VORTEX CORE ANOMALY FROM THE GAPLESS FERMIONS IN THE CORE

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The massless chiral fermions, localized in the core of quantized vortex in fermi superfluids and superconductors, produce the anomaly in the vortex core structure at low temperature. The result obtained by Kramer and Pesch [4] for axisymmetric vortex in s-wave superconductor is generalized to the more complicated vortices.

1. Introduction

The gapless fermions interacting with the Bose fields of the order parameter lead to the anomalous behavior of superfluids and superconductors at low temperature, $T \ll T_c$. In the superfluid ^{3}He-A, where the gapless fermions are chiral, two classes of phenomena take place: 1) the chiral anomaly - nonconservation of the linear momentum of the coherent condensate motion due to the spectral flow; and 2) singularity in the gradient expansion of the order parameter field, which is equivalent to the zero-charge effect in particle physics, since it comes from the logarithmical polarization of fermionic vacuum (see Review [1]).

In conventional superconductors the massless chiral fermions appear in cores of quantized vortices [2]. They also lead to similar phenomena: 1) the spectral flow along the branches of chiral fermions in the vortex core gives rise to the momentum transfer from the superfluid component to the normal one [3]; and 2) singularity in the order parameter field in the vortex core region - an anomalous increase in the slope of the order parameter at $T \ll T_c$ - was analytically found in Ref.[4] and numerically confirmed in Ref.[5]. The slope of the gap $\Delta(r)$ near the origin, $(d\Delta/dr)|_{r=0}$, increases as T_c/T at low T, [4] while the core size remains to be of the order of coherence lehgth ξ [5]. The latter singularity was calculated for particular case of the axisymmetric vortex in the s-wave isotropic superconductor and we want to find out how the singularity is modified in the case of more complicated vortices including vortices in uncoventional superfluids and superconductors, such as superfluid ^{3}He and high-T$_c$ materials.

2. Anomalous branches of localized fermions

The quantized vortices in superfluids and superconductors contain the anomalous branches of the low-energy fermions localized in the vortex core [2]. The energy spectrum of these fermions in the semiclassical approximation is characterized by two quantum numbers: the momentum k_z along the vortex axis and the impact parameter $\tilde{y} = \hat{z} \cdot (\mathbf{q} \times \mathbf{r})/q = r\sin(\alpha - \phi)$ where $\mathbf{q}$ is the projection of $\mathbf{k}$ onto $x - y$ plane with $q^2 = k_F^2 - k_z^2$; α and ϕ are the angles in $x - y$ plane of $\mathbf{q}$ and $\mathbf{r}$ correspondingly.

In the conventional singly-quantized Abrikosov vortex in conventional s-wave superconductor, there are two identical branches corresponding to two spin projections with the spectrum

$$E(k_z, \tilde{y}) = \tilde{y} q \omega_l(q) \quad . \qquad (2.1)$$

In the quantum limit which takes into account the quantization of the angular momentum, $q\tilde{y} = \hbar n_l$, the quantity $\omega_l(q)$ is the distance between the levels with neighbouring n_l. Usually this interlevel distance is of order $\Delta^2(\infty)/E_F \ll \Delta(\infty)$ ($\Delta(\infty)$ is the gap far from the vortex and E_F is the Fermi energy), and we consider the region $T \gg \omega_l$, where this quantization can be neglected. These branches are anomalous since, if they are considered as continuous function of $\tilde{y}$, they cross zero energy level. The crossing occurs at $\tilde{y} = 0$ for all k_z (Fig.1a), and thus the one-dimensional Fermi-surface (Fermi-line) is formed. At low temperature the sharp Fermi-distribution of the chiral fermions in the vicinity of the Fermi-line leads to the anomaly in the vortex core, which is strengthened by the unique situation that the position of the Fermi-line does not depend on k_z.

