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The anisotropic conductivity of two-dimensional electrons on a half-filled high Landau level

I. S. Burmistrov[1)]

L.D. Landau Institute for Theoretical Physics RAS, 117940 Moscow, Russia

Institute for Theoretical Physics, University of Amsterdam, 1018 XE Amsterdam, The Netherlands

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We study the conductivity of two-dimensional interacting electrons on the half-filled Nth Landau level with $N \gg 1$ in the presence of the quenched disorder. The existence of the unidirectional charge-density wave state at temperature $T < T_c$, where T_c is the transition temperature, leads to the anisotropic conductivity tensor. We find that the leading anisotropic corrections are proportional to $(T_c - T)/T_c$ just below the transition in accordance with the experimental findings. Above T_c the correlations corresponding to the unidirectional charge-density wave state below T_c result in the corrections to the conductivity proportional to $\sqrt{T_c/(T - T_c)}$.

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1. *Introduction.* Two-dimensional electrons in a perpendicular magnetic field was a subject of intensive studies, both theoretical and experimental, for several decades [1, 2]. It has been found that the properties of two-dimensional electrons in the magnetic field are strongly affected by the presence of electron-electron interaction as well as by impurities. The behaviour of the system in a strong magnetic field where only the lowest Landau level is occupied has been investigated in great details [2]. But only several attempts were made to consider the system in a *weak* magnetic field (large number of Landau levels $N \gg 1$ are occupied) where the Coulomb energy at distances of the order of the magnetic length exceeds the cyclotron energy [3].

The progress in understanding the clean two-dimensional electrons in a weak magnetic field was achieved by Aleiner and Glazman who, by using the small parameter $1/N \ll 1$, have derived the successive theory that describes electrons on the partially filled Nth Landau level [4]. By treating the effective electron-electron interaction on the Nth Landau level within the Hartree-Fock approximation, Koulakov, Fogler, and Shklovskii [5] predicted a unidirectional charge-density-wave (UCDW) state (stripe phase) for the half-filled high Landau level at zero temperature and in the absence of disorder. Moessner and Chalker [6] showed the existence of the UCDW state on the half-filled high Landau level without disorder below some temperature T_0. In the presence of disorder the UCDW state on the half-filled high Landau level can exist if the Landau level broadening $1/2\tau$ does not exceed the critical value $1/2\tau_c = 4T_0/\pi$ [7]. (We use the system of units with $\hbar = 1$, $c = 1$ and $k_B = 1$ throughout the Letter.)

The anisotropic magnetoresistance discovered near half-fillings of Landau levels at low temperatures was attributed to the existence of the UCDW state [8]. This stimulates an extensive study of the properties of two-dimensional electrons in a weak magnetic field [9]. However up to date, the magnetoresistance of the UCDW state have been theoretically considered in the zero temperature limit only where stripes have well-defined edges [9].

The main objective of the present Letter is to present the results for the conductivity tensor of the UCDW state developed on the half-filled high Landau level in the presence of the quenched disorder just below the transition temperature T_c where the expansion in the CDW order parameter Δ is justified.

2. *UCDW state.* The two-dimensional electrons in a weak perpendicular magnetic field H occupy large number ($N \gg 1$) of the Landau levels. We assume that disorder is *weak* so it leads to the Landau level broadening $1/2\tau$ that satisfies the condition $1/2\tau \ll \omega_H$, where $\omega_H = eH/m$ is the cyclotron frequency with e and m being the electron charge and the effective electron mass respectively. Simultaneously, we imply that the number of impurities exceeds the number of states on Landau level, i.e impurities lift the degeneracy of the Landau level completely [11]. It means that the Landau level broadening $1/2\tau$ should be larger than $\omega_H|a|/z_0$, where a stands for the impurity scattering length and $z_0(\gg |a|)$ is the decay length of the electron wave function in direction perpendicular to the two-dimensional plane. Thereby, we assume that the Landau

