = e^{in\phi}|\Delta(r)|$ in the vortex with winding number n one obtains $N_{zm} = n$ which should be multiplied by two if one takes into account two spin projections. So the general relation, which does not depend on the detailed structure of the vortex, is

$$N_{zm} = 2n. \tag{4.4}$$

Note that the quantity N_{zm} changes sign for vortices with negative winding number. This is because N_{zm} is algebraic quantity, since it also shows if $E(n_l)$ increases or decreases upon crossing zero energy.

5. *Conclusion.* The Magnus force acting on the vortex from the superfluid motion results from the reversible flux of the linear momentum from the vortex to infinity. On the contrary the reactive force from the normal component is the consequence of the reversible flux of momentum from the vortex into the region near the axis, i.e. into the core region. Within the core the linear momentum of the vortex transforms to the linear momentum of the fermions when the fermionic levels on anomalous branches cross the chemical potential. This type of the Callan-Harvey effect does not depend on the detailed structure of the vortex core and even on the type of pairing. It is the same for the singular and continuous vortices. The topological result (4.4) for the number of anomalous branches, which as functions of the impact parameter ρ cross zero energy, remains to be valid also for vortices in the p-wave superfluids, e.g. in ^{3}He-A and ^{3}He-B. In the case of singly-quantized vortices in ^{3}He-B two anomalous branches have been obtained by Schopohl[13]. While for the most symmetric vortex they cross zero at $\rho = 0$ like in Eq.(2.5), for the vortex with broken symmetry in the core the crossing occurs at finite ρ. This however does not change the Eq. (3.2) for the spectral flow and the D' value (3.4).

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