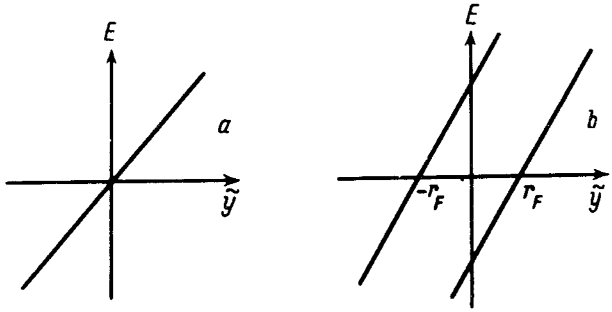

Fig.1. Energy spectrum of fermionic zero modes localized in the vortices in terms of the impact parameter: a – one doubly-degenerate branch of fermions in conventional singly-quantized vortices in s-wave superconductor; b – two branches in doubly quantized vortex or in the ^{3}He-B vortex with broken parity, the Fermi-surface occurs at finite impact parameter $\tilde{y} = \pm r_F$

The number of fermionic zero modes, $N_{\rm zm}$, is completely defined by the topology of the vortex, i.e. by the winding number n of the vortex [3]: $N_{\rm zm} = 2n$. The similar relation between the number of fermionic zero modes in the core of the string and the string winding number n takes place in the relativistic field theories [6,7]. The difference is that in the core of the string the branches cross zero as functions of k_z, while in condensed matter vortices - as functions of $\tilde{y}$.

The fact that the anomalous branch in Eq.(2.1) crosses zero energy at zero impact parameter $\tilde{y}$ is the result of the symmetry of the vortex which requires that $-E(-\tilde{y})$ should be also the branch of the fermionic states in the vortex core. Therefore, if there is only one anomalous branch, then, according to the equation $E(-\tilde{y}) = -E(\tilde{y})$ for this unique branch, the crossing should occur at $\tilde{y} = 0$. The situation is different in the case of vortices with winding number $n = 2, 3$, etc., which contain several branches of zero modes. Several nonidentical branches can occur even for singly-quantized vortices, e.g. in the triplet-paired states, such as ^{3}He-A and ^{3}He-B, where the degeneracy over the spin states is lifted and the modes with different spin projections are not identical.

According to the relation between the winding number and the number of branches [3], in doubly-quantized vortex ($n = 2$) in conventional s-wave superconductors there should be two anomalous branches (each being degenerate over spin). It is evident from the symmetry consideration, that there is no reason for two branches to cross zero at $\tilde{y} = 0$. Instead the equation $E_1(-\tilde{y}) = -E_2(\tilde{y})$ is satisfied if the branches cross zero at two different points $\tilde{y} = \pm r_F(q)$ (Fig.1b), which are symmetric with respect to the origin:

$$E_1(k_z,\tilde{y}) = (\tilde{y} - r_F(q))q\omega_l(q) \quad , \quad E_2(k_z,\tilde{y}) = (\tilde{y} + r_F(q))q\omega_l(q) \quad . \tag{2.2}$$

So the position of the Fermi-surfaces, $\pm r_F(q)$, of zero modes starts to depend on q, which should lead to the smoothing of singularity in the vortex core. In the vortex with winding number $n = 3$ two branches also cross zero at mutually symmetric points $\tilde{y} = \pm r_F(q)$, while the third one ought to be antisymmetric and cross zero at $\tilde{y} = 0$.

In the case of singly-quantized vortices in the triplet-paired ^{3}He-B two anomalous branches (corresponding to two different spin components) have been calculated by Schopohl [8]. While for the most symmetric vortex, o-vortex, the branches cross zero at $\tilde{y} = 0$, for the vortex with broken parity in the core, the v-vortex, the crossing occurs at finite $\tilde{y} = \pm r_F(q)$. The finite value of r_F takes place also in the ^{3}He-A continuous vortices, where the parity is also broken [9]. This r_F depends on the angle in $x - y$ plane if in addition the axial symmetry is broken in the vortex core [8].

3. Order parameter jump in the conventional axisymmetric vortices

Let us consider the influence of the zero modes on the gap function within the vortex core. The gap equation can be found from the BCS action which contains two terms:

$$S = \int d^3r dt \frac{|\Delta|^2}{g} + \mathrm{Tr}\ \ln\ (i\omega - \mathrm{H}) \quad , \tag{3.1}$$

where g is the interaction constant and the second term, with H being the Hamiltonian for fermions in the presence of the vortex, is the contribution of fermions. The trace is over all the fermionic states ν and also over the thermal frequencies. Variation over the gap function, $\delta S/\delta\Delta^* = 0$, gives the self consistent equation for the gap function:

$$\Delta(\mathbf{r}) = g\sum_{\nu} u_\nu(\mathbf{r})v_\nu^*(\mathbf{r})\tanh\frac{E_\nu}{2T} \quad . \tag{3.2}$$

where u_ν and v_ν are the Bogoliubov wave functions.