[1)]e-mail: burmi@itp.ac.ru

level broadening $1/2\tau$ satisfies the condition $\omega_H|a|/z_0 \ll 1/2\tau \ll \omega_H$. As the temperature decreases transition from the homogeneous state to the UCDW state occurs. Vector $\mathbf{Q}$ that characterizes a period of the UCDW can be oriented along spontaneously chosen direction. Usually, the orientation is fixed either by the intrinsic anisotropy of the crystal or by the small external in-plane magnetic field [9]. Hereinafter, we assume that the vector $\mathbf{Q}$ is directed under angle ϕ with respect to the x axis. The period of the UCDW is seemed to be of the order of the cyclotron radius $R_c = l_H\sqrt{2N+1}$, where $l_H = 1/\sqrt{m\omega_H}$ denotes the magnetic length. More precisely, the modulus of the vector $\mathbf{Q}$ equals $Q = r_0/R_c$, where $r_0 \approx 2.4$ is the first zero of the zeroth order Bessel function of the first kind $\mathcal{J}_0(z)$ [5, 7]. In the mean-field approximation the temperature T_c of the second-order transition from the homogeneous to UCDW state is determined as the solution of the following equation [7],

$$\frac{T_c}{T_0} = \frac{2}{\pi^2}\zeta\left(2, \frac{1}{2} + \frac{1}{4\pi T_c\tau}\right), \tag{1}$$

where $\zeta(2, z)$ is the generalized Riemann zeta function and T_0 the transition temperature in the *clean* case $(1/\tau \ll T_0)$. We notice that Eq.(1) has the solution for T_c only if the Landau level broadening is smaller than the critical one $1/2\tau \le 1/2\tau_c = 4T_0/\pi$. According to Refs. [5, 6],

$$T_0 = \frac{r_s\omega_H}{4\pi\sqrt{2}}\left[\ln\left(1+\frac{c}{r_s}\right) - \frac{c}{\sqrt{2}+r_s}\right], \quad \frac{1}{N} \ll r_s \ll 1, \tag{2}$$

where $c = 1/(\sqrt{2}r_0) \approx 0.3$, and $r_s = \sqrt{2}e^2/\varepsilon v_F \ll 1$ with v_F and ε being the Fermi velocity and the dielectric constant of a media respectively. It is worth mentioning that the T_0 is determined by the characteristic energy $e^2/R_c \sim r_s\omega_H \ll \omega_H$ of the screened electron-electron interaction on the N-th Landau level.

If we take into account the fluctuations of the UCDW order parameter the transition becomes of the first order at lower temperature $T_c - \delta T$ where $\delta T/T_c \propto h_1(\eta_c)N^{-2/3}$ [7]. Here for convenience we introduce the dimensionless parameter $\eta_c = 1/4\pi T_c\tau$ that we use throughout the Letter. The function $h_1(\eta_c)$ decreases monotonically from the value 0.46 at $\eta_c = 0$ to zero at $\eta_c \to \infty$. As it is known the fluctuations of the UCDW order parameter produce the power divergences and therefore destroy it in the infinite system. In the finite system of size L the UCDW state survives provided that the condition $L/R_c \ll h_2(\eta_c)N^{1/3}$ is hold. Here the function $h_2(\eta_c)$ monotonically increases from the value 0.55 at $\eta_c = 0$ to infinity at $\eta_c \to \infty$. Therefore, in the considered case of the weak magnetic field $(N \gg 1)$ the effect of the fluctuations on the transition is negligible and the mean-field picture is well justified.

3. *Results.* The conductivity tensor σ_{ab} of the two-dimensional electrons on the half-filled high Landau level above the transition temperature T_c, i.e. in the homogeneous state, is known to be isotropic [1]. To this end we show that in the presence of the *quenched weak* disorder the existence of the UCDW state on the half-filled high Landau level below T_c results in the *anisotropic* corrections to the isotropic conductivity tensor σ_{ab}. For the temperature slightly below T_c, where the condition $T_c - T \ll T_c$ is hold, the anisotropic corrections are given as

$$\left.\begin{array}{c}\delta\sigma_{xx}^{(\text{anis})}\\ \delta\sigma_{yy}^{(\text{anis})}\end{array}\right\} = \mp N f(\eta_c)\,\mathcal{G}(\eta_c)\cos[2\phi]\frac{T_c - T}{T_c}, \tag{3}$$

and

$$\left.\begin{array}{c}\delta\sigma_{xy}^{(\text{anis})}\\ \delta\sigma_{yx}^{(\text{anis})}\end{array}\right\} = N f(\eta_c)\,\mathcal{G}(\eta_c)\sin[2\phi]\frac{T_c - T}{T_c}. \tag{4}$$

