THE ANGULAR DISTRIBUTION OF ULTRACOLD NEUTRONS PRODUCES BY SCATTERING COLD NEUTRONS IN SUPERFLUID ^{4}He

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Submitted 19 October 1993

We calculate the angular distribution of ultracold neutrons (UCN) produced by the inelastic scattering of cold (8.9 Å) neutrons in superfluid ^{4}He and show that it is isotropic at zero UCN energy, while at the limiting velocity of UCN (8 m/s), there is a 6% asymmetry in the forward- vs. backward-directed UCN production rate. Although this is a relatively weak effect, it should be included in accurate Monte Carlo simulations of proposed and existing superfluid ^{4}He UCN sources.

The production of ultracold neutrons (UCN) by scattering 8.9 Å neutrons to zero energy by emission of a single excitation in superfluid ^{4}He, as proposed by Golub and Pendlebury [1, 2], has received some experimental attention. The primary features of the theoretical treatment of the system have been verified [3, 4].

One so-far unrecognized feature of the system, as we will theoretically demonstrate, is that the production rate of UCN is not isotropic. Although the effect is only on the order of 10% at the highest UCN energies, it will be important to include this effect in the Monte Carlo studies of existing [5] and proposed UCN sources based on scattering in superfluid ^{4}He ("superthermal" sources) [4].

We consider first the kinematics of UCN production. Incident neutrons with a "critical momentum" (where the free neutron dispersion curve $E = \hbar^2 k_i^2/2m_n$ crosses the Landau–Feynman dispersion curve for the elementary excitations in superfluid ^{4}He) can be inelastically scattered to near-rest by emission of a single phonon. The critical momentum q^* corresponds to a wavelength of ≈ 8.917 Å and an energy of 11 K. Both energy and momentum are conserved in this process:

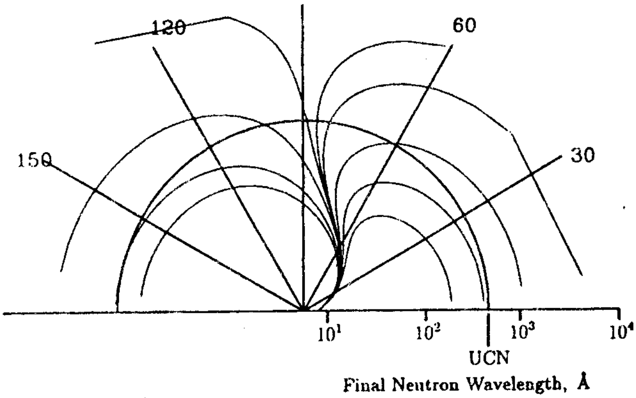
$$\mathbf{q} = \mathbf{k}_i - \mathbf{k}_f, \tag{1}$$

$$\hbar^2 k_i^2/2m = \hbar^2 k_u^2/2m + E(q), \tag{2}$$

where k_i is the incident neutron wavenumber, k_u is the final (UCN) neutron wavenumber, and $E(q)$ is the elementary excitation dispersion relation. For a fixed initial momentum k_i and a given final UCN momentum magnitude k_u, the direction θ of the UCN momentum relative to the incident neutron momentum and q are uniquely determined.

Figure is a result of a numerical calculation of $\lambda_u = 2\pi/|\mathbf{k}_u|$ and θ, given $k_i \approx q^*$ and using $E(q)$ as parameterized by Maris [6]. This plot is to be compared with Fig. 2 of Ref. [7] where k_u as a function of k_i and θ is plotted;

we are interested in the curves for incident wavenumber $k_i \approx 0.68\,\text{Å}^{-1}$ for which $k_u \approx 0$ at $\theta = 90°$. However, in our case, we have plotted λ_u which more readily shows the angular dependence of the kinematically-allowed scattering to near zero final energy. Figure indicates that UCN can be produced at any θ, while the complexity of the kinematics for $k_u \to 0$ simply is not evident on the scale at which Fig. 2. of Ref. [7] is drawn.



Polar plot of kinematically allowed UCN wavelengths produced by down-scattering from a specified incident cold neutron wavelength. The incident wavelengths, from left-to-right near the origin, are as follows: 8.8, 8.85, 8.9, 8.916, 8.920, 8.925, 9.95, 9, 9.1 Å. The semicircle at 500 Å indicates the threshold of UCN production

We next consider the dynamics of UCN production. Following Ref. [7], the cross section for producing neutrons in a given final state is given by the Born approximation as

$$\sigma = \frac{2a^2}{k_i}\int S(\mathrm{q})\delta(k_u^2 - k_i^2 + \frac{2m}{\hbar^2}E(q))d^3k_u \tag{3}$$

where $a \approx 1$ barn is the ^{4}He-n coherent scattering length, $S(\mathrm{q})$ (= $S(q)$ for a liquid) is the structure function, and $d^3k_u = k_u^2 dk_u d\Omega$. We are interested in the production rate to a given angle for a fixed incident neutron momentum. Following Cohen and Feynman [7], Eq. (3) can be integrated over k_u first to give

$$\frac{d\sigma}{d\Omega} = \frac{2a^2}{k_i}S(q)\frac{k_u^2}{|f'(k_u)|}, \tag{4}$$

where

$$f(k_u) = k_u^2 - k_i^2 + \frac{2m}{\hbar^2}E(q), \tag{5}$$

$$f'(k_u) = 2(k_u + \frac{m}{\hbar^2}E'(q)\frac{\partial q}{\partial k_u}) \tag{6}$$

and m is the neutron mass. From Eq. (1) we have

$$q^2 = k_u^2 + k_i^2 - 2k_u k_i \cos\theta \tag{7}$$

which can be differentiated to yield

$$qdq = (k_u - k_i\cos\theta)dk_u \tag{8}$$

or

$$\frac{\partial q}{\partial k_u} = \frac{k_u}{q}(1 - \frac{k_i}{k_u}\cos\theta). \tag{9}$$

We thereby obtain [7]

$$\frac{d\sigma}{d\Omega} = \frac{a^2 k_u S(q)}{k_i \left| \left(1 + \frac{mE'(q)}{\hbar q}(1 - \frac{k_i}{k_u}\cos\theta)\right) \right|} \tag{10}$$

where k_i is determined by $f(k_u) = 0$.