Let us first consider the conventional vortices with the energy spectrum in Eq.(2.1). The low-energy anomalous branches (zero modes) lead to the singular contribution to the gap function:

$$\Delta_{\mathrm{sing}}(\mathbf{r}) = g\sum_{\mathbf{q}} u_{\mathbf{q}}(\mathbf{r})v_{\mathbf{q}}^*(\mathbf{r})\tanh\frac{rq\omega_l(q)\sin(\alpha-\phi)}{2T}, \tag{3.3}$$

which comes from the fact that at the intermediate asymptote region, $T_c \gg T \gg \omega_l$, the sharp coordinate dependence of the Fermi-function near the fermi-surface of chiral fermions produces the sharp behavior of the order parameter near the vortex axis. This behavior is characterized by new scale

$$\xi_2 = k_F^{-1}\frac{T}{\omega_l} \sim \xi\frac{T}{T_c} \quad . \tag{3.4}$$

At low $T \ll T_c$, when the Fermi-function is narrow and close to the step function, this scale ξ_2 becomes less than ξ and starts to define the properties of the vortex core near the origin. In this limit of low temperature one can use the step function:

$$\frac{rq\omega_l(q)\sin(\alpha-\phi)}{2T} \approx \Theta(r\sin(\alpha-\phi)) = \Theta(r)\Theta(\sin(\alpha-\phi)) \quad , \tag{3.5}$$

and neglect the coordinate dependence of the Bogoliubov functions which have the characteristic length scale of order $\xi \gg \xi_2$:

$$u_{\mathbf{q}}(0)v^*_{\mathbf{q}}(0) \equiv \lambda(q)e^{i\alpha} \sim e^{i\alpha}|\Delta(\infty)| \quad . \tag{3.6}$$

The oscillations on the scale k_F^{-1} and the resulting Friedel oscillations of the gap function in the vortex core become important only at $T \sim \omega_l$ [4,5].

As a result the Fermi-liquid distribution of quasiparticles occupying the gapless branch results in the step-wise behavior of the order parameter with the infinite slope at the origin:

$$\Delta_{\rm sing}(\mathbf{r}) = g\Theta(r)\int_{-k_F}^{k_F}\frac{dk_z}{2\pi}\lambda(q)\int_0^{2\pi}\frac{d\alpha}{2\pi}\; e^{i\alpha}\Theta(\sin(\phi-\alpha)) \quad ,$$

or

$$\Delta_{\rm sing}(\mathbf{r}) = \Theta(r)e^{i\phi}|\Delta_{\rm sing}(\infty)| \quad , \tag{3.7}$$

where the contribution of the zero modes to the gap far from the vortex

$$|\Delta_{\rm sing}(\infty)| = g\frac{2}{\pi}\int_{-k_F}^{k_F}\frac{dk_z}{2\pi}\lambda(q) \quad , \tag{3.8}$$

is of order $g|\Delta(\infty)$. It is small compared with the regular contribution only due to the small coupling constant g, which is of order $1/\ln(E_F/T_c)$. Therefore the gap $\Delta(\vec{r})$ has a jump at the origin from zero value to $\Delta_{\rm sing}(\infty)$, this jump is smoothed out over the distance of order $\xi_2 \ll \xi$ if one takes into account the finite T. Then due to the regular contribution to the order parameter from the fermions with the gap, the order parameter continuously increases from $\Delta_{\rm sing}(\infty)$ until the value $\Delta(\infty)$ at the distance of order of the conventional coherence length ξ (see Fig.2a). This double-scale behavior of the order parameter is in agreement with the numerical calculations [5].

The radial derivative of the order parameter has the δ-functional singularity corresponding to the infinite slope at the origin:

$$\partial_r\Delta_{\rm sing}(\mathbf{r}) = \delta(r)e^{i\phi}|\Delta_{\rm sing}(\infty)| \quad . \tag{3.9}$$

4. Zero modes with Fermi-surface at finite impact parameter

Let us now consider how the result obtained above is modified when the Fermi-surface of zero modes takes place at finite r_F in Eq.(2.2). In the axially symmetric vortex the distance of Fermi-surfaces from the origin, $r_F(q)$, does not depend on ϕ and is of order of coherence length in the ^{3}He-B vortices with broken parity [8]. It should be of the same order in doubly quantized vortices in conventional superconductors.