The functions $f(z)$ and $g(z)$ are defined as

$$f(z) = \frac{32\mathcal{J}_1^2(r_0)z^3}{\left(\frac{1}{2}+z\right)^5}, \; g(z) = \frac{16\pi z}{\left(\frac{1}{2}+z\right)\left(z^2 + \left(\frac{1}{2}+z\right)^2\right)}, \tag{5}$$

where $g(z)$ will be used below. The other function $\mathcal{G}(z)$ is given as

$$\mathcal{G}(z) = \frac{\zeta(2, \frac{1}{2}+z) - z\zeta(3, \frac{1}{2}+z)}{-3\zeta(4, \frac{1}{2}+z) + 4\Phi_0(z) + 2\Phi_2(z)}, \tag{6}$$

with

$$\Phi_n(z) = \frac{\zeta(2, \frac{1}{2}+z)}{z^2\mathcal{J}_0^2(nr_0)} - \frac{\operatorname{Im}\psi(\frac{1}{2}+z+i\,z\mathcal{J}_0(nr_0))}{z^3\mathcal{J}_0^3(nr_0)}. \tag{7}$$

The $\psi(z)$ stands for the digamma function and symbol Im denotes the imaginary part.

There are several features of the main results (3) and (4). First of all, the anisotropic corrections $\delta\sigma_{ab}^{(\text{anis})}$ are proportional to $(T_c - T)/T_c$. Although, the Eqs.(3) and (4) are derived only for the case of a short-range random potential (quenched disorder), it can be shown that the anisotropic corrections remain proportional to $(T_c - T)/T_c$ in the case of a long-range random potential that corresponds to high mobility samples used in experiments [8] as well. We emphasize that such temperature dependence of the developing anisotropy in magnetoresistance was observed in the experiments [8].

The angle dependence of the anisotropic correction (3) to conductivity σ_{xx} has the minimum for $\phi = 0$, that corresponds to the vector $\mathbf{Q}$ directed along the x axis and stripe oriented along the y axis. From Eq.(3) we see that the conductivity σ_{yy} along the stripe is enhanced whereas the conductivity σ_{xx} across the stripes (along the modulation of the order parameter) is suppressed as it should be according to the experiments [8]. At the same time, the anisotropic correction (4) to σ_{xy} vanishes. If the vector $\mathbf{Q}$ is oriented at angle $\phi = \pi/4$ with respect to the x axis, the anisotropic correction (3) to σ_{xx} becomes zero due to the symmetry between x and y axes. Conversely, the anisotropic correction (4) to σ_{xy} is attained the minimum.

The behavior of the anisotropic corrections (3) and (4) as the functions of the parameter η_c for fixed temperature T and angle ϕ are shown in Fig.1.

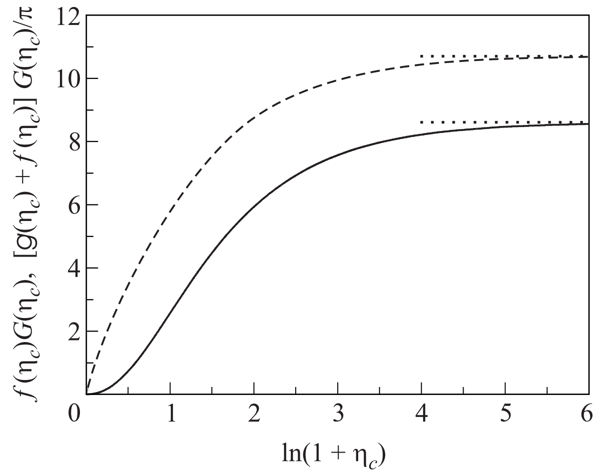

Fig.1. The $f(\eta_c)\mathcal{G}(\eta_c)$ and $[g(\eta_c) + f(\eta_c)]\mathcal{G}(\eta_c)/\pi$ as functions of $\ln(1+\eta_c)$

In addition, the existence of the UCDW state on the half-filled high Landau level leads to the isotropic correction that for $T_c - T \ll T_c$ is as follows

$$\delta\sigma_{xx}^{(\mathrm{isot})} = -\frac{N}{\pi}\left[g\left(\eta_c\right) + f\left(\eta_c\right)\right]\mathcal{G}\left(\eta_c\right)\frac{T_c - T}{T_c}. \qquad (8)$$

The behavior of the isotropic correction (8) as the functions of η_c for fixed temperature T is shown in Fig.1.