Since the incident neutron beam is an incoherent mixture of plane wave states (specified by a spectral density $d\Phi(k_i)/dk_i$ with units neutrons/$\text{Å}^{-1}\text{s}\cdot\text{cm}^2$), to calculate the differential UCN production rate at a specified k_u within a range dk_u, we must multiply the incident spectral density by the kinematic factor which is, using Eqs. (1) and (2),

$$\left|\frac{\partial k_i}{\partial k_f}\right|_\theta = \frac{|k_u + \frac{mE'(q)}{\hbar^2 q}(k_u - k_i\cos\theta)|}{|-k_i + \frac{mE'(q)}{\hbar^2 q}(k_i - k_u\cos\theta)|}. \tag{11}$$

The differential production rate is thus

$$\frac{d^2P}{d\Omega dk_u} = Na^2 S(q)\frac{d\Phi(k_i)}{dk_i}\left|\frac{\partial k_i}{\partial k_u}\right|_\theta \frac{d\sigma}{d\Omega} = Na^2\frac{d\Phi(k_i)}{dk_i}\frac{k_u^2}{k_i^2}\frac{1}{|-1 + \frac{mE'(q)}{\hbar^2 q}(1 - \frac{k_u}{k_i}\cos\theta)|} \tag{12}$$

where N is the ^{4}He density. The same result is obtained when Eq. (3) is integrated over k_i first.

Since only those incident neutrons in a very narrow range of momentum near q^* are scattered to UCN, $k_i, q \approx q^*$. Defining the group velocity and critical velocity, respectively, as

$$v_g = E'(q^*)/\hbar, \tag{13}$$

$$v_n^* = \hbar q^*/m, \tag{14}$$

and integrating over the azimuthal angle, we find the differential production rate, in the limit $k_u << k_i, q^*$, and keeping all terms first order in k_u/q^*,

$$\frac{d^2P}{d\cos\theta dk_u} \approx 2\pi Na^2 S(q^*)\frac{d\Phi(q^*)}{dk_i}\left(\frac{k_u}{q^*}\right)^2 \frac{v_n^*}{v_n^* - v_g}\left(1 + \left(\frac{k_u}{q^*}\right)\frac{v_n^*}{v_n^* - v_g}\cos\theta\right), \tag{15}$$

where have taken the phonon dispersion as linear and assumed that $S(q)$ and $d\Phi(k_i)/dk_i$ are approximately constant with $k_i, q = q^*$. ¿From the parameterization of the dispersion curve given in Ref. [6], we find

$$\alpha = \frac{v_n^*}{v_n^* - v_g} = 1.437 \tag{16}$$

The integration of Eq. (15) over $\theta = 0$ to π yields the same total UCN production rate as was obtained previously [1, 2].

If we consider Eq. (12) directly without assuming a linear phonon dispersion, we find (numerically)

$$\frac{d^2P}{d\cos\theta dk_u} \approx 2\pi Na^2 S(q^*)\frac{d\Phi(q^*)}{dk_i}(1.438)\left(\frac{k_u}{q^*}\right)^2\left(1 + (1.56)\left(\frac{k_u}{q^*}\right)\cos\theta\right) \tag{17}$$

where the extra angular dependence reflects the nonlinearity of the phonon dispersion relation.

For a UCN of wavelength 500 Å, the fractional difference in the UCN production rate between $\theta = 0°$ and $\theta = 180°$ is approximately 6%. This asymmetry falls off as $1/\lambda_u$, the UCN wavelength.

In conclusion, we have shown that the angular distribution of UCN produced by inelastic scattering in superfluid ^{4}He is not isotropic. Although the effect is relatively small, including such effects in the Monte Carlo modelling of existing and proposed sources is very important, particularly when a high-accuracy comparison between experimental results and theory is desired [8, 9].

SKL was supported by NSF grant 9100719.

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1. R.Golub and J.M.Pendlebury, Phys. Lett. **53A**, 133 (1975).
2. R.Golub and J.M.Pendlebury, Phys. Lett. **62A**, 337 (1977).
3. R.Golub, C.Jewell, P.Ageron et al., Z. Phys. **B51**, 187 (1983). A.I.Kilvington, R.Golub, W.Mampe, and P.Ageron, Phys. Lett. **125A**, 416 (1987).
4. Ultracold Neutrons, R.Golub, D.J.Richardson, and S.K.Lamoreaux, (Adam-Hilger, 1991).
5. H.Yoshiki, K.Sakai, M.Ogura et al., Phys. Rev. Lett. **68**, 1323 (1992).
6. H.Maris, Rev. Mod. Phys. **49**, 341 (1977).
7. M.Cohen and Richard P. Feynman, Phys. Rev. **107**, 13 (1957).
8. R.Golub and S.K.Lamoreaux, Phys. Rev. Lett. **70**, 517 (1993).
9. H.Yoshiki et al., Phys. Rev. Lett. **70**, 518 (1993).