The contribution to the order parameter from the zero modes contains now two Fermi-steps symmetrically shifted from the origin:

$$\Delta_{\rm sing}(\mathbf{r}) = ge^{i\phi}\int_{-k_F}^{k_F}\frac{dk_z}{2\pi}\;\lambda(q)\;\int_0^{2\pi}\frac{d\alpha}{2\pi}\;\sin\alpha[\Theta(r\sin\alpha - r_F(q)) + \Theta(r\sin\alpha + r_F(q))] =$$

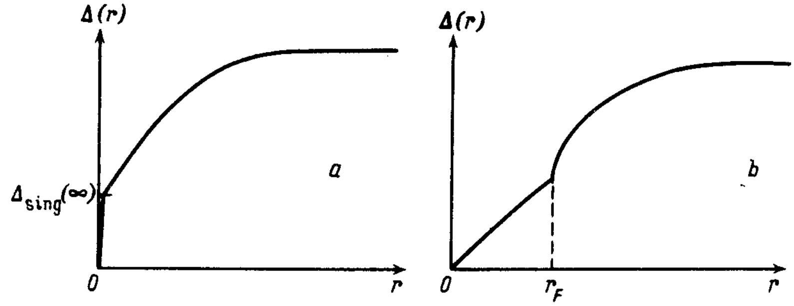

Fig.2. Singularity in the order parameter within the vortex core due to zero modes: *a* – step-wise discontinuity at the vortex axis in conventional singly-quantized vortices in *s*-wave superconductor with zero modes in Fig.1*a*; *b* – square-root singularity at finite distance r_F in 2-dimensional vortices with zero modes behavior in Fig.1*b*

$$= ge^{i\phi} \int_{-k_F}^{k_F} \frac{dk_z}{2\pi} \lambda(q) \sqrt{1 - \frac{r_F^2(q)}{r^2}}\ \Theta(r - r_F(q)) \qquad (4.1)$$

We see that the singularity is essentially smoothed out due to the dependence of the Fermi-surface on k_z (or q). The singularity can survive only for 2-dimensional vortices, i.e. for the vortices in very thin films with dimensional quantization, or in the layered materials, like high-T_c superconductors. In the first case k_z is quantized, i.e. $q = k_F$ is fixed, as a result the Fermi-line of zero modes becomes the Fermi-point, and one has

$$\Delta_{\rm sing}(\mathrm{r}) = e^{i\phi}|\Delta_{\rm sing}(\infty)|\sqrt{1 - \frac{r_F^2}{r^2}}\ \Theta(r - r_F) \quad . \qquad (4.2)$$

This is the square-root singularity with the infinite slope at the Fermi-surface $r = r_F$, which appears on the background of the regular contribution (see Fig.2*b*). At $r_F = 0$ the Eq.(4.2) transforms into Eq.(3.7).

If the axial symmetry is spontaneously broken, like in superfluid ^{3}He vortices, or externally broken in superconductors due to the crystal field, the Fermi-surface depends on ϕ. The anisotropy of the Fermi-surface leads to the smoothing of the square-root singularity even in two-dimensional vortices.

5. Conclusion

The anomalous behavior of the order parameter within the vortex core in the temperature region $T_c \gg T \gg T_c^2/E_F$ results from the Fermi-surface (Fermi-line) formed by the chiral fermions localized in the vortex core. The sharp distribution function of the fermions in the vicinity of the Fermi-surface leads to singularity in the order parameter. In the case of the simplest axisymmetric vortices in s-wave superconductors the Fermi-surface occurs just at the vortex axis, which results in the sharp step-wise distribution of the order parameter near the vortex axis. For the more complicated vortices, in which the Fermi-surfaces of chiral fermions take place far from the vortex axis, the singularity is integrated over the Fermi-surface and is smoothed out. The only exclusion is represented by the 2-dimensional systems (or the quasi-2-dimensional layered superconductors), where the Fermi-line of chiral fermions becomes the Fermi-point. In this case the square-root singularity in the order parameter field takes place far from the vortex axis.

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