4. *Model.* The grand canonical partition function of the two-dimensional interacting electrons in the random potential $V(\boldsymbol{r})$ subjected to the perpendicular constant magnetic field H and the time-dependent external vector potential $\mathbf{A}$ is given by

$$\mathcal{Z} = \int \mathcal{D}[\Psi,\Psi^{\dagger}]\mathcal{D}[V]\,\mathcal{P}[V]\,\exp\mathcal{S}_0[\Psi,\Psi^{\dagger},V], \qquad (9)$$

where the action $\mathcal{S}_0[\Psi,\Psi^{\dagger},V]$ in the Matsubara representation has the form

$$\mathcal{S}_0 = \int\limits_r \Psi^{\dagger}(\mathbf{r})\Big[i\omega + \mu - \mathcal{H}_0 + \hat{K} - V(\mathbf{r})\Big]\Psi(\mathbf{r}) - $$
$$- \frac{T}{2}\int\limits_{rr'}\sum_{\alpha\nu_n}\Psi^{\dagger}(\mathbf{r})I_n^{\alpha}\Psi(\mathbf{r})\frac{e^2}{\varepsilon|\mathbf{r}-\mathbf{r}'|}\Psi^{\dagger}(\mathbf{r}')I_{-n}^{\alpha}\Psi(\mathbf{r}'). \quad (10)$$

Here we use the matrix notation $\Psi^{\dagger}(\cdots)\Psi = \Psi_{\omega_n}^{\dagger\alpha}(\cdots)_{nm}^{\alpha\beta}\Psi_{\omega_m}^{\beta}$ for the electron annihilation $\Psi_{\omega_n}^{\alpha}(\mathbf{r})$ and creation $\Psi_{\omega_n}^{\dagger\alpha}(\mathbf{r})$ operators. Superscripts $\alpha,\beta = 1,\cdots,2N_r$ stands for replica indices combined with spin ones. We introduce the replica indices in order to average over the random potential $V(\mathbf{r})$. The subscripts ω_n, ω_m denote the Matsubara fermionic frequencies $\omega_n = \pi T(2n+1)$. The one-particle hamiltonian $\mathcal{H}_0$ for a two-dimensional electron in the presence of the magnetic field $H = \epsilon_{ab}\partial_a A_b^{\mathrm{st}}$ is defined as $\mathcal{H}_0 = -\mathbf{D}^2/(2m)$ with the covariant derivative $\mathbf{D} = \nabla - ie\,\mathbf{A}^{\mathrm{st}}$. The matrix ω has only diagonal elements $\omega_{nm}^{\alpha\beta} = \omega_n\delta^{\alpha\beta}\delta_{nm}$ that represent the Matsubara frequencies ω_n. The matrix I_n^{α} with elements $(I_n^{\alpha})_{lm}^{\beta\gamma} = \delta^{\beta\alpha}\delta^{\gamma\alpha}\delta_{m-l,n}$ is a generator of the $U(1)$ gauge transformation. The time-dependent external vector potential $\mathbf{A}$ is involved through the matrix $\hat{K} = \sum_{\alpha,n}K(\nu_n)I_n^{\alpha}$ where

$$K(\nu_n) = -\frac{e}{m}\mathbf{A}(\nu_n)\mathbf{D} + \frac{e^2}{2m}\sum_{\nu_m}\mathbf{A}(\nu_{n-m})\mathbf{A}(\nu_m). \qquad (11)$$

Here $\mathbf{A}(\nu_n)$ is the Fourier component of the external vector potential $\mathbf{A}$ with frequency $\nu_n = 2\pi T n$.

We assume the white-noise distribution for the random potential $V(\mathbf{r})$

$$\mathcal{P}[V(\mathbf{r})] = \frac{1}{\sqrt{2\pi g}}\exp\left[-\frac{1}{2g}\int_r V^2(\mathbf{r})\right]. \qquad (12)$$

5. *Method.* To proceed we integrate over the random potential $V(\mathbf{r})$ in Eq.(9). As usual, it leads to the quartic interaction that we decouple by introducing the matrix field $Q(\mathbf{r})$ [12]. The annihilation $\Psi(\mathbf{r})$ and creation $\Psi^{\dagger}(\mathbf{r})$ operators written in the basis of the eigenfunctions $\phi_{pk}(\mathbf{r})$ of the hamiltonian $\mathcal{H}_0$

$$\Psi(\mathbf{r}) = \sum_{p,k}\psi_{pk}\phi_{pk}(\mathbf{r}),\ \ \Psi^{\dagger}(\mathbf{r}) = \sum_{p,k}\psi_{pk}^{\dagger}\phi_{pk}^{*}(\mathbf{r}) \qquad (13)$$

involve the electron states on all Landau levels. Therefore, the term with the electron-electron interaction in the action (10) contains the interactions of electrons from different Landau levels. In general, to treat the problem (10) analytically seems to be impossible. However, as it was shown in Ref. [4], if the $N-1$ Landau levels are filled whereas the Nth Landau level is partially occupied, one can obtain the description of the system in terms of electrons on the Nth Landau level only provided that the relative strength of the bare Coulomb interaction is *small*, $r_s \ll 1$, and the magnetic field is rather *weak*, $Nr_s \gg 1$.

Following the same strategy as in Ref. [10], we obtain the grand canonical partition function $\mathcal{Z}$ as

$$\mathcal{Z} = \int \mathcal{D}[\Psi, \Psi^\dagger]\mathcal{D}[Q] \exp \mathcal{S}[\Psi, \Psi^\dagger, Q], \qquad (14)$$

where

$$\mathcal{S} = \int_r \Psi^\dagger(\mathbf{r})\left[i\omega + \mu - \mathcal{H}_0 + \hat{K} + iQ\right]\Psi(\mathbf{r}) - \frac{1}{2g}\mathrm{Tr}Q^2 - \\ - \frac{T}{2}\int_{rr'}\sum_{\alpha\nu_n}\psi^\dagger(\mathbf{r})I_n^\alpha\psi(\mathbf{r})U_{\mathrm{scr}}(\mathbf{r},\mathbf{r}')\psi^\dagger(\mathbf{r}')I_{-n}^\alpha\psi(\mathbf{r}'). \qquad (15)$$

Here symbol Tr denotes the trace over the Matsubara, replica combined with spin and spatial indices. The electron-electron interaction is written in terms of electron operator $\psi(\mathbf{r}) = \sum_k \psi_{Nk}\phi_{Nk}(\mathbf{r})$ on the Nth Landau level only. The screened interaction $U_{\mathrm{scr}}(\mathbf{r})$ of electrons on the Nth Landau level takes into account the effects of electrons on the other levels and has the following form [4, 10]

$$U_{\mathrm{scr}}(q) = \frac{2\pi e^2}{\varepsilon q}\frac{1}{1 + \dfrac{2\left(1 - \frac{\pi}{6\omega_H\tau}\right)}{qa_B}\left(1 - \mathcal{J}_0^2(qR_c)\right)}. \qquad (16)$$

It is worth mentioning that the range of the screened electron-electron interaction (16) is determined by the Bohr radius $a_B = \varepsilon/me^2$. We assume that the magnetic field is so weak that the condition $Nr_s^2 \gg 1$ is hold. That means that the range a_B of the screened electron-electron interaction (16) is much less than the magnetic length l_H. It allows us to treat the interaction in the Hartree-Fock approximation [6].

In the absence of the external vector potential $\mathbf{A}$ one can project the first line in Eq.(15) onto the Nth Landau level, i.e. substitute $\Psi(\mathbf{r}) \to \psi(\mathbf{r})$. Then the action becomes to involve electrons on the Nth Landau level only and, evidently, it simplifies the analysis. The accuracy of such projection is of the order of $\max\{T, \tau^{-1}\}/\omega_H \ll 1$. It is worthwhile to mention that the correction $\pi/6\omega_H\tau \ll 1$ in the screened interaction (16) results in the correction of the same order to the T_0. For reasons to be explained shortly we neglect this effect. However, in order to investigate the response of the system to the external vector potential $\mathbf{A}$ such projection onto the Nth Landau level is not appropriate. We should leave the action (15) as it stands because the matrix elements $D^a_{p_1p_2} = \int_r \phi^*_{p_1k}(\mathbf{r})D^a\phi_{p_2k}(\mathbf{r})$ of the covariant derivative $\mathbf{D} = (D^x, D^y)$ involve the electron states on the adjacent Landau levels. As the last step we mention that the electrons on the Nth Landau level should be regarded as spin-polarized according to the numerical findings [14].

The action (15) involves the unitary matrix field $Q(\mathbf{r})$. There exists the saddle-point solution $Q(\mathbf{r}) = W^{-1}U_{\mathrm{sp}}W$ in the absence of the electron-electron interaction. Here the constant unitary matrix W describes the global rotation, whereas $(U_{\mathrm{sp}})^{\alpha\beta}_{nm} = \delta^{\alpha\beta}\delta_{nm}\mathrm{sgn}\omega_n/2\tau$ with $1/2\tau = \sqrt{g/2\pi l_H^2}$. Being motivated by the form of the saddle-point solution we split the matrix field $Q(\mathbf{r})$ in transverse $W(\mathbf{r})$ and longitudinal $U(\mathbf{r})$ components as $Q(\mathbf{r}) = W^{-1}(\mathbf{r})(U_{\mathrm{sp}} + U(\mathbf{r}))W(\mathbf{r})$. As it is well-known the transverse field $W(\mathbf{r})$ is responsible for weak localization corrections [12] but in the case of interest they are of the order of $\ln N/N \ll 1$. Therefore we eliminate the transverse field from the future considerations by formally putting $W(\mathbf{r}) = 1$. The transformation of the variable $Q(\mathbf{r})$ discussed above leads to the additional measure in the functional integral [13]

$$\ln I[U] \sim \int_r \sum_{\omega_n\omega_m}^{\alpha\beta}\left[1 - \Theta(nm)\right]U^{\alpha\alpha}_{nn}(\mathbf{r})U^{\beta\beta}_{mm}(\mathbf{r}), \qquad (17)$$

where $\Theta(x)$ stands for the Heaviside step function.

To describe the UCDW state we introduce the CDW order parameter Δ that is related with a distortion of the electron density on the Nth Landau level

$$\langle\delta\rho(\mathbf{q})\rangle = 2\pi l_H^{-2}F_{NN}(\mathbf{q})[\delta(\mathbf{q}-\mathbf{Q}) + \delta(\mathbf{q}+\mathbf{Q})]\Delta, \qquad (18)$$

where the $F_{p_1p_2}(\mathbf{q})$ is defined as

$$F_{p_1p_2}(\mathbf{q}) = 2\pi l_H^2\sum_k \phi^*_{p_1k}(0)\phi_{p_2k}(\mathbf{q}l_H^2)e^{i\frac{1}{2}q_xq_yl_H^2}. \qquad (19)$$

In particularly, the form-factor $F_{NN}(q) \approx \mathcal{J}_0(qR_c)$ for $qR_c \ll 2N$. The presence of the distortion of the electron density by the charge-density wave on the Nth Landau level results in the additional periodic potential $\lambda(\mathbf{r})$ that is related with the UCDW order parameter as

$$\lambda(\mathbf{q}) = (4\pi)^2T_0F^{-1}_{NN}(q)\left[\delta(\mathbf{q}-\mathbf{Q}) + \delta(\mathbf{q}+\mathbf{Q})\right]\Delta. \qquad (20)$$

After the Hartree-Fock decoupling [15] of the interaction term in the action (15) and integration over electrons, we obtain

$$\mathcal{Z} \sim \int \mathcal{D}[U] I[U] \exp \mathcal{S}[U], \tag{21}$$

where the action becomes

$$\mathcal{S} = -\frac{1}{2g}\mathrm{Tr}U^2 + \mathrm{Tr}\ln\left(1 + (iU + \hat{K} + P_N \lambda P_N)G\right). \tag{22}$$

The projection operator $P_N(\mathbf{r},\mathbf{r}') = \sum_k \phi_{Nk}(\mathbf{r})\phi^*_{Nk}(\mathbf{r}')$ in the action (22) indicates that the potential $\lambda(\mathbf{r})$ exists on the Nth Landau level only. In general, the UCDW state on the Nth Landau level creates the potential not only on the same level but also on the adjacent ones. We neglect this effect since it results in the corrections to the conductivity tensor σ_{ab} of the order of $1/N^2$ beside expressions (3),(4), and (8).

The saddle-point Green function $G^{\alpha\beta}_{nm}(\boldsymbol{r},\boldsymbol{r}')$ is determined as

$$G^{\alpha\beta}_{nm}(\mathbf{r},\mathbf{r}') = \delta^{\alpha\beta}\delta_{nm}\sum_{pk}\phi^*_{pk}(\mathbf{r})\phi_{pk}(\mathbf{r}')G^{-1}_p(\omega_n),$$
$$G^{-1}_p(\omega_n) = i\omega_n + \omega_H(p-N) + i\frac{\mathrm{sgn}\omega_n}{2\tau}, \tag{23}$$

We notice that the Green function (23) coincides with the Green function averaged over disorder in the self-consistent Born approximation [1].

6. *Conductivity tensor.* With the action (22) in hands we can evaluate the contributions to the conductivity tensor σ_{ab} due to the presence of the UCDW state on the half-filled Landau level. As one can verify the contributions of the first order in the UCDW induced potential $\lambda(\mathbf{r})$ vanish. In order to find the contributions to σ_{ab} of the second order in λ we should expand the action (22) upto the second order both in λ and K. Then, integrating over the U fields, we obtain several contributions. We present the diagrams that correspond to them in the standard perturbative technique in Fig.2.

The first three diagrams (Fig.2a) yield only the isotropic contribution

$$\sigma^{(a)}_{ab}(\nu_n) =$$
$$= -T\sum_{\omega_n}\frac{G^3_N(\omega_n)}{1+g\pi^{\omega_n}(0,0)}\frac{T_0^2\Delta^2}{(1+g\pi^{\omega_n}(0,Q))^2}\times$$
$$\times\frac{4\pi\omega_H}{\nu_n}\left(\frac{2}{m}\sum_p D^a_{Np}D^b_{pN}G_p(\omega_n+\nu_n) - \delta_{ab}\right). \tag{24}$$

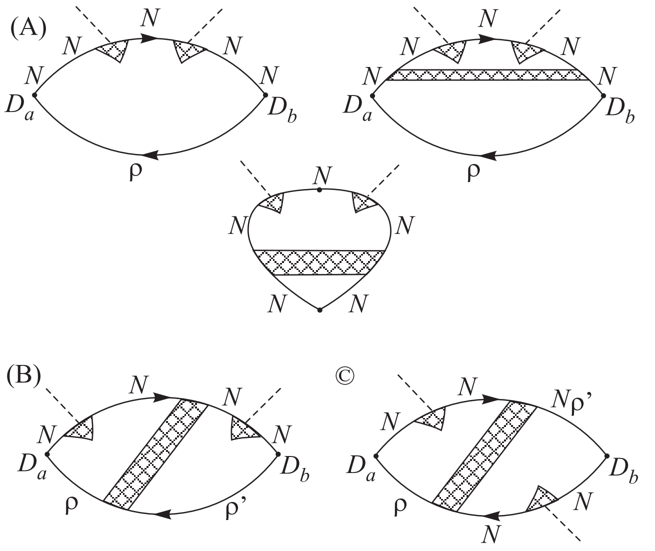

Fig.2. Diagrams for the corrections to σ_{ab}. The solid lines are the Green functions, the $N/p/p'$ symbols denote the Landau level, the dashed lines are the UCDW induced potential $\lambda(\boldsymbol{r})$ and the shaded blocks are impurity ladders

Here the polarization operator $\pi^{\omega_n}(\nu_n, q)$ on the Nth Landau level is defined as

$$\pi^{\omega_n}(\nu_n,q) = -n_L G_N(\omega_n+\nu_n)G_N(\omega_n)F^2_{NN}(q). \tag{25}$$

The contribution of diagram Fig.2b is seemed to be proportional to $\mathcal{J}_0(r_0)$ and vanishes therefore. The last diagram Fig.2c is as follows

$$\sigma^{(c)}_{ab}(\nu_n) =$$
$$= \frac{8\pi\omega_H}{\nu_n m}T\sum_{\omega_n}\frac{G^2_N(\omega_n)G^2_N(\omega_n+\nu_n)T_0^2\Delta^2}{(1+g\pi^{\omega_n}(0,Q))(1+g\pi^{\omega_n}(\nu_n,Q))}$$
$$\times\sum_{pp'}G_p(\omega_n)G_{p'}(\omega_n+\nu_n)\frac{D^a_{pN}D^b_{p'N}I_{NpNp'}(\mathbf{Q})}{1+g\pi^{\omega_n}(0,Q)}, \tag{26}$$

where a single impurity line is written in the Landau level indices representation as

$$I_{p_1p_2p_3p_4}(\mathbf{Q}) = g\int_q F_{p_1p_2}(\mathbf{q})F_{p_3p_4}(-\mathbf{q})e^{-i\mathbf{q}\mathbf{Q}l_H^2}. \tag{27}$$

The contribution (26) contains the anisotropic as well as isotropic corrections to σ_{ab}. If we take $p = p' = N\pm1$ we obtain the anisotropic contribution due to the structure of matrix elements $D^a_{p_1p_2}$. The opposite case $p = N\pm1$ and $p' = N\pm1$ results in the isotropic correction.

Now with a help of the results $I_{N,N\pm1,N,N\pm1} \approx ge^{\mp 2i\phi}\mathcal{J}_1^2(r_0)$ and $I_{N,N\pm1,N,N\mp1} \approx g\mathcal{J}_1^2(r_0)$ we perform the summation over the Landau level indices as well as over the Matsubara frequency. As the last step, we express the UCDW order parameter Δ via the temperature difference $T_c - T \ll T_c$ as [7]

$$\Delta = \zeta\left(2, \frac{1}{2} + \eta_c\right)\sqrt{\mathcal{G}(\eta_c)}\sqrt{\frac{T_c - T}{T_c}} \qquad (28)$$

and obtain the results (3), (4) and (8).

7. *Fluctuations of the order parameter.* There is a legitimate question about the effect of the fluctuations on the conductivity tensor above and below the transition temperature T_c. Below we consider the former case as more interesting. At $T > T_c$ the mean-field order parameter $\Delta = 0$ in average, but the average of its square $\langle\Delta^2\rangle$ is non-zero. It leads to the appearance of the corrections to the conductivity tensor σ_{ab} above T_c due to the presence of the UCDW correlations. We can find the contributions of the order parameter fluctuations to σ_{ab} by substituting $\langle\Delta^2\rangle$ for Δ^2 in Eqs.(24) and (26).

Generally, the angle ϕ and modulus Q of the CDW vector $\mathbf{Q}$ can fluctuate simultaneously. Naturally, only the isotropic correction can appear in this case. Then, the result for the correction one can obtain from Eq.(8) with a help of the following substitutions

$$\frac{T_c - T}{T_c} \to \frac{r_0}{4\pi N}\sqrt{\frac{T_c}{T - T_c}}, \qquad (29)$$

$$\mathcal{G}(z) \to \sqrt{\frac{\zeta^{-1}\left(2, \frac{1}{2} + z\right)}{\gamma\zeta\left(2, \frac{1}{2} + z\right) + \mathcal{J}_1^2(r_0)z^2\zeta\left(4, \frac{1}{2} + z\right)}},$$

where $\gamma = \partial \ln T_0/\partial r_0 \approx 2.58$. It is worthwhile to mention that these fluctuational contribution (29) to σ_{ab} above T_c is analogous to the correction for conductivity of a normal metal due to superconducting pairing [16]. The fluctuation correction (29) has square-root divergence at $T \to T_c$. This fact indicates that the result (29) is not applicable in the vicinity of the transition temperature T_c. The limit of applicability is determined by the requirement that the fluctuational correction should be much smaller than σ_{ab} it self.

8. *Conclusion.* Summarizing, we calculated the anisotropic as well as isotropic corrections to the conductivity tensor of the two dimensional electrons on the half-filled high Landau level just below the transition to the UCDW state. The corrections obtained are proportional to $(T_c - T)/T_c$ that is in agreement with that found in the experiments. Also we calculated the fluctuational correction to the conductivity tensor of the two-dimensional electrons above the transition